Indian Institute of Management Calcutta
Working Paper Series
WPS No 852 / September 2020

The Consequences of Deteriorating Public-provision Regimes

Arijit Sen*
Professor, Economics Group
IIM Calcutta, Joka, Kolkata 700104, India
Email: arijitsen@iimcal.ac.in

*Corresponding Author

Indian Institute of Management Calcutta, Joka, D.H. Road, Kolkata 700104

URL: http://facultylive.iimcal.ac.in/workingpaper

The Consequences of Deteriorating Public-provision Regimes

Arijit Sen[1]

Abstract

We consider a model of 'public provision of private goods supplemented by private topping-up' *a la* Epple and Romano (1986a), in which majority-voting determines the government's capacity of public provision, and unsatisfied private demand determines the equilibrium level of 'consumption topping-up' *via* purchase of market-supplied substitute goods. In such a model of public-private provision, we study the impact of changes in production efficiencies in the public and the private sectors on the volume of public provision, on the extent of private topping-up, and on changes in the welfare of citizens in different income classes. We focus on the consequences of a deteriorating public-provision regime on overall equilibrium outcomes, and identify the following effects: (*a*) such deterioration raises 'utility inequality' as richer citizens engage in increased topping-up; (*b*) incremental changes in public sector efficiency can generate large discontinuous changes in demand for market-supplied substitute goods and in individual welfare; and (*c*) any improvement in production technologies of such substitute goods can discretely lower the welfare of poor citizens.

Keywords: public provision, private supplement, majority voting

JEL Classification: H4

[1] Professor , Economic Group , Indian Institute of Management Calcutta, Email – arijitsen@iimcal.ac.in

Date of current draft: 10 July 2020.

Acknowledgements: I thank Bhaskar Dutta and Kai Konrad, and seminar participants at Ashoka University and at the Max Planck Institute of Tax Law and Public Finance, for comments. I also thank Bharat Goel for excellent research assistance and IIM Calcutta for research support.

The Consequences of Deteriorating Public-provision Regimes

1. Introduction

In almost all economies in the world, there are many goods and services that are 'rival in consumption' – like potable water, electricity, public transportation, and healthcare – that are publicly provided by the government / public sector. At the same time, many close substitutes of these goods are market-supplied (by private sellers) in these economies, and certain sections of affluent consumers 'top up / supplement' their consumption of the publicly-provided goods by purchasing such market-substitutes – bottled water, privately-produced electricity, private transport modes, privately-supplied healthcare, and the like.

Epple and Romano (1996a) and Gouveia (1997) are two early papers that carried out formal analyses of this scenario of 'public provision supplemented by private topping-up' in related models, where majority-voting determined government capacity for public provision, and then unsatisfied private demand determined the equilibrium level of 'consumption topping-up' *via* purchase of market-supplied substitute good(s).[2]

In studying such a model of public-private provision, the current paper focuses on the impact of intertemporal changes in production efficiencies in the public and the private sectors on (*i*) the volume of majority-determined public provision, (*ii*) the extent of 'private topping-up / supplementation', and (*iii*) the changes in individual welfare of citizens in different income classes.

A detailed study of such comparative-statics issues in a fully-specified model of 'public provision supplemented by private topping-up' is important precisely because of deep concerns about the welfare implications of intertemporally deteriorating public provision regimes. Such concerns have been expressed over the last couple of decades by many social scientists – in the context of developed as well as less-developed countries of the world.[3]

Furthermore, many developing countries have, in the recent past, experienced phenomenal growth in sales of various kinds of private supplements to a large set of publicly-provided goods. For instance, in India, while the average annual growth of household final consumption has been about 8.5% over the last five years, the average annual growth rate of sales in the water-purifier industry has been about 17%, and that in the power-inverter industry has been

[2] Related research was also begun at that time regarding a class of 'opting out' models. Initiated by Epple and Romano (1996b) and Glomm and Ravikumar (1998), these models studied the scenario where a set of rich citizens in an economy can opt out of a publicly-provided system – e.g., a system of public education or public housing – and join a fully private system – private schooling or gated communities. Lülfesmann and Myers (2011) is a more recent contribution to this strand of research.

[3] In 1991, Robert Reich wrote an article in the New York Times Magazine, titled "Secession of the Successful"; and in 2012, Mike Lofgren wrote "Have the Super-Rich Seceded from the United States?" Regarding India, see, for instance, a 2008 article in the Outlook Magazine by Anjali Puri, where the author writes about how the urban rich in India are becoming "free from India" by building gated communities.

about 15%. Our analysis aims to explain such growth spurts in the sales of private supplements that seem to accompany intertemporal deterioration in public provision regimes.

In our theoretical analysis, we aim to study how a deteriorating public provision regime creates the space for private supplements industries to grow, and how the burgeoning private supply dampens the public demand for improvements in the public provision system. We study a model where citizens consume a publicly-provided rival good and a closely-related private supplement, where the resources for the production of the former good is determined by majority-voting by citizens over a linear tax rate, and the latter good is produced and sold in the market under Bertrand competition.

In our study, we focus mainly on answering the following questions: In an economy where the efficiency of the public sector declines over time, how do citizens respond in terms of their public choice over the public provision regime, and their individual demands for private top-ups / supplements? How are citizens in different income classes differentially impacted in terms of individual welfare due to such public sector decline? In an environment where the private supplement good is supplied at cost in equilibrium, we address these questions by studying the effects of a *ceteris paribus* rise in the public production cost, as well as a *ceteris paribus* fall in the private production cost. In doing so, we explore the possibility of incremental relative cost changes leading to large discontinuous changes in economic outcomes and welfare.

We uncover the following effects of a decline in public-system efficiency in an economy:

- Incremental increase in production costs in the public sector vis-à-vis the private sector can cause discontinuously large changes in public provision and private supplementation levels, thus explaining observed sales growth spurts in supplement goods industries in many countries. Specifically, starting from a scenario where few rich citizens consume the private supplement, an incremental fall in the relative efficiency of the public system causes small changes in outcomes: consumption of the publicly-provided good falls and that of the private supplement rises incrementally. But, if such decline continues and the relative efficiency of the public system falls below a threshold value, discrete changes occur: a small rise in public cost or a small fall in private cost causes a discontinuously large reduction in public provision and a discrete increase in private topping-up – due to a drastic change in the citizens' majority-preferred public provision regime. These changes have the following important implications for citizens' welfare.
- A relative decline in the public system raises 'utility inequality' in the following ways. Starting from a situation where few rich citizens consume the private supplement, a rise in public production cost (respectively, a fall in supplement cost) leads to the following welfare changes: for every citizen richer than a threshold income level, maximized utility *falls less* (respectively, *rises more*) than the maximized utility of a citizen poorer than the

threshold income level, thereby raising utility-inequality between the rich and the poor.[4] This happens because the rich – *via* topping up – *lose less* (respectively, *gain more*) from a rise in public cost (respectively, a fall in supplement cost).

- Changes in individual utility and in utility-inequality take dramatic forms when the public cost rises above a specific threshold. In that case, an incremental rise in public cost causes the utilities of all citizens richer than an income threshold to rise discontinuously, and the utilities of all poorer citizens to fall discontinuously, thereby generating a discrete increase in utility-inequality in the population. This result has the following counterpart: a small reduction in the supplement production cost beyond a threshold value can lead to a discrete worsening of the welfare of a large number of citizens located in the bottom tail of the income distribution.[5]

The rest of the paper is organized as follows. Section 2 presents our model of public provision with the possibility of private topping-up. In Section 3, we determine the majority-voting equilibrium outcome as a function of the citizens' common conjecture about the market-price of the supplement good. Then, in Section 4, we determine the overall equilibrium of the public-private system under the assumption that the supplement is priced in the private sector under Bertrand competition. We carry out our central comparative-statics exercises in Section 5, and determine the positive and normative consequences of a deteriorating public provision regime. Finally, some concluding remarks are presented in Section 6.

2. The Model

Consider an economy inhabited by a continuum of citizens of measure one. The citizens have identical preferences, but differ in their incomes x. Across the population, individual income x is uniformly distributed on the support $[x^-, x^+]$, where $x^+ > x^- > 0$. Let $X \in (x^-, x^+)$ denote the aggregate income in the economy.

Individual utility depends on the consumption of three goods: A, B, and C. Good A is the numeraire good – a private good that is traded in the market at a price of unity.

Good B is a rival-in-consumption good that is publicly provided in the following manner: there

[4] A rise in the public unit-cost raises utility-inequality in the following precise sense: The gap in maximized utility between a rich and a poor citizen, relative to the gap that would exist if the supplement good was unavailable, rises in the unit-cost of public production.

[5] Intuitively, this outcome can be understood in terms of the following story: A fall in the unit cost (and therefore the price) of a competitively-supplied water-purifying tablet can encourage a majority of the more affluent citizens to vote for a large reduction in the public provision of potable water (given its high cost of production) precisely because these affluent citizens can now afford the tablets. This regime shift leads to a discrete improvement in individual well-being for all affluent citizens, but causes a discrete fall in the welfare of a large number of poor citizens who miss out on the free public supply of potable water (and are then forced to buy water-purifying tablets).

is majority voting over the total amount of money, M, to be spent by the public sector in producing B. The citizens vote on a linear income tax rate t, and M is determined by the total tax collection $t.X$. The public sector uses $M = t.X$ to produce the private good B (e.g., public water supply, public healthcare, publicly-provided electricity) employing a linear technology: $b = M/g$. The total output b is then distributed freely and equally among all citizens. As a result, each citizen receives b units of good B under the public provision regime.

Each citizen can then top up its B consumption by consuming a perfect substitute good C that is produced and sold in the private sector (bottled water, private healthcare, private electricity generation). Such topping up of B by C is valuable to a citizen precisely when the publicly-provided quantity b of B is considered *insufficient* by her. The homogeneous good C is produced in the private sector under Bertrand competition, given constant unit cost of production h.

If a citizen consumes a units of A, b units of B, and c units of C, then her utility is: $U(a, b, c) = [b + c]^{(k)}[a]^{(1-k)}$, where $0 < k < 1$. The subsequent results that we derive for this Cobb-Douglas utility specification remain valid in an alternative utility specification where the sub-utilities from the numeraire good (A) and the other two goods (B and C, they being perfect substitutes) are *separable*: $U(a, b, c) = u(a) + v(b + c)$.[6]

We aim to study the 'public-private equilibrium' in which the public provision volume is determined by majority-voting by the citizens, and the price of the privately-supplied good is set competitively by the C-sellers. Specifically, we focus on equilibrium outcomes that arise from the following sequence of public and private decisions. Taking as given the income distribution, the efficiency of public provision as captured by the value of g, and the unit cost h of producing C in the private sector (along with the citizen's utility specification), we focus on three stages of decision-making: In the first stage, the citizens vote over the income tax rate t to determine the total tax revenues to be made available for the public provision of B, thus determining the public provision regime $\{b, t\}$. Then, in the second stage, the C-sellers simultaneously set the price p of C under Bertrand competition. Finally, in the third stage, each citizen decides whether, and to what extent, to top up (in consumption) the publicly-provided amount of B by the privately-purchased amount of C.

In the following analysis, we will identify the public-private equilibrium by a two-step analysis. We will first determine the public provision regime $\{b, t\}$ that will be generated by majority voting as a function of the citizens' 'common conjecture' regarding the price p of C (that will be set by the C-seller(s)). This will be our re-doing of the Epple-Romano (1986a) analysis for the Cobb-Douglas utility specification. We will then determine the overall equilibrium by identifying the equilibrium C-price p, and the citizens' equilibrium consumption vectors.

[6] The analysis for this alternative utility specification is available from the author, and follows along the lines indicated in the analysis of Buckley *et. al.* (2015).

Our main objective will be to understand how the public-private equilibrium *varies* with changes in public and private production costs. To that end, we will conduct comparative statics exercises to determine the variation in the equilibrium levels of public provision of B, private supplementation of C, and individual welfare, with changes in the public and private cost parameters g and h.

3. Majority Voting Outcomes

In this section, we study majority voting outcomes regarding public provision of B given the the citizens' common conjecture about the price p^e of the supplement good C. To that end, we begin by determining individual decisions to 'top-up / supplement B with C' given a public provision regime $\{b, t\}$ and an announced price p of C, and then backward induct to determine the $\{b, t\}$ regime that will emerge *via* majority voting.

Once the public provision regime $\{b, t\}$ is established (and given an announced price p of C), the utility maximization problem of 'citizen x' (i.e., a citizen with income x) involves choosing $c \geq 0$ and $a \geq 0$ to maximize $U(a, b, c)$ subject to the budget constraint: $p.c + a \leq (1-t)x$. Given the Cobb-Douglas utility function specified above, it is easy to verify that citizen x will top-up B with C if and only if the established b is strictly smaller than the threshold $b^+(x \mid p, t) \equiv (k/1-k)(1-t)(x/p)$. Further, for any $b \in [0, b^+(.))$, the optimal C-consumption amount for citizen x will be $[k(1-t)(x/p) - (1-k)b]$.

Resultantly, the indirect utility of citizen x given regime $\{b, t\}$ and price p of C will be:

$$V(x \mid p, b, t) = \begin{cases} K.(p)^{(1-k)}\{(1-t)(x/p) + b\} & \text{for } b \in [0, b^+(y \mid p, t)) \text{ where } K \equiv [k^k(1-k)^{1-k}], \\ [b]^{(k)}[(1-t)x]^{(1-k)} & \text{for } b \geq b^+(x \mid p, t). \end{cases}$$

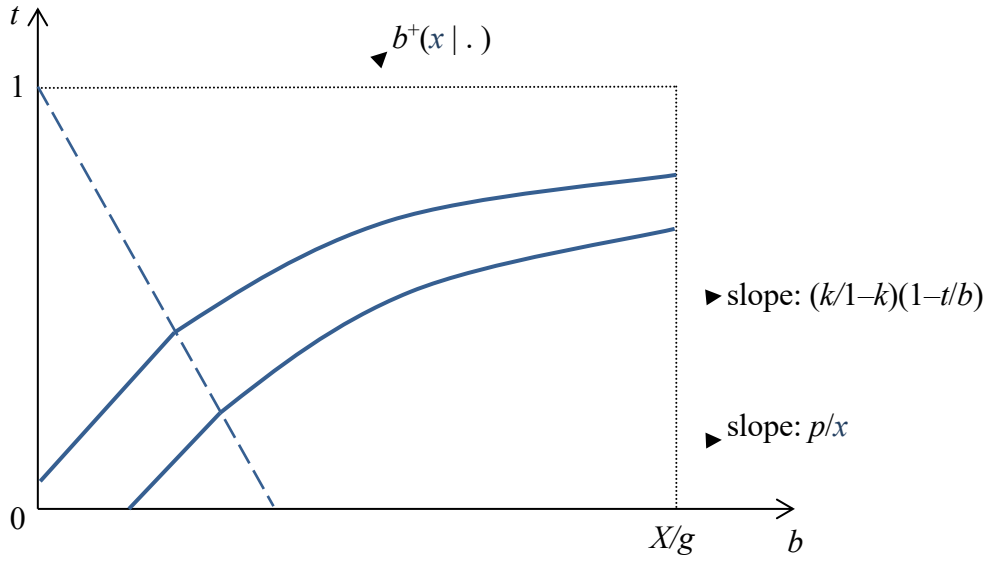

Figure 1. Citizen x's indifference map in the (b, t) space

The indifference map – in the (b, t) space – associated with the indirect utility function $V(x \mid .)$ is presented in Figure 1. Consider any point in the (b, t) space such that $b \in [0, b^{+}(x \mid p, t))$. The indifference curve of citizen x through that point is a straight line with slope p/x; this straight line extends to the point where $b = b^{+}(x \mid p, t)$, and beyond that point, the indifference curve becomes strictly concave with slope $(k/1-k)(1-t)/x$. As a result, in the $(b \geq 0, t \geq 0)$ space, every indifference curve of citizen x, for all x, is increasing, concave, and differentiable, and has a slope that is no greater than p/x.

Note that any positive $\{b, t\}$ regime will be technologically feasible if and only if it satisfies the 'B- production constraint': $b = (t.X)/g$. Substituting this constraint into citizen x's indirect utility function $V(x \mid .)$ gives her induced preferences over b: $W(b \mid x, p) \equiv V(x \mid p, b, t = g.b/X)$. Concavity of citizens' indifference curves in the $(b \geq 0, t \geq 0)$ space, and convexity of the B-production constraint, imply that the induced preferences $W(b \mid x, p)$ of each citizen x over b will be single-peaked. Therefore, when the provision regime $\{b, t\}$ is chosen under majority rule (given a conjectured p^{e}), a voting equilibrium will exist, and the point most preferred by the voter with the median most-preferred level of public provision will be selected.

Given the common conjecture p^{e}, recognize the following facts: For citizen x, if $x/p^{e} < X/g$, then among all feasible points on the B-production constraint, her optimal choice will be the point where the strictly concave part of one of her indifference curves is tangent to the B-production constraint. Note that this will be the point $\{b^{*} = k.X/g, t^{*} = k\}$. Alternatively, if $x/p^{e} \geq X/g$ for citizen x, then among all feasible regimes $\{b, t\}$, her optimal choice will be at the 'corner' where $\{b = 0, t = 0\}$. These choices are depicted in Figure 2.

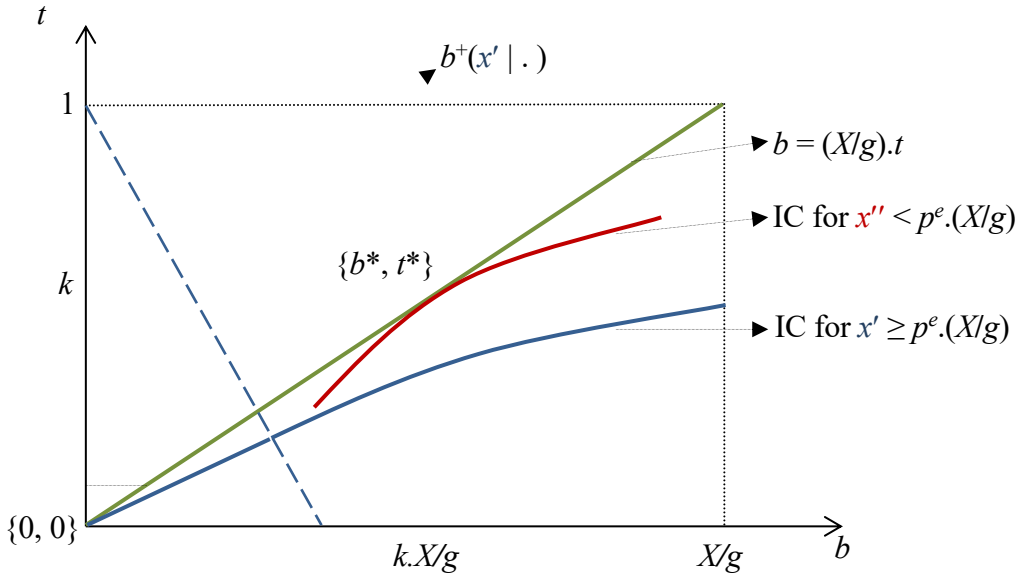

Figure 2. Citizens' optimal $\{b, t\}$ choices

The above facts lead to the following result regarding majority voting: Given a C-price conjecture p^{e}, let $\mu(p^{e}, g)$ be the measure of citizens with income $x < p^{e}.X/g$. If $\mu(p^{e}, g) > 0.5$,

the public provision regime $\{b^* = k.X/g,\ t^* = k\}$ will be the majority voting outcome. Alternatively, if $\mu(p^e, g) \leq 0.5$, the regime $\{b = 0, t = 0\}$ will be the majority voting outcome.[7]

Thus, we have the following result that identifies the majority voting outcome as a function of the conjectured price p^e of C, and the citizens' resultant topping-up choices.

Proposition 1. **[i]** If the C-price conjecture p^e is no bigger than the public cost parameter g, then majority voting will lead to the public provision regime $\{b = 0,\ t = 0\}$; and given that, if the C-sellers set the price of C at p, then C-consumption of citizen x will be $[k.(x/p)]$.

[ii] Alternatively, if the C-price conjecture p^e is strictly greater than g, then majority voting will lead to the public provision regime $\{b^* = k.(X/g),\ t^* = k\}$; and given that, if the C-sellers set the price of C at p, then C-consumption of citizen x will be $max\{0, k(1-k)[(x/p)-(X/g)]\}$.

We will refer to the $\{b^* = k.(X/g),\ t^* = k\}$ regime as one of 'adequate public provision', and the $\{b = 0,\ t = 0\}$ regime as one of 'null provision'. Proposition 1 establishes the discontinuity in the majority-voting outcome with respect to the (conjectured) C-price relative to the public cost parameter g. When this relative price is above a threshold, a majority of the citizens – all with incomes *below* the cut-off $[p^e.X/g]$ – will vote for adequate public provision. In contrast, if the relative price falls below the threshold, a majority of the citizens – all with incomes *above* $[p^e.X/g]$ – will switch to voting for the null-provision regime.

Having determined the majority voting outcome as a function of the conjectured C-price p^e, we now proceed to characterizing the overall equilibria in the economy, when the C-price is determined under Bertrand competition. As we will see, the discontinuity in the citizens' voting behavior described in Proposition 1 will have important implications for the impact of changes in the public and private cost parameters (g and h) on overall equilibrium outcomes and on citizens' welfare.

4. Overall Equilibrium Outcome

We consider the C industry to be a homogeneous-product Bertrand oligopoly (with free entry). In such an industry, the equilibrium price p of C will equal its unit production cost h.[8] Recognize that it does not matter whether Bertrand price-competition among C-sellers occur before, after, or at the same time that the citizens vote over public provision. In all cases, rational citizens will hold the 'correct conjecture' that p will equal h. This fact enables us to use

[7] When $\mu(p^e, g) = 0.5$, the richer half of the population will vote for $\{0, 0\}$ and the poorer half of the population will vote for $\{b^* = k.X/g,\ t^* = k\}$. We assume that in case of such a tie, the choice of the richer half of the population will prevail.

[8] Extension of our current analysis by considering a more complex C-industry, involving unit-cost heterogeneity, product heterogeneity, and/or positive industry entry costs, will be worthwhile future projects.

Proposition 1, with $p^e = h$, in a straight-forward manner to characterize the public-private equilibrium outcome. That, in turn, will facilitate our carrying out comparative statics exercises to determine how the overall equilibrium outcome and citizens' welfare will respond to changes in public and private production costs.

Using Proposition 1, we record the overall public-private equilibrium outcomes and payoffs in three distinct parameter configurations that are distinguished on the basis of the unit costs of producing *B* and *C*.

Proposition 2. Under Bertrand price-competition in the *C*-market, the equilibrium price *p* of *C* will equal its unit cost *h*, and each *C*-seller will make zero economic profits. The equilibrium public provision regime, top-up consumption, and citizens' welfare will be as specified below.

Parameter Configuration I: $g/h \leq X/x^+$.
In this configuration, majority voting will lead to the adequate provision regime; no citizen will consume *C*; and each citizen with income *x* will have maximized utility $K.[X/g]^{(k)}[x]^{(1-k)}$.

Parameter Configuration II: $X/x^+ < g/h < 1$.
Here, majority voting will lead to the adequate provision regime; each citizen with income $x \leq h.X/g$ will not consume *C* and will have maximized utility $K.[X/g]^{(k)}[x]^{(1-k)}$, while each citizen with income $x > h.X/g$ will consume $[k(1-k)[(x/h) - (X/g)]$ amount of *C* and will have maximized utility $K.[p]^{(1-k)}[(1-k)(x/h) + k(X/g)]$; and the aggregate *C* consumption will be $[k(1-k)/2\Delta](1/h)[X^+ - (h.X/g)]^2$.

Parameter Configuration III: $g/h \geq 1$.
Here, majority voting will lead to the null provision regime; each citizen with income *x* will consume $[k(x/h)]$ amount of *C* and will have maximized utility $K.(h)^{(1-k)}.(x/h)$; and the aggregate *C* consumption will be $(k/h).X$.

Proposition 2 clearly demonstrates how the aggregate consumption of the supplement good *C* increases in the economy as the relative efficiency of public provision falls vis-à-vis that of private supply. Furthermore, it clarifies how the topping-up behavior of each individual citizen changes with changes in the relative production efficiency in the two sectors. But most importantly, it highlights the presence of certain significant discontinuities in the public-private system – incremental changes in the relative production efficiency in the public and the private sectors cause discrete changes in provision regimes and consumption profiles. In the next section, we make use of Proposition 2 to carry out our comparative static exercises.

5. Cost Comparative Statics

In this section, our first goal will be to identify the changes in equilibrium consumption of publicly-provided good *B* and the private supplement *C*, and in individual welfare, as the level of public efficiency (as measured by *g*) varies, holding all other parameters of the economic

environment constant. We will then consider the equilibrium changes brought about by a *ceteris paribus* variation in the private C-cost parameter h.

5.1 *Effects of declining public efficiency*

In an economy where the efficiency of the public sector monotonically declines over time, how do citizens respond in terms of their voting over the public provision regime and their individual demand for a private top-up? How are citizens in different income classes impacted in terms of individual welfare? In an environment where the private top-up good is supplied at cost, we address these questions by studying the effects of a *ceteris paribus* fall in the public production cost g on the overall public-private equilibrium outcomes.

Specifically, we consider increases in g in a range $[g_{min}, g_{max}]$ where $g_{min} < h.(X/x^{+}) < h < g_{max}$, and the resultant impact on the level of public provision of B, on the set of citizens who choose to top up B with C, on the aggregate consumption of C, and on the level of well-being of citizens with different income levels x. Studying the effect of an increase in g on the different equilibrium outcomes in the three parameter configurations in Proposition 2 allows us to derive the comparative statics results presented below. Note that $g \in [g_{min}, h.X/x^{+}]$ corresponds to Parameter Configuration I, $g \in (h.X/x^{+}, h)$ corresponds to Parameter Configuration II, and $g \in [h, g_{max}]$ corresponds to Parameter Configuration III.

Impact on public provision level: For all $g \in [g_{min}, h]$, $b = k.X/g$. Thus, when g increases from g_{min} towards h, the public provision level b falls continuously at a decreasing rate. However, there is a discrete drop in b from $k.X/h$ to 0 as g rises from 'a bit below h' to h – as the majority voting outcome shifts discretely from adequate public provision to null provision. Thereafter, the provision level stays at 0 for all higher g. See Figure 3.

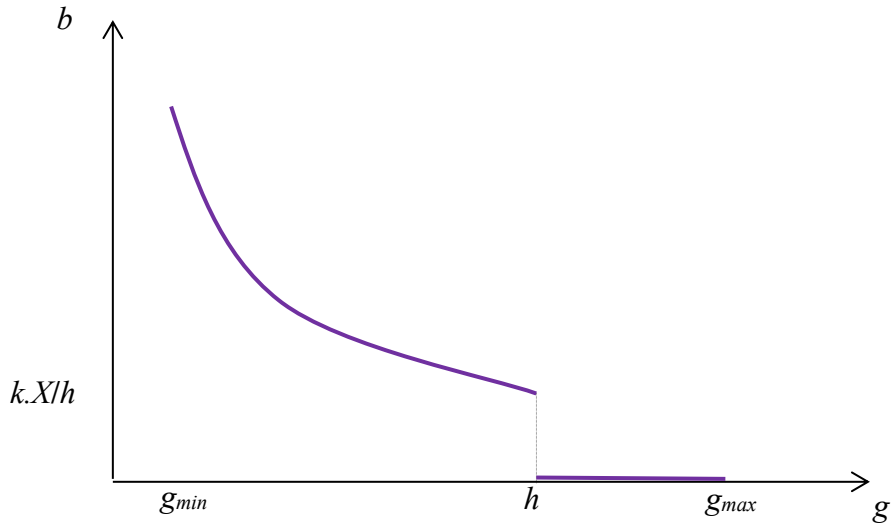

Figure 3. Declining public efficiency and public provision

Impact on the extent of topping-up: No citizen tops up B with C when $g \in [g_{min}, h.X/x^{+}]$. Individual topping-up begins when g increases above $h.X/x^{+}$, with a richer citizen opting to top-up B with C before a poorer citizen. The measure of citizens who engage in such topping-up rises continuously (from zero) as g increases from $h.X/x^{+}$ towards h, with the richer sections of the population topping up B with C as g rises to h from below. When g rises from 'a bit below h' to h, the measure of supplementors jumps discontinuously to unity. All citizens top up B with C for all $g \geq h$.

A relatively poor citizen with income $x < X$ tops up B with C if and only if $g \geq h$; her top-up level jumps from zero to a positive amount when g increases from 'a bit below h' to h; further, when g increases beyond h, her topping-up level stabilizes in that she consumes a constant level of the private good C given that the public provision volume remains unchanged at zero in that scenario. On the other hand, a relatively rich citizen with income $x > X$ tops up B with C if and only if $g > h.X/x$. For such a citizen, topping-up level rises continuously from zero as g rises from $h.X/x$ towards h; there is an upward jump in the topping-up level when g rises from 'a bit below h' to h; and when g increases beyond h, the richer citizen's C-consumption level becomes constant for the same reason as for a relatively poor citizen.

Changes in the set of supplementors as well as changes in the level of individual supplementation / topping-up imply the following impacts of increases in g on the aggregate level of supplementation. The aggregate supplementation level is zero for all $g \in [g_{min}, h.X/x^{+})]$; this level rises continuously from zero as g increases from $h.X/x^{+}$ towards h; there is an upward jump in the aggregate supplementation level when g rises from 'a bit below h' to h; and when g increases beyond h, the aggregate consumption of C stabilizes to a constant level. See Figure 4. It is this theoretical prediction that is consistent with the observed sales growth spurts in private supplements to publicly-provided goods in many developing countries.

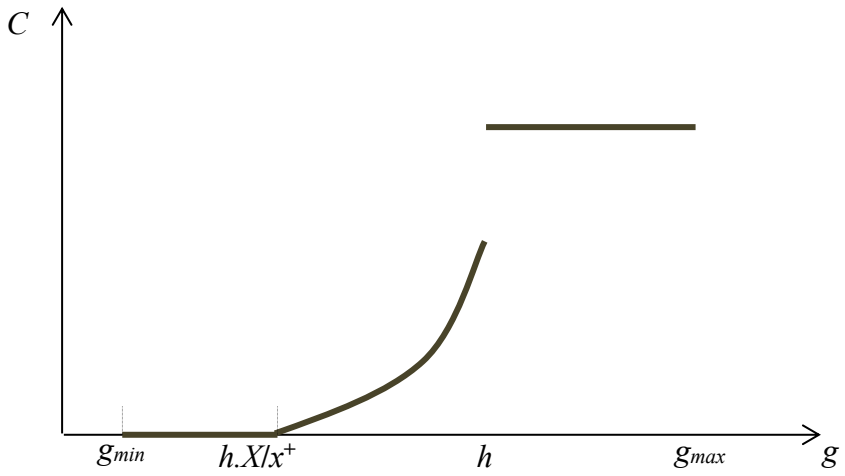

Figure 4. Declining public efficiency and aggregate supplementation

Impact on individual welfare: Consider the impact of an increase in g on individual welfare when g is strictly smaller than h – i.e., when the public provision regime is more efficient than the private supply regime. In this case, the maximized utility of a relatively poor citizen with income $x < X$ is $\{K.[X/g]^{(k)}[x]^{(1-k)}\}$, which falls continuously at a decreasing rate for any increase in $g \in [g_{min}, h]$. In contrast, for a relatively rich citizen with income $x > X$, her maximized utility is $\{K.[X/g]^{(k)}[x]^{(1-k)}\}$ for all $g \in [g_{min}, h.X/x]$ and is $\{K.[h]^{(1-k)}[(1-k)(x/h) + k.(X/g)]\}$ for all $g \in (h.X/x, h)$. This means that while her utility falls continuously at a decreasing rate for any increase in $g \in [g_{min}, h]$, the rate of fall is arrested once g rises above $k.(EX/x)$ (from which point the citizen starts topping up). As a result, starting from any point $h.X/x \in (h.X/x^{+}, h)$, an incremental increase in g causes a *larger* utility fall for all citizens with income $x \in (x^{-}, h.X/x^{+})$ than for those with $x \in (h.X/x^{+}, x^{+})$; while the richer citizens find it worthwhile to top up B with C, the act of topping-up limits the fall in their utility as the public provision regime becomes more inefficient. This implies that the 'gap' in maximized utility between a citizen richer than $h.X/g$ and one poorer than X, relative to the 'utility gap' that would exist if the privately-supplied good C was unavailable, rises in g. It is in this sense that a deterioration in public provision causes *utility dispersion* among citizens when $g < h$.

The adverse impact of a fall in public efficiency on citizens' welfare takes a dramatic turn when g rises from 'a bit below h' to h. Specifically, there exists a threshold income level $x^0 \in (x^{-}, X)$ such that this small rise in g causes (*i*) the utility of every citizen with income $x > x^0$ to *rise discontinuously*, and (*ii*) the utility of every citizen with income $x < x^0$ to *fall discontinuously* around $g = h$. This generates a discrete increase in the *utility inequality* among citizens. As g rises beyond h, the utility of every citizen remains constant as the null-provision regime is established through majority-voting. See Figure 5.

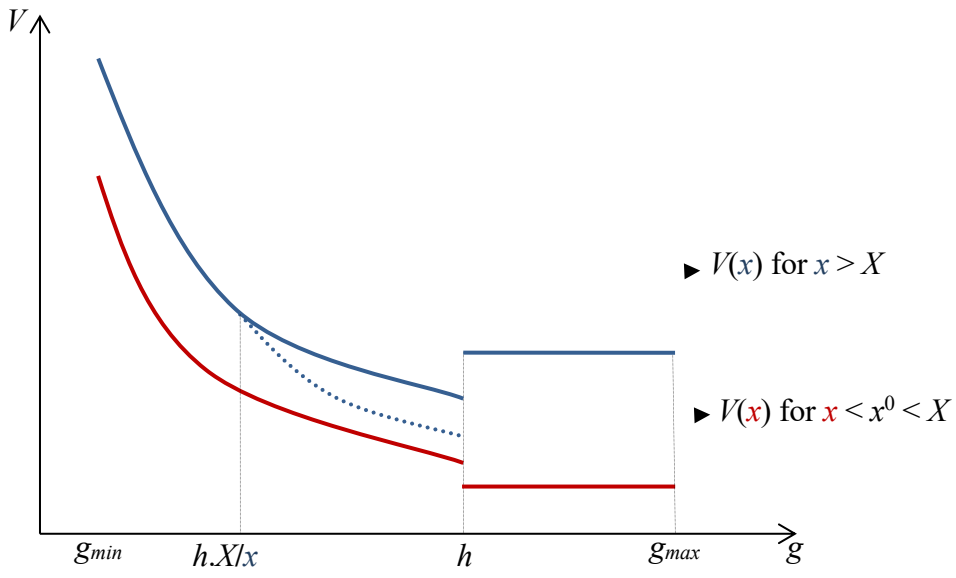

Figure 5. Declining public efficiency and citizens' welfare

The comparative statics results imply the following features of the public-private equilibrium under Bertrand competition in the C-market. As long as public efficiency is above a threshold ($g < h$), an incremental decline in public efficiency causes incremental changes in outcomes – public provision falls incrementally, the extent of topping-up rises incrementally, and individual utility falls incrementally for all citizens. With respect to the *distribution* of well-being, an incremental rise in g – for $g \in (h.X/x^+, h)$ – causes utility-dispersion between rich and the poor citizens, which is a consequence of the fact that the former engage in (increased) topping-up.

In contrast, discontinuous changes occur when public efficiency falls below a threshold – and g rises from 'just below' to 'just above' h. Then public provision falls discretely (due to a discontinuous change in the majority voting outcome), private topping-up rises discontinuously (with jumps in the number of supplementors as well as in the level of individual supplementation); and most importantly, individual utility rises discretely for all citizens richer than a threshold income level x^0, and falls discretely for all citizens poorer than the threshold income level. These discontinuous changes lead to a discrete increase in utility-inequality among the population.

A striking aspect of the above-described changes is that a deterioration in the public provision regime can cause a substantial set of rich citizens to be strictly better off. There exists a precise counterpart to this result: an improvement in private technology of C-production can cause a substantial set of poor citizens to be strictly worse off. In what follows, we establish this result by studying the consequences of improvements in private-sector productivity.

5.2 *Effects of improving private efficiency*

We now consider a *ceteris paribus* fall in the unit cost h of C-production in the private sector. Specifically, we consider decreases in h in a range $[h_{min}, h_{max}]$ where $h_{min} < g < h_{max} \leq g.(x^+/X)$.[9] Given that C is sold at cost in equilibrium, when $h \in (g, h_{max}]$ and h falls incrementally, the public provision level remains unchanged, private topping-up rises incrementally (the number of supplementors as well as the level of individual supplementation rise incrementally); the utility of the poorer non-supplementors is unaffected while that of the richer supplementors rise, causing a utility-dispersion (but without making any citizen strictly worse-off).

Again, something dramatic happens when the private productivity improvement crosses a threshold – when h falls from 'just above g' to 'just below g'. Note that here exists $x^* \in (x^-, X)$ such that the incremental fall in h causes the utility of every citizen with income $x > x^*$ to *rise discontinuously*, and the utility of every citizen with $x < x^*$ to *fall discontinuously*, thereby generating a discrete upward jump in utility-inequality among the population.

[9] Note that $h \in [h_{min}, g]$ corresponds to Parameter Configuration III, and $h \in (g, h_{max}]$ corresponds to Parameter Configuration II.

Intuitively, a fall in the unit cost (and thus the price) of the competitively-supplied good *C* induces the rich citizens (who constitute a majority) to vote for a drastic reduction in public provision. This causes a discrete change in the majority-voting outcome from 'adequate provision' to 'null provision' (the majority compensates for this change by increased supplementation). While this discontinuous regime change causes a discrete improvement in individual well-being for all citizens richer than an income threshold, the noteworthy fact is that an incremental improvement in private-sector productivity leads to a discrete reduction in the welfare of many citizens located in the bottom-tail of the income distribution.

Note that if the supplement good *C* came in multiple quality levels, with a higher-quality good being a better supplement for the publicly provided good (e.g., generator sets *vs.* inverters), then cost and price decreases in each of these variants of the supplement good could generate outcomes of the following nature. A fall in the cost/price of the lowest-quality supplement below a threshold level could lead to some discrete reduction in public provision and a consequent discrete reduction in the welfare of the poorer citizens. Next, a fall in the cost/price of penultimate-quality supplement below its threshold level could lead to further discrete reduction in public provision and in the welfare of the poorer citizens. Thus, in the presence of multiple quality levels of the supplement good *C*, there can be many stages at which improvements in the private technology can discontinuously lower the level of well-being of the poorer sections of the population.

6. Concluding Remarks

In a scenario where citizens consume publicly provided goods as well as private supplements, this paper has aimed to determine the nature of changes in equilibrium outcomes that occur when the relative efficiency of the public sector (*vis-à-vis* the private sector) deteriorates over time.

We have identified the possibility of large discontinuous changes in the majority-voted public provision regime arising from small changes in fundamentals. Small decreases in supplement prices and / or small increases in public costs can cause the majority-voting outcome to shrink discretely from an adequate public provision regime to a null provision regime. In essence, this paper argues that a public system decline generates behavioural changes among citizens leading to (discrete) changes in private as well as in public decisions and complex welfare effects, often hurting the poor disproportionately. This theoretical finding accords well with the recent evidence of explosive sales growth in various supplements markets following intertemporal decline in public provision regimes in many (developing) countries.

In this context, note that the current paper models a deteriorating public provision regime as the source of increasing dissatisfaction among the affluent households regarding the public provision regime. Recognize that rising incomes at the top-end of the income distribution (due

to unequal economic growth) given an unchanged public provision system can be another reason why the rich will demand private supplements to public provision.

Further, there are enough reasons to believe that widespread use of private supplementation by the rich generate economy-wide negative externalities – private electricity generators pollute, plastic water bottles fill up landfills, and so on. It will be fruitful to study the extent to which citizens (fail to) internalize these externalities in their voting and consumption decisions, and the resultant impact of the negative externalities on societal welfare. Furthermore, for publicly provided goods like primary education, citizens' participation and involvement are critical inputs in the public production process. These are likely to be reduced when citizens engage in increased supplement consumption. A process of public system decline will then have negative magnification effects that need to be studied, along the lines initiated by Gurgur (2016).

References

Buckley, N. and Cuff, K. and Hurley, J. and Mestelman, S. and Thomas, S. and Cameron, D. 2015. "Support for public provision of a private good with top-up and opt-out: A controlled laboratory experiment." *Journal of Economic Behavior & Organization*, 111, 177–196.

Epple, D. and R. Romano. 1996a. "Public provision of private goods." *Journal of Political Economy*, 104, 57–84.

Epple, D. and R. Romano. 1996b. "Ends against the middle: Determining public service provisions when there are private alternatives." *Journal of Public Economics*, 62, 297–325.

Glomm, G. and B. Ravikumar. 1998. "Opting out of publicly provided services: A majority voting result." *Social Choice and Welfare*, 15, 187–99.

Gouveia, M. 1997. "Majority rule and the public provision of a private good." *Public Choice*, 93, 221–44.

Gurgur, T. 2016. "Voice, exit and local capture in public provision of private goods." *Economic Governance*, 17, 397–424.

Lülfesmann, C. and G. Myers. 2011. "Two-tier public provision: Comparing public systems." *Journal of Public Economics*, 95, 1263–71.