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An Integrated VIKOR-TOPSIS-Regression based Methodology for Evaluating the Performance and Exploring the determinants of Primary and Secondary education: Evidence from India

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Abstract

In the context of primary and secondary education, designing a robust performance assessment methodology remains a matter of concern for developing countries such as India. Motivated by this problem, in this article, we propose an integrated VIKOR-TOPSIS-Regression based methodology to measure the input-level performance of 82930 primary and secondary schools that come under 20 districts of West Bengal, a state of India, and to investigate the impact of this performance along with the contextual factors such as medium of instruction and location of the school on the school output level performance, i.e., student pass rate. To evaluate the performance of the schools, we select two prevalent MCDM methods, viz., VIKOR and TOPSIS due to their intrinsic advantages in the presence of conflicting decision-making criteria, i.e., cost and benefit criteria, to evaluate the input-level performance of the schools. After obtaining the scores of a school as per VIKOR and TOPSIS method, we integrate them into a single score using the Shannon entropy-based weighting technique and devise both conservative as well as optimistic integrated MCDM performance valuation framework of schools. We perform a rigorous comparative analysis on district-level as well as state-level performance across the methods and discuss the insights. Finally, we investigate the impact of the input-level performance of the schools, medium of instruction, and location of the school on the student pass

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rate separately for boys and girls using multiple linear regression analysis. Through the hypotheses development, we conclude that all the factors have a significant impact on boys' pass rate whereas only input-level performance of the school and location of the school have a significant influence on the girls' pass rate.

Keywords: Primary and secondary education, TOPSIS, VIKOR, Regression, Integrated multicriteria decision making

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1. Introduction

Primary and secondary education remain the basic building blocks of a person's development as well as the key to better livelihood irrespective of countries. Effective primary and secondary education play an instrumental role in the growth, development, and poverty reduction for any nation in the world. However, according to the world bank, around 250 million people in the world still lack basic literacy skill despite more than three years of schooling.[5] The situation aggravates for the developing countries such as India, Pakistan, Bangladesh, and so on where more than 60% of the students of the secondary schools are unable to acquire the basic reading and writing skill.[6] Specially in India, the quality of primary and secondary education remains a matter of concern.

As per the EFA Global Monitoring Report published in 2010 by UNESCO, India secured 105th rank among 128 countries from the perspective of quality of education[7]. India has been under-performing among developing countries in ensuring education for all children at the elementary level, even in Asia. Though the literacy rate of India has grown from 64.84% in 2001 to 74.04 % in 2011[8], it still has the largest number of illiterate populations in the world, indicating unsatisfactory performance in the primary and secondary education. Despite the introduction of several programmes such as Operation Black Board (OBB), Shiksha Karmi Project (SKP), Andhra Pradesh Primary Education Project (APPEP), Bihar Education Project (BEP), U.P Basic Education Project (UPBEP), and Sarva Shiksha Abhiyan, the progress is not steady.[9] A recent survey on 6 lakh children between the ages of 3-16 conducted by Pratham, an NGO indicates that nearly half of the grade V students were not able to read, and nearly same proportion of grade V students did not have the basic arithmetic skills, which they should have

[5] Source: http://www.worldbank.org/en/topic/education/brief/primary-and-secondary-education
[6] Source: https://www.bmz.de/en/issues/Education/hintergrund/bildungsituation/index.html
[7] Source: http://unesdoc.unesco.org/images/0018/001866/186606E.pdf
[8] Source: http://niti.gov.in/content/literacy-rate-7years
[9] Source: http://ssashagun.nic.in/docs/SSA-Frame-work.pdf

learned by the end of grade II (Pratham 2013). It signifies that the presence of a proper performance monitoring of the education emerges as an order of the day in the context of India.

Our analysis indicates that the Government of India allocates a substantial percentage of the education budget for the improvement of primary and secondary education. For instance, around 80% of the planned budget, i.e., 350 crores (in Indian Rs.) is allotted for primary and secondary education in 2015-16.[10] Further, we observe that this funding is utilized on several developmental factors such as the development of schools' physical infrastructure, internal management, quality of education, etc.[11] Efficient management of these input parameters plays an instrumental role to improve the schools' output level performance, i.e., students' education. Several scholars such as Branham (2004), Altonji (2005), Asiabaka (2008) indicate the importance of input-level performance evaluation of the schools through capturing their performance in the parameters mentioned above. In this context, application of various multicriteria decision making (MCDM) techniques can be very useful.

According to Rezaei (2016), multicriteria decision making can be defined as a complex and dynamic process that facilitates decision-making at the managerial level in the presence of conflicting criteria, i.e., cost and benefit. Our exploration of existing scholarly works reveals the presence of Multicriteria decision making (MCDM) methods such as data envelopment analysis (DEA) (Thanassoulis and Dunstan 1994), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), (Nisel 2014) Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) (Chen and Chen 2010), Analytic Hierarchy Process (AHP) (Badri and Abdulla 2004), etc. for the performance measurement process in the domain of education. Now, each of these methods has its unique advantages and biasness toward performance measures. It signifies that the design of an integrated MCDM method in a scientific manner can be helpful to bring multiple advantages into a single framework and to measure input-level performances in a more effective manner. Existing literature indicates the absence of a substantial number of works that focus on developing an integrated method.

From the perspective of schools' output level performance, students' pass rate is mostly used as the measurement criteria (Umashankar and Dutta 2007, Lavy 2009, Kassile 2014). Here, researchers opine that contextual factors such as location of a school (Reeves and Bylund 2005,

[10] Source: https://mhrd.gov.in/sites/upload_files/mhrd/files/statistics-new/ABE2013-16.pdf
[11] Source: https://www.qcin.org/PDF/Comman/Quality-in-School-Education.pdf

Mitra et al. 2008) and medium of instruction, i.e., vernacular or foreign language (Heugh 1999, Pathan and Shiakh 2012) influence the students' pass rate apart from schools' input-level performance. Our exploration reveals that the impact of these factors along with schools' input level performance on the students' pass rate has not been paid enough attention.

The above-mentioned issues motivate us to address following research questions:

- What are relevant MCDM methods that can be useful to measure the input-level performance of the schools in the presence of conflicting criteria?
- How can an integrated method be developed by combining these methods through a scientific approach?
- What is the impact of a school's input-level performance on its output-level performance, i.e., student outcome?
- How do the contextual factors, i.e., location of a school and the medium of instruction influence the output-level performance of a school?

In this article, we propose an integrated VIKOR-TOPSIS-Regression based methodology to assess input-level performance of 82930 primary and secondary schools that come under 20 districts of West Bengal, a state of India, and to investigate the impact of input-level performance of schools, medium of instruction, and location of the school, on the schools' output performance, i.e., student pass rate. Here, we select two prevalent MCDM methods, viz., VIKOR and TOPSIS to evaluate the input-level performance of the schools because of their intrinsic advantages in the presence of conflicting decision-making criteria. For each of the methods, first, we determine the score of a school in each of the parameters, i.e., infrastructure, school management, and quality of education. The weights of the criteria under a parameter are determined using Shannon entropy-based approach. Then, we aggregate these parameter scores into a single score for each of the methods using Shannon entropy-based approach. The application of Shannon entropy brings two advantages. First, it facilitates scientific weight allocation to the different criteria instead of arbitrary weight assignment. Second, it ensures more robustness through matrix comparison compared to the pairwise comparison. We also devise both conservative as well as optimistic integrated methodology. We perform a rigorous comparative analysis on district-level as well as state-level performance across the methods and discuss the insights. Finally, we investigate the impact of the input-level performance of the schools, medium of instruction, and location of the school on the output level performance, i.e., student pass rate separately for boys and girls using multiple linear regression analysis. Through the hypotheses development and testing, we conclude that all the factors have a significant impact on boys' pass rate whereas only input-level performance of the school and location of the school have a significant influence on the girls' pass rate. Although our analysis primarily addresses concerns in a real-life situation, the framework, methodology, and policy-level implications obtained from analysis can be applicable to the similar global scenarios, specially in case of developing economy countries.

The article is organized as follows. Section 2 presents a summarized description of the scholarly works relevant to our work. In Section 3, we demonstrate the proposed methodology. In Section 4, we perform a rigorous comparative analysis on the district as well as state-level performance across the methods. In section 5, we explain the effect of the school's input-level performance, medium of instruction, and location of the school on the student outcome. Finally, the article concludes by discussing contributions and future research avenues.

2. Literature Review

Our exploration of existing literature reveals that the relevant scholarly articles can be classified into three categories; Performance evaluation in the education sector, application of VIKOR and TOPSIS in the education sector, and integrated MCDM methods.

2.1. Performance evaluation in the education sector

Our exploration of the research articles reveals that several scholars focus on the performance evaluation of the schools. Thanassoulis and Dunstan (1994) apply data envelopment analysis (DEA) model to assess the performance of the schools and to guide the secondary schools for achieving better performance through setting the target as well as the benchmark. Similarly, Bradley et al. (2001) extend the DEA-based performance evaluation of the schools to the exploration of determinants influencing efficiency and conclude that the competition plays an instrumental role in the performance of the secondary schools. Overton et al. (2016) investigate whether the presence of teacher unions can have an impact on the efficiency of the schools or not through DEA and statistical analysis. Aparicio et al. (2017) adopt a non-radial DEA based approach to evaluate the performance of the schools for the countries of the organisation for economic co-operation and development (OECD). Johnson and Ruggiero (2018) apply Malmquist productivity index to measure the performance of the schools as well as factors influencing the efficiency. Badri and Abdulla (2004) develops an AHP based method to evaluate the performance of the faculty members. In a similar way, Badri et al. (2016) apply AHP to evaluate the quality of education for the schools of Abu Dhabi.

2.2. Application of VIKOR and TOPSIS in the education sector

According to the scholars such as Opricovic and Tzeng (2004), Peng et al. (2011), Franek and Kashi (2014), VIKOR and TOPSIS method facilitates more effectiveness and robustness to handle the conflicting criteria compared to the other MCDM methods. Due to the presence of conflicting criteria in this study, we select VIKOR and TOPSIS. Ghosh (2011) exhibits application of both VIKOR and TOPSIS to assess faculty performance in engineering education.

TOPSIS (technique for order preference by similarity to an ideal solution), proposed by Chen and Hwang (1992), is one of the prevalent MCDM techniques. Several scholars have applied this method to investigate the sustainability of government bonds, (Bilbao-Terol et al. 2014), housing affordability (Mulliner et al. 2016), performance of the third-party logistics service providers etc. (Singh et al. 2017), and so on. In the context of the education sector, Ertuğrul and Karakaşoğlu (2007) develop a TOPSIS method to facilitate faculty selection. Similarly, Li et al. (2016) adopt Fuzzy TOPSIS based approach to evaluate the higher vocational education development levels. Ding and Zeng (2015) apply TOPSIS method to evaluate the performance 68 Chinese universities.

VIKOR, proposed by Opricovic (1998) and advanced by Opricovic and Tzeng (2004), is considered as another useful MCDM techniques to evaluate the performance of alternatives under conflicting criteria. It is the Serbian abbreviation for the VlseKriterijumska Optimizacija I Kompromisno Resenje (means Multicriteria Optimization and Compromise Solution). Scholars apply VIKOR method in various problems such as assessment of online health information (Afful-Dadzie 2016), selection of third-party logistics partner (Sasikumar and Haq 2011), improvement of airlines service quality (Liou 2011) and so on. From the perspective of education, Wu et al. (2011) asses the performance of extension education centers of three universities in Taiwan using the VIKOR method. Nisel (2014) presents an extended VIKOR based methodology to evaluate the performance of the top hundred online MBA programmes for the year 2013. Sarkar and Sarkar (2016) propose a VIKOR-based ranking method to assess the performance of Indian Technical Institutes.

2.3. Integrated MCDM methods

In recent times, there is a rising interest among scholars to design integrated MCDM methodology to bring advantages of different MCDM methods into a single framework as well as to achieve higher robustness. For example, scholars such as Tzeng, and Huang (2012), Seitz and Torre (2014), propose an integrated approach comprising different MCDM methods such as analytic network process (ANP), grey relational analysis (GRA), goal programming, and so on. In the context of education, Chen and Chen (2010) integrate DEMATEL and TOPSIS to measure the innovation performance of Taiwanese higher educational institutions. Song and Zheng (2015) develop a hybrid TOPSIS -grey correlation model for assessing the teaching quality in higher education. Similarly, Chakraborty et al. (2017) adopt an integrated preference ranking organisation method for enrichment evaluations (PROMETHEE) and geographical information systems (GIS) framework to evaluate the educational performance of Indian states. Similarly, Sarkar (2016) proposes a mixed principal component analysis and data envelopment analysis (PCA-DEA) for the performance assessment of primary schools. Chen et al. (2017) propose a hybrid DEA method based on input-oriented bounded-and -discrete data DEA model and context-dependent DEA model to evaluate the college graduation rate of the higher education institutes. We present the summarized literature in Table 1, that clearly highlights our contribution.

Table 1 Summarized description of literature review

Scholarly works	Nature of the method adopted in the work (Single or integrated)	Description of the method	Performance Measurement		Study on the impact of contextual factors and input-level performance on the output-level performance
			Input level	Output level	
Aparicio et al.	Single	DEA	—	✓	—

(2017)					
Johnson and Ruggiero (2018)			—	✓	✓
Badri et al. (2016)		AHP	✓	—	—
Li et al. (2016)		TOPSIS	—	✓	—
Wu et al. (2011)		VIKOR	—	✓	—
Chen and Chen (2010)	Integrated	DEMATEL and TOPSIS (Weighting technique: Fuzzy ANP)	—	✓	—
Song and Zheng (2015)		TOPSIS and Grey Correlation Model (Weighting technique: Shannon Entropy)	✓	—	—
Chakraborty et al. (2017)		PROMETHEE and GIS (Weighting technique: Shannon Entropy)	—	✓	—
Our paper	**Integrated**	**VIKOR and TOPSIS (Weighting technique: Shannon Entropy)**	✓	—	✓

It is evident that most of the existing works are restricted to performance measurement using a single method. Here, we incorporate a robust integrated framework and extend our work to investigate the impact of input-level performance along with other contextual factors such as the location of the school and the medium of instruction on the output-level performance, i.e., student outcome through regression analysis.

3. Methodology

In this section, we demonstrate the proposed methodology to evaluate the input-level performance of the schools in terms of providing infrastructural support and learning facilities to the students. First, we present a summarized description of the criteria as well as parameters used in the evaluation, selected MCDM methods for assessment, the method used for integration, and rationale behind the selection of these methods. Then, we present a summarized description of the proposed method to facilitate practical implementation. Finally, we demonstrate our method in detail.

3.1. Selection of parameters, criteria, methods, integration mechanism, and summarized description of the proposed method

Our exploration of the existing literature reveals that the performance of a school can be evaluated on three input parameters: physical infrastructure, school management, and quality of education (Branham, 2004; Altonji, 2005; Asiabaka, 2008). The rationale behind the selection of these parameters emerges from existing scholarly works as well as the data provided by the primary education department of West Bengal. Now, the performance of a school considering these parameters can be measured through different criteria. Depending on criteria chosen, we may have conflicting objectives, i.e., minimization or maximization from the perspective of the better performance of a school. For this reason, we classify the criteria into two categories; ‘cost’ and ‘benefit’ where minimization and maximization, respectively is preferred for that specific parameter depending on the nature of that parameter. The detailed description of parameters, along with the cost and benefit criteria, is presented in Table 2.

Table 2 Parameters selected for the evaluation of the schools

Parameters	Criteria	Nature
Physical infrastructure	Distance from block head quarters	Cost
	Distance from cluster resource center	Cost
	Total classrooms used for instructional purposes	Benefit
	Number of classrooms in good condition	Benefit
	Number of classrooms that require major repair	Cost
	Number of classrooms that require minor repair	Cost
	Number of toilet seats constructed/available for boys	Benefit
	Number of toilet seats constructed/available for girls	Benefit
	Number of books in library	Benefit
	Number of computers in library	Benefit

School management	Number of pre-primary students	Benefit
	Number of working days	Benefit
	Number of inspections	Benefit
	Pre-primary teachers	Benefit
	Number of visits by block resource center officer	Benefit
	Number of visits by cluster resource center officer	Benefit
	Amount of school development grant receipt	Benefit
	Amount of school development grant expenditure	Benefit
	Amount of teacher learning material receipts	Benefit
	Amount of teacher learning material expenditure	Benefit
Quality of education	Total number of male teachers	Benefit
	Total number of female teachers	Benefit
	Total number of head teachers in schools	Benefit
	Total number of teachers graduate and above	Benefit
	Total number of teachers with professional qualification	Benefit
	Total number of working days spent to non-teaching assignments	Cost
	Total number of teachers involved in (non-teaching assignments)	Cost
	Number of instructional days (previous year)- primary	Cost
	Number of instructional days (previous year)- upper primary	Benefit
	Teacher working hours (per day) –primary	Benefit
	Number of hours children stay in school (current year)- upper primary	Benefit
	Teacher working hours (per day) –upper primary	Benefit
	No. of children enrolled special training in current year – boys	Benefit
	No. of children enrolled special training in current year– girls	Benefit
	No. of children provided special training in current year – boys	Benefit
	No. of children provided special training in current year– girls	Benefit
	No. of children enrolled special training in previous year – boys	Benefit

In this work, we select VIKOR and TOPSIS method to measure the input level performance. According to the scholars such as Opricovic and Tzeng (2004), Peng et al. (2011), and Franek and Kashi (2014), VIKOR and TOPSIS method facilitates more effectiveness and robustness to handle the conflicting criteria compared to the other MCDM methods. It acts as the rationale behind our selection. Now, TOPSIS and VIKOR methods assign the best rank to the school with the maximum and the minimum score, respectively. Now, the difference in data normalization technique, i.e., vector and linear normalization in case of TOPSIS and VIKOR, respectively, may result in the two different ranking lists for above-mentioned MCDM methods. It motivates us to design a ranking list based on an aggregated score by integrating scores obtained from TOPSIS and VIKOR.

Here, we adopt the Shannon entropy (Shannon, 1948) based approach to determine the weights of different criteria as well as parameters, and to integrate the selected MCDM methods. Shannon entropy-based weighting technique exhibits several advantages. It allocates weights based on variation in the values, thus leading to a more scientific weight assignment Compared to the equal weight assignment. Also, application of matrix-based comparison instead of pairwise comparison yields higher robustness. Several scholars such as Soleimani-Damaneh and Zarepisheh, (2009), Wu et al. (2012), and Adhikari et al. (2018) recently incorporate this approach in integrating scores obtained from different Data Envelopment Analysis (DEA) into a single score. Extending their approach, we apply this technique to integrate two completely different MCDM method.

After the selection of the parameters, we evaluate the input-level performance of the schools in the following manner:

- *Stage 1:* We determine the weights of each of these criteria using Shannon entropy concept.
- *Stage 2:* Using the weights of the criteria determined in stage 1, first, we apply the MCDM methods, i.e., VIKOR and TOPSIS to determine the score of the each of the parameters of a school.
- *Stage 3:* Next, we calculate the weights of these parameters using Shannon entropy method for each of VIKOR and TOPSIS methods. Then, we determine the scores of a school for these methods.
- *Stage 4:* After getting the scores of each of the schools as per the methods described above, we compute the weights of scores obtained from each of the methods using Shannon entropy and integrate it into a single score.

To facilitate practical implementation, we present the method as a flow diagram presented in figure 1.

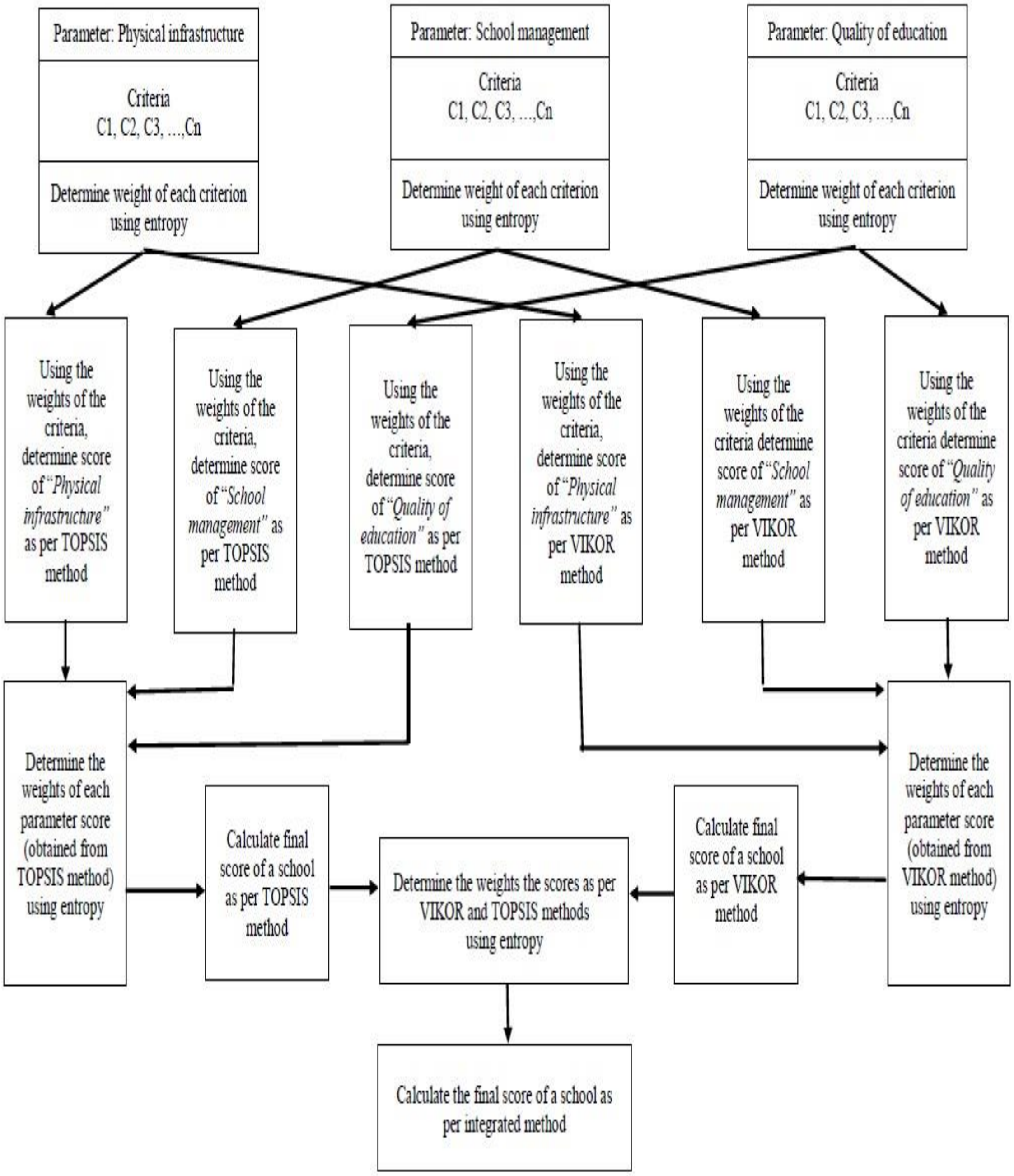

Figure 1 Proposed methodology for input-level performance evaluation

3.2. Application of Shannon entropy in the weight calculation of the parameters, criteria and aggregation of methods

As mentioned earlier, we apply Shannon entropy (Shannon, 1948) concept to determine the weights of different criteria and parameters for aggregating them into a single parameter score and final score of a school, respectively. The method is demonstrated as follows:

Let parameters of school performance (par) viz. physical infrastructure, quality of education, and school management be represented as phy, qua, and scm, respectively. $par \in \{phy, qua, scm\}$. Here we assume there are m schools are under evaluation and k criteria under any parameter. Now, the value of criterion i of parameter par of school j can be written as $x_j\left(f_i^{par}\right)$, where $i \in \{1, 2, \ldots, k\}$, $j \in \{1, 2, \ldots, m\}$, and $par \in \{phy, qua, scm\}$.

$$X\left(f^{par}\right) = \begin{array}{c} \begin{array}{cccc} f_1^{par} & f_2^{par} & \cdots & f_k^{par} \end{array} \\ \begin{bmatrix} x_1\left(f_1^{par}\right) & x_1\left(f_2^{par}\right) & \cdots & x_1\left(f_k^{par}\right) \\ x_2\left(f_1^{par}\right) & x_2\left(f_2^{par}\right) & \cdots & x_2\left(f_k^{par}\right) \\ \vdots & \vdots & \vdots & \vdots \\ x_m\left(f_1^{par}\right) & x_m\left(f_2^{par}\right) & \cdots & x_m\left(f_k^{par}\right) \end{bmatrix} \end{array}$$

Next, we form a normalized matrix, denoted by $X^N\left(f^{par}\right)$ where values of each row of this normalized matrix can be calculated as follows:

$$x_j^N\left(f_i^{par}\right) = \left(\frac{x_j\left(f_i^{par}\right)}{\sum_{j=1}^{m} x_j\left(f_i^{par}\right)}\right),$$

where $i \in \{1, 2, \ldots, k\}$, $j \in \{1, 2, \ldots, m\}$, and $par \in \{phy, qua, scm\}$.

Here the normalized matrix can be expressed as follows:

$$
\begin{array}{cc}
 & \begin{array}{cccc} f_1^{par} & f_2^{par} & \dots & f_k^{par} \end{array} \\
X^N\left(f^{par}\right)= & \begin{bmatrix} x_1^N\left(f_1^{par}\right) & x_1^N\left(f_2^{par}\right) & \dots & x_1^N\left(f_k^{par}\right) \\ x_2^N\left(f_1^{par}\right) & x_2^N\left(f_2^{par}\right) & \dots & x_2^N\left(f_k^{par}\right) \\ \vdots & \vdots & \vdots & \vdots \\ x_m^N\left(f_1^{par}\right) & x_m^N\left(f_2^{par}\right) & \dots & x_m^N\left(f_k^{par}\right) \end{bmatrix}
\end{array}
$$

Next, we determine the entropy value $\left(e\left(f_i^{par}\right)\right)$ as well as the weights $\left(u\left(f_i^{par}\right)\right)$ of criterion i can be calculated in the following manner:

$$
e\left(f_i^{par}\right)=-e_0\sum_{j=1}^{m}x_j^N\left(f_i^{par}\right)\ln\left(x_j^N\left(f_i^{par}\right)\right),\quad u\left(f_i^{par}\right)=\left\{\frac{\left(1-e\left(f_i^{par}\right)\right)}{\sum_{i=1}^{k}\left(1-e\left(f_i^{par}\right)\right)}\right\}
$$

where $i\in\{1,2,\dots,k\}$, $j\in\{1,2,\dots,m\}$, and $par\in\{phy, qua, scm\}$.

Here, these weights are used to determine a single score for a parameter of a school. Similarly, we determine the weights of different parameters for integrating them into a final score of a school. Finally, following the same approach, we determine the weights of the scores obtained from TOPSIS and VIKOR method to aggregate them into a single score.

3.3. Determining the score of a school using TOPSIS method

In this sub-section, we apply the TOPSIS method to evaluate the performance of the schools. From the perspective of decision-makers, TOPSIS shows user-friendliness to determine the score of the alternatives in the presence of conflicting criteria (Opricovic and Tzeng, 2004). The main idea behind this method is to determine the closeness of an alternative from the ideal solution, viz., the shortest distance from the ideal solution and the farthest distance from the negative ideal solution using the concept of Euclidian distance. The higher value of closeness parameter signifies the more accurate solution and vice-versa. Here, we determine the score of a school in any specified parameter by combining the scores of the criteria along with the weights of these criteria calculated with the help of the TOPSIS method and entropy concept. Similarly, after getting the scores of a school in various parameters, we compute the weights of these parameters and integrate them into a single performance score applying TOPSIS method. The method is illustrated below:

3.3.1. Creation of normalized data matrix of the criteria for different parameters

First, we create a normalized matrix for the criteria of school j, denoted by $X^N\left(f^{par}\right)_{TOP}$ where values of each row of this normalized matrix can be calculated as follows:

$$x_j^N\left(f_i^{par}\right)_{TOP} = \left(\frac{x_j\left(f_i^{par}\right)}{\sqrt{\sum_{j=1}^{m}\left\{x_j\left(f_i^{par}\right)\right\}^2}}\right),$$

Where, $i \in \{1, 2, \ldots, k\}$, $j \in \{1, 2, \ldots, m\}$, and $par \in \{phy,\ qua,\ scm\}$.

Here the normalized matrix can be expressed as follows:

$$X^N\left(f^{par}\right)_{TOP} = \begin{array}{c} \begin{array}{cccc} f_1^{par} & f_2^{par} & \cdots & f_k^{par} \end{array} \\ \begin{bmatrix} x_1^N\left(f_1^{par}\right)_{TOP} & x_1^N\left(f_2^{par}\right)_{TOP} & \cdots & x_1^N\left(f_k^{par}\right)_{TOP} \\ x_2^N\left(f_1^{par}\right)_{TOP} & x_2^N\left(f_2^{par}\right)_{TOP} & \cdots & x_2^N\left(f_k^{par}\right)_{TOP} \\ \vdots & \vdots & \vdots & \vdots \\ x_m^N\left(f_1^{par}\right)_{TOP} & x_m^N\left(f_2^{par}\right)_{TOP} & \cdots & x_m^N\left(f_k^{par}\right)_{TOP} \end{bmatrix} \end{array}$$

3.3.2. Creation of weighted normalized data matrix of the criteria for different parameters

We next create a weighted normalized matrix of school j, denoted by $y^N\left(f^{par}\right)_{TOP}$ where values of each row of this weighted normalized matrix can be calculated as follows:

$$y_j^N\left(f_i^{par}\right)_{TOP} = u\left(f_i^{par}\right)x_j^N\left(f_i^{par}\right)_{TOP}$$

Where, $i \in \{1, 2, \ldots, k\}$, $j \in \{1, 2, \ldots, m\}$, and $par \in \{phy,\ qua,\ scm\}$.

$$Y^N\left(f^{par}\right)_{TOP} = \begin{array}{c} \begin{array}{cccc} f_1^{par} & f_2^{par} & \cdots & f_k^{par} \end{array} \\ \begin{bmatrix} y_1^N\left(f_1^{par}\right)_{TOP} & y_1^N\left(f_2^{par}\right)_{TOP} & \cdots & y_1^N\left(f_k^{par}\right)_{TOP} \\ y_2^N\left(f_1^{par}\right)_{TOP} & y_2^N\left(f_2^{par}\right)_{TOP} & \cdots & y_2^N\left(f_k^{par}\right)_{TOP} \\ \vdots & \vdots & \vdots & \vdots \\ y_m^N\left(f_1^{par}\right)_{TOP} & y_m^N\left(f_2^{par}\right)_{TOP} & \cdots & y_m^N\left(f_k^{par}\right)_{TOP} \end{bmatrix} \end{array}$$

3.3.3. Determining the score of a school in a specific parameter

Now, ideal solution $\left(y_i\right)_+^*$ and negative-ideal solution $\left(y_i\right)_-^*$ for criterion i can be expressed as:

$$\left(y_i\right)_+^* = \left\{\left(\max_j y_j^N\left(f_i^{par}\right)_{TOP} / \left(i \in I^{'}\right),\right),\left(\min_j y_j^N\left(f_i^{par}\right)_{TOP} / \left(i \in I^{"}\right),\right)\right\}.$$

$$\left(y_i\right)_-^* = \left\{\left(\min_j y_j^N\left(f_i^{par}\right)_{TOP} / \left(i \in I^{'}\right),\right),\left(\max_j y_j^N\left(f_i^{par}\right)_{TOP} / \left(i \in I^{"}\right),\right)\right\}.$$

Where $I^{'}$ and $I^{"}$ represents cost and benefit criteria, respectively.

Next, we determine the Euclidean distance from ideal and negative ideal solutions across the criteria for school j, i.e., $\left(d_j\right)_+^*$ and $\left(d_j\right)_-^*$, respectively.

$$\left(d_j\right)_+^* = \sqrt{\sum_{i=1}^{k}\left[y_j^N\left(f_i^{par}\right)_{TOP} - \left(y_i\right)_+^*\right]^2}, \left(d_j\right)_-^* = \sqrt{\sum_{i=1}^{k}\left[y_j^N\left(f_i^{par}\right)_{TOP} - \left(y_i\right)_-^*\right]^2}.$$

Finally, the score of the school j in parameter par, i.e., $\left(\theta_j^{par}\right)_{TOP}$ can be captured through the closeness parameter of school j from ideal solution, as follows:

$$\left(\theta_j^{par}\right)_{TOP} = \left[\frac{\left(d_j\right)_-^*}{\left(d_j\right)_+^* + \left(d_j\right)_-^*}\right]$$

3.3.4. Determining the final score of a school and final Ranking

Following the approach described in sub-section 3.2, we determine the weight of parameters. Let the weight of parameter par can be represented as $\left(w^{par}\right)_{TOP}$. In a similar fashion, we apply TOPSIS method to combine scores of a school across different parameters into a single score. Now, the score of a school j as per TOPSIS method can be expressed as:

$$\left(\theta_j\right)_{TOP} = \sum_{par \in \{phy, qua, scm\}} \left(w^{par}\right)_{TOP} \left(\theta_j^{par}\right)_{TOP}, \quad \sum_{par \in \{phy, qua, scm\}} \left(w^{par}\right)_{TOP} = 1, \quad par \in \{phy, qua, scm\}.$$

According to the TOPSIS method, the higher value of $\left(\theta_j\right)_{TOP}$ signifies the better performance of the school j. So, the school with highest $\left(\theta_j\right)_{TOP}$ will be considered as the best school and the ranking will be prepared in descending order.

As opined by Lai and Hwang (1994), the vector normalization technique used by TOPSIS method may cause variation in the normalized values for different evaluation units. Further, the calculation of the school scores based on the without considering the relative importance of the best and worst solutions may lead to the scenario where the score of a school determined by

TOPSIS is not always closest to the ideal solution. For this reason, we also incorporate the VIKOR method to determine the scores of the schools to ensure more robustness.

3.4. Determining the score of a school using VIKOR method

In this sub-section, we demonstrate the application of VIKOR method for the performance assessment of the schools. VIKOR is considered as another useful MCDM techniques to assess the performance of alternatives under conflicting criteria. In a similar line with TOPSIS, it also measures the performance of an alternative through the closeness to the ideal solution. On the contrary, VIKOR incorporates linear normalization technique instead of vector normalization and considers the relative importance of the ideal solutions into the process. As per this approach, we first determine two merit scores of school j, i.e., $(s_j)_{VIK}$ and $(R_j)_{VIK}$ that captures the performance of a school based on maximization of the group utility and minimizing the individual regret, respectively. These scores are aggregated into a single score $(Q_j)_{VIK}$. Following the approach of Opricovic and Tzeng (2004), we assign equal weight (v) to each of the merit scores, i.e., 0.5. The method is described below:

3.4.1. Creation of normalized data matrix of the criteria for different parameters

First, we determine the solution $(x)^*_+$ and negative-ideal solution $(x)^*_-$ for criterion i as follows:

$$(x)^*_+ = \left\{ \left(\max_j x_j\left(f_i^{par}\right) / \left(i \in I^{'}\right), \right), \left(\min_j x_j\left(f_i^{par}\right) / \left(i \in I^{''}\right), \right) \right\}.$$
$$(x)^*_- = \left\{ \left(\min_j x_j\left(f_i^{par}\right) / \left(i \in I^{'}\right), \right), \left(\max_j x_j\left(f_i^{par}\right) / \left(i \in I^{''}\right), \right) \right\}.$$

Where $I^{'}$ and $I^{''}$ represents cost and benefit criteria, respectively.

3.4.2. Determining the merit scores of the school

Here, we calculate the values of two merit scores of school j, i.e., $(s_j)_{VIK}$ and $(R_j)_{VIK}$ in the following manner:

$$(S_j)_{VIK} = \sum_{i=1}^{k} \left[u\left(f_i^{par}\right) \left\{ \frac{(x)^*_+ - x_j\left(f_i^{par}\right)}{(x)^*_+ - (x)^*_-} \right\} \right], (R_j)_{VIK} = \max_i \left[u\left(f_i^{par}\right) \left\{ \frac{(x)^*_+ - x_j\left(f_i^{par}\right)}{(x)^*_+ - (x)^*_-} \right\} \right].$$

3.4.3. Determining the final score of a school and final compromise ranking

Here, we compute the values of $(Q_j)_{VIK}$. First, we define $(S)^*_+$, $(S)^*_-$, $(R)^*_+$, and $(R)^*_-$ in the following manner:

$$(S)^*_+ = \max_j \left(S_j\right)_{VIK},\ (S)^*_- = \min_j \left(S_j\right)_{VIK},\ (R)^*_+ = \max_j \left(R_j\right)_{VIK},\ (R)^*_- = \min_j \left(R_j\right)_{VIK}$$

Now, the values of $\left(Q_j\right)_{VIK}$ can be expressed as:

$$\left(Q_j\right)_{VIK} = v\left[\frac{\left(S_j\right)_{VIK} - (S)^*_-}{(S)^*_+ - (S)^*_-}\right] + (1-v)\left[\frac{\left(R_j\right)_{VIK} - (R)^*_-}{(R)^*_+ - (R)^*_-}\right]$$

Next, we prepare the ranking lists based on $\left(S_j\right)_{VIK}$, $\left(R_j\right)_{VIK}$, and $\left(Q_j\right)_{VIK}$ values. Here, lower value of $\left(Q_j\right)_{VIK}$ signifies the better performance of the school j. Now, the school j with minimum $\left(Q_j\right)_{VIK}$ will be considered as the best school if following conditions are satisfied:

- Condition 1: Acceptable Advantage:

Let school $j^{(2)}$ is the second-best school and its score is minimum $\left(Q_{j^{(2)}}\right)_{VIK}$. Now, following condition should be satisfied:

$$\left(Q_{j^{(2)}}\right)_{VIK} - \left(Q_j\right)_{VIK} \geq DQ,\ DQ = \frac{1}{(n-1)}.$$

- *Condition 2: Acceptable Stability in Decision Making*

The school j with minimum $\left(Q_j\right)_{VIK}$ should be best ranked in the lists prepared based on $\left(S_j\right)_{VIK}$ and $\left(R_j\right)_{VIK}$, i.e., should have minimum $\left(S_j\right)_{VIK}$ and $\left(R_j\right)_{VIK}$.

If any of the above-mentioned conditions is not satisfied, then a set of compromise solutions is proposed comprising

- All schools $j,\ j^{(2)}, \ldots, j^{(n)}$ will be considered as the best if first condition is not satisfied where $\left(Q_{j^{(L)}}\right)_{VIK} - \left(Q_j\right)_{VIK} < DQ,\ DQ = \dfrac{1}{(n-1)}$ for maximum L.
- Both schools j and $j^{(2)}$ will be considered as the best if the second condition is not fulfilled.

3.5. Designing the ranking list of the school based on integrated TOPSIS and VIKOR score

In this sub-section, we propose a ranking list based on the scores of the schools obtained from integrating TOPSIS and VIKOR scores. First, we apply the entropy concept to calculate the weight of scores obtained from TOPSIS and VIKOR and integrate them into a single score. Here, we propose two ranking lists; conservative and optimistic ranking method focussing on the maximization of the utility and the minimization of the regret, respectively. The main goal behind designing two lists is to investigate whether there is any difference when the objectives are different. The method is proposed below:

Let, weight of TOPSIS and VIKOR score can be represented as $(w)_{TOP}$ and $(w)_{VIK}$, respectively. The score of the school j as per TOPSIS and VIKOR can be expressed as $\left(\theta_j\right)_{TOP}$ and $\left(\theta_j\right)_{VIK}$, respectively where $\left(\theta_j\right)_{VIK} = \left(Q_j\right)_{VIK}$. Now, the score of school j as per conservative and optimistic ranking method, i.e., $\left(\theta_j\right)_{final}^{Cons}$ and $\left(\theta_j\right)_{final}^{opti}$, can be expressed as:

$$\left(\theta_j\right)_{final}^{Cons} = (w)_{TOP}\left(1-\left(\theta_j\right)_{TOP}\right) + (w)_{VIK}\left(\theta_j\right)_{VIK},$$

$$\left(\theta_j\right)_{final}^{opti} = (w)_{TOP}\left(\theta_j\right)_{TOP} + (w)_{VIK}\left(1-\left(\theta_j\right)_{VIK}\right),$$

$$(w)_{TOP} + (w)_{VIK} = 1.$$

The school with the maximum and the minimum score is ranked as the best school according to the conservative and optimistic ranking method, respectively. It signifies that the ranking list is prepared in descending and ascending order in case of conservative and optimistic method, respectively.

4. Numerical Analysis

In this section, we evaluate the performances of 82930 primary and upper primary schools that come under 20 districts of West Bengal, a state of India, using the proposed methodology. Department of primary and secondary education of West Bengal provides the data for the year 2014-15. Firstly, we present a description of state-level performance based on the scores of schools according to TOPSIS, VIKOR, Integrated (Optimistic), and Integrated (Conservative). Next, we extend our analysis to the district-level and present comparative analysis on the mean as well as standard deviations of the scores for all above-mentioned methods. Finally, we demonstrate the importance of Shannon entropy-based approach in our study.

4.1. State level performance

In this sub-section, we present a summarized description of the scores of schools under evaluation. The details are provided in Table 3. We observe that there exists a difference in scores obtained through different methods. The average score of the schools is slightly higher in case of VIKOR method than that of TOPSIS method. On the other hand, the average score of the schools is higher in case of the integrated (optimistic) method than the integrated (conservative) approach. It signifies that the average score is higher for the method with minimization objective, i.e., VIKOR. Within integrated methods, score is higher for the method with maximization objective, i.e., integrated (optimistic). It signifies that the integrated method demonstrates the ranking method in a more effective manner.

Table 3 State-level descriptive statistics of the scores of schools across the methods

	TOPSIS	VIKOR	Integrated (Optimistic)	Integrated (Conservative)
Mean	0.84421	0.84452	0.51431	0.50015
Standard Deviation	0.05695	0.05327	0.00419	0.00191
Best score	0.90802	0.62260	0.51823	0.49650
Worst score	0.62219	0.90574	0.50493	0.50311
Best school	Srikhola Junior Basic School (Darjeeling)	NAPO SSK (Paschim Medinipur)	Srikhola Junior Basic School (Darjeeling)	Adarsha Vidyapith (North twenty-four Pargana)
Lowest ranked school	NAPO SSK (Paschim Medinipur)	Srikhola Junior Basic School (Darjeeling)	NAPO SSK (Paschim Medinipur)	Gutinagori H.P School

4.2. District level performance

In this sub-section, we compare means and standard deviations of the scores, obtained using the above-mentioned methods, across the districts of West Bengal. We report summary of the results in Table 4. Our analysis reveals that Cooch Behar, Purba Medinipur, Howrah, and

Maldah have the highest average scores of the schools according to TOPSIS, VIKOR, integrated (optimistic), and integrated (conservative), respectively. Also, Purba Medinipur has the lowest score according to TOPSIS and integrated (optimistic) method whereas Cooch Behar and Kolkata obtain the lowest average score of the schools as per VIKOR and integrated (conservative), respectively. In similar fashion with state-level study, the ranking list provided by The TOPSIS method is completely reverse of the list proposed by VIKOR method and vice-versa. It indicates the difference in objective, i.e., maximization and minimization in case of TOPSIS and VIKOR, respectively, yields two different lists. Further, we find that TOPSIS method and integrated (optimistic) assigns same ranks 11 out of 20 districts. The comparative studies between TOPSIS and Integrated (Conservative), integrated (optimistic) and integrated (Conservative), TOPSIS and VIKOR, indicates the low percentage of same ranks, viz., 30%, 25%, and 10%, respectively. On the contrary, the ranking lists suggested by the integrated (optimistic) and integrated (conservative) are completely distinct. From the perspective of standard deviation, the highest variation in the scores of the schools has been observed in case of Darjeeling across the methods. Lowest variability is observed in case of Kolkata for TOPSIS, VIKOR, and integrated (optimistic). According to integrated (Conservative), the lowest variation is seen in case of Cooch Bihar. In summary, the four methodologies do not converge to provide a consistent set of results. Hence, we recognize the importance of an aggregation method by combining results from the four methods.

Table 4 Comparative studies on average and standard deviation of the scores across the districts for all methods.

	Mean score							
	TOPSIS	**Rank**	**VIKOR**	**Rank**	**Integrated (Optimistic)**	**Rank**	**Integrated (Conservative)**	**Rank**
Alipurduar	0.76871	16	0.7744	5	0.50856	16	0.50285	19
Bankura	0.87605	12	0.87396	9	0.51679	12	0.49896	12
Bardhaman	0.88332	8	0.88066	12	0.51737	9	0.49867	5
Birbhum	0.88774	7	0.88514	14	0.51753	6	0.4987	7
Dakshin Dinajpur	0.89683	1	0.89427	20	0.51789	3	0.49872	8

Darjeeling	0.89516	3	0.89242	18	0.51791	2	0.49863	3
Howrah	0.81666	14	0.81978	7	0.51181	14	0.50156	14
Hooghly	0.88331	9	0.8811	13	0.51716	11	0.49889	11
Jalpaiguri	0.89597	2	0.89327	19	0.51793	1	0.49865	4
Cooch Behar	0.88995	4	0.88707	17	0.51776	4	0.49856	2
Kolkata	0.77239	15	0.77822	6	0.50865	15	0.50291	20
Maldah	0.88304	10	0.88004	10	0.51752	7	0.4985	1
Murshidabad	0.88939	5	0.88673	16	0.51763	5	0.49867	5
Nadia	0.88783	6	0.88546	15	0.51742	8	0.49882	10
North 24 Parganas	0.88246	11	0.88005	11	0.51722	10	0.49879	9
Paschim Medinipur	0.76511	18	0.77067	3	0.50847	18	0.50278	17
Purba Medinipur	0.76144	20	0.76687	1	0.50838	20	0.50272	15
Purulia	0.86668	13	0.86535	8	0.51604	13	0.49933	13
Uttar Dinajpur	0.76388	19	0.7694	2	0.50844	19	0.50276	16

District	**Standard Deviation**							
	TOPSIS	**Rank**	**VIKOR**	**Rank**	**Integrated (Optimistic)**	**Rank**	**Integrated (Conservative)**	**Rank**
Alipurduar	0.00666	16	0.0069	16	0.00016	16	0.00012	16
Bankura	0.01191	4	0.01217	4	0.00038	3	0.00014	15
Bardhaman	0.01119	5	0.01159	5	0.00028	5	0.0002	3
Birbhum	0.0105	6	0.01089	6	0.00025	8	0.0002	3
Cooch Behar	0.00456	19	0.00471	19	0.00012	19	0.00007	20
Dakshin Dinajpur	0.00612	17	0.00634	17	0.00015	17	0.00011	17
Darjeeling	0.04151	1	0.03828	1	0.00329	1	0.00162	1
Hooghly	0.01011	10	0.0105	10	0.00024	10	0.00019	5
Howrah	0.0092	12	0.00953	12	0.00023	11	0.00017	9

Jalpaiguri	0.0081	15	0.00839	15	0.0002	14	0.00015	13
Kolkata	0.00434	20	0.00449	20	0.00011	20	0.00008	19
Maldah	0.00592	18	0.00612	18	0.00015	17	0.0001	18
Murshidabad	0.01033	8	0.0107	8	0.00026	6	0.00018	7
Nadia	0.00826	14	0.00856	14	0.0002	14	0.00015	13
North 24 Parganas	0.01296	2	0.01344	2	0.00032	4	0.00024	2
Paschim Medinipur	0.00933	11	0.00966	11	0.00023	11	0.00017	9
Purba Medinipur	0.00863	13	0.00894	13	0.00021	13	0.00016	12
Purulia	0.01283	3	0.01306	3	0.00045	2	0.00017	9
Uttar Dinajpur	0.01022	9	0.01058	9	0.00025	8	0.00018	7
South 24 Parganas	0.01041	7	0.01079	7	0.00026	6	0.00019	5

4.3. Importance of Shannon-entropy based weighting technique.

In this sub-section, we demonstrate the importance of Shannon entropy concept in weighting technique. The details related to the weights of criteria, parameter, and methods are presented in Table 5. Unlike the equal weighting method, the weight in the Shannon-entropy concept varies as it is assigned based on the variation in the values under that criterion. For this reason, Shannon-entropy based weighting helps to capture the importance of the criteria in a more effective way than equal weighting method. To determine the score of the parameter physical infrastructure, benefit criterion ‘number of computers in library,’ and cost criteria such as ‘number of classrooms that require major repair and minor repair’ emerge as the most important factors. For the parameter school management, ‘number of working days’, ‘amount of teacher learning material receipts’, and ‘amount of teacher learning material expenditure’ act as influential factors. For the parameter quality of education, ‘the number of children stay in the school’, ‘the number of students provided special training for both primary and upper primary’ play critical roles. While equal weightage is assigned to all parameters under TOPSIS, VIKOR method emphasizes more on parameters such as physical infrastructure and quality of education by giving them higher weightage than school management. The final integrated scores obtained for optimistic and conservative scenarios also rely on assigning different weightages on TOPSIS and VIKOR methods.

Table 5 Description of weights of criteria, parameter, and methods under Shannon entropy concept

Parameters	**Criteria**	**Weights**
Physical infrastructure	Distance from block head quarters	0.026
	Distance from cluster resource center	0.066
	Total classrooms used for instructional purposes	0.003
	Number of classrooms in good condition	0.045
	Number of classrooms that require major repair	0.208
	Number of classrooms that require minor repair	0.225
	Number of toilet seats constructed/available for boys	0.038
	Number of toilet seats constructed/available for girls	0.026
	Number of books in library	0.083
	Number of computers in library	0.280
School management	Number of pre-primary students	0.052
	Number of working days	0.161
	Number of inspections	0.083
	Pre-primary teachers	0.130
	Number of visits by block resource center officer	0.124
	Number of visits by cluster resource center officer	0.070
	Amount of school development grant receipt	0.036
	Amount of school development grant expenditure	0.044
	Amount of teacher learning material receipts	0.149
	Amount of teacher learning material expenditure	0.151
Quality of education	Total number of male teachers	0.016
	Total number of female teachers	0.020
	Total number of head teachers in schools	0.034
	Total number of teachers graduate and above	0.019
	Total number of teachers with professional qualification	0.025
	Total number of working days spent to non-teaching	0.065
	Total Number of teachers involved in (non-teaching	0.065
	Number of instructional days (previous year)- primary	0.015

	Number of instructional days (previous year)- upper primary	0.063
	Teacher working hours (per day) –primary	0.074
	Number of hours Children stay in school (current year)- upper primary	0.075
	Teacher working hours (per day) –upper primary	0.075
	No. of children enrolled special training in current year – boys	0.075
	No. of children enrolled special training in current year– girls	0.077
	No. of children provided special training in current year – boys	0.077
	No. of children provided special training in current year– girls	0.075
	No. of children enrolled special training in previous year – boys	0.076

MCDM	Parameters	Weight
TOPSIS	Physical Infrastructure	0.333
	School Management	0.333
	Teacher Quality	0.333
VIKOR	Physical Infrastructure	0.345
	School Management	0.310
	Teacher Quality	0.345
Integration	**MCDM Methods**	**Weight**
Integrated (Optimistic)	TOPSIS	0.521
	VIKOR	0.479
Integrated (Conservative)	TOPSIS	0.478
	VIKOR	0.522

5. Discussion

In Section 4, we evaluate the input-level performance of the school using entropy integrated VIKOR-TOPSIS method. Here, we investigate the effect of this input-level performance on the school's output-level performance, i.e., student outcome. We also inquire whether the other relevant factors such as the medium of education and location of school have a substantial impact on the student outcome or not. We formulate hypotheses and investigate through multiple linear regression analysis.

5.1. Hypothesis Development

Here, we measure student outcome of a school using its pass rate for both boys and girls. Several scholars such as Umashankar and Dutta (2007), Lavy (2009), Kassile (2014), have shown context-specific implementation of pass rate as a metric of performance evaluation. The data provided by the department of education reveals that 15931 and 14489 schools among the 82930 schools are not able to achieve the 100% pass rate for boys and girls, respectively. Also, around 1% of these schools exhibit a low pass rate, i.e., less than 70%. It signifies the variation in the output-level performance of the schools.

Existing scholarly works (Heugh, 1999; Mitra et al., 2008) indicate that medium or the language of instruction, i.e., vernacular or others (mostly English) as well as location of the school, i.e., urban or rural play instrumental roles in students' performance. For this reason, we study the impact of school performance along with two contextual factors i.e., medium of instruction, and location of the school on students' performance.

Branham (2004) concludes that school infrastructure plays a crucial role in increasing student attendance as well as student performance. Duflo (2001) opines that enrolment as well as test scores are directly proportional to the performance of the school in different parameters. Several other researchers e.g., Hallack (1990), Ajayi (2002) etc. investigate on the availability of infrastructural facilities along with their effect on the student and the school performance. Thus, we propose the following hypotheses:

H1a. The final school score positively influences the overall pass rate of boys of the school.

H1b. The final school score positively influences the overall pass rate of girls of the school.

Location of the school is another crucial factor from the perspective of a student's performance. Due to the better access to the resources and other facilities, it seems students from the school of urban region exhibits better performance than that of rural areas. Supporting this claim, Nigeria, Owoeye and Yara (2011) opine that students from urban area are better performers than their rural counterparts. In a similar way Mitra et al. (2008) conduct a study on north Indian schools and conclude that students' performance deteriorates with the increasing distance of the school from the urban area. However, scholars are divided on this issue. Researchers such as Cotton (1996), Reeves and Bylund (2005) infer that students from the schools of the rural area often show same or better performance than the students of the urban area. Thus, it leads to the following hypotheses:

H2a. Location of a school significantly influences its overall pass rate of boys**.**

H2b. Location of a school significantly influences its overall pass rate of girls.

Medium of instruction remains an important factor in students' learning. A group of experts identifies that vernacular medium of instruction facilitates the students learning in an easier and effective way, whereas other experts suggest that students should adopt bilingual mode where instruction though the English language is preferable. In the context of Africa, Heugh (1999) argues the importance of incorporation of African languages in their medium of instruction. Khan (2017) discusses how appropriate medium of instruction is crucial in facilitating a student's learning for the rural schools of India. Hence, we propose the following hypotheses.

H3a. Medium of instruction of a school significantly influences its pass rate of boys.

H3b. Medium of instruction of a school significantly influences its pass rate of girls.

Figure 2 depicts the proposed hypotheses, i.e., H1-H3.

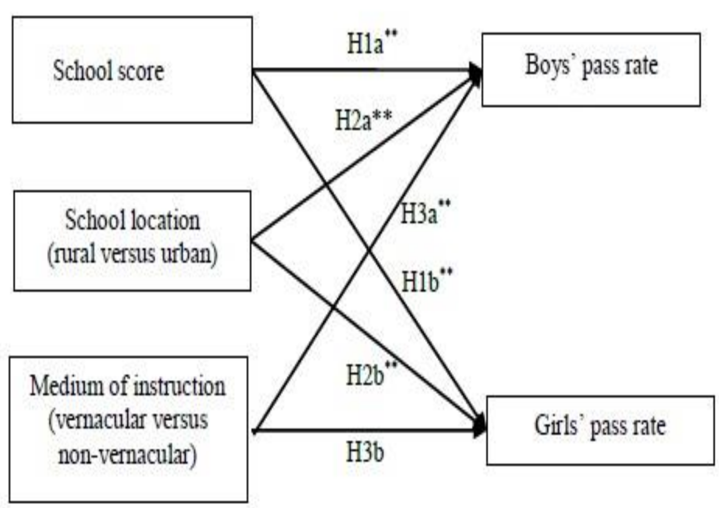

Figure 2 Impact of school score, medium of instruction, and school location on student performance

5.2. Analyses and Results

Here, we apply a multiple linear regression (MLR) analysis to study the impact of school score, medium of instruction, and school location on student performance. We incorporate a log transformation of the decision variables, viz., boys' pass rate and girls' pass rate to adjust for the skewness (Cameron and Trivedi, 2013). We also control for the medium of instruction and location of the school by including dummy variables representing 'medium of instruction' and 'school location,' respectively. The medium of instruction takes value 1 for vernacular medium and 2 for others. Similarly, the schools located at rural and urban rea are represented by 1 and 2, respectively. The regression equations given below depict our conceptual model. Hypotheses 1a, 2a, and 3a are tested using equation 1, whereas equations 2 tests hypotheses 1b, 2b, and 3b. We also investigate the correlation of independent variables and find no significant multicollinearity among them (Kumar, 1975).

Regression Equation 1

$$\log\,(boys\;\;pass\;rate) = \alpha_0 + \alpha_1 * (School\;location) + \alpha_2 * (Med\;of\;\;instruction) + \alpha_3 * (school\,score)_{Method} + \in$$

Regression Equation 2

$$\log\,(girls\;\;pass\;rate) = \alpha_0 + \alpha_1 * (School\;location) + \alpha_2 * (Med\;of\;\;instruction) + \alpha_3 * (school\,score)_{Method} + \in$$
$$Method \in \left[TOPSIS,\;VIKOR,\,Integrated\;\;(Optimistic),\,Integrated\;\;(Conservative)\right]$$

The results obtained from models 1and 2 indicate that hypotheses H1a, H2a, H3a, and H1b, and H2b are supported, whereas H3b is not supported in case of any of the methods. It implies that school location and school score emerge as significant factors for both boys' pass rate and girls' pass rate. However, medium of instruction is significant only for boys' pass rate, and does not have any significant impact on girls' pass rate. Further, we observe that school score is positive for both TOSPSIS and integrated optimistic method. On the other hand, school score is negative for VIKOR and integrated conservative method. As minimization is preferred in case of VIKOR and integrated conservative method, the coefficient for the school score is negative. The coefficients of school location and medium of instruction are positive for all the methods.

Our analysis yields several interesting insights. First, better input level performance of a school results in higher pass rate of boys and girls. It highlights the importance of improved physical infrastructure, school management, and quality of education on the student outcome. Second, pass rate of boys and girls of urban schools is higher than that of rural schools. In a similar line with Othman and Muijs (2013) and Opoku-Asare (2015), studies conducted in the context of developing countries such as Malaysia and Ghana, respectively, we can conclude that better infrastructure, adequate resources, higher student-teacher ratio, and other facilities of urban schools often play an instrumental role to improve students' performance. Third, from the perspective of medium of instruction, pass rate of boys is higher for non-vernacular languages as compared to the vernacular language (Bengali in our case). The non-vernacular medium of instruction often compels the students to be bilingual, which according to extant research positively affects the cognitive development of a child as compared to their monolingual counterparts (Ben-Zeev 1977), thus positively influencing their pass rate. Interestingly, medium of instruction does not have any significant impact on the pass rate of girls. It implies overall performance for girls is similar for schools with vernacular and non-vernacular medium of instruction. Marks (2008) and Eriksson et al. (2012) opine that girls exhibit better proficiency in learning languages compared to boys. Similarly, in the context of West Bengal secondary education, Gupta (2010) conclude that girls perform better than boys in the subjects related to languages. Thus, we can infer that the medium of instruction does not act as a hindrance to girls' learning.

Table 6 Results of Hypothesis Testing (Results H1-H3)

Method	**Independent Variables**	**Model 1 (Dependent variable: boys pass rate)**	**Model 2 (Dependent variable: girls pass rate)**
Hypothesis		H1a, H2a, H3a	H1b, H2b, H3b
TOPSIS	TOPSIS Score of school	0.015044**	0.01661**
	School Location	0.004403**	0.003698**
	Medium of instruction	0.00003**	0.00002
	R-square	0.00096	0.00083
	Adjusted R-square	0.00093	0.00079
VIKOR	VIKOR Score of school	-0.01596**	-0.017664**
	School Location	0.00439**	0.003692**
	Medium of instruction	0.00003**	0.00002
	R-square	0.00096	0.00082
	Adjusted R-square	0.00092	0.00079
Integrated (optimistic)	Integrated (optimistic) Score of school	0.21274**	0.23114**
	School Location	0.00444**	0.00373**
	Medium of instruction	0.00003**	0.00002

	R-square	0.00098	0.00084
	Adjusted R-square	0.00095	0.00081
Integrated (conservative)	Integrated (conservative) Score of school	-0.47856**	-0.5097**
	School Location	0.004481**	0.003779**
	Medium of instruction	0.00003**	0.00002
	R-square	0.001	0.00085
	Adjusted R-square	0.00096	0.00081

6. Conclusion

In the context of primary and secondary education, performance monitoring is an important issue around the world. Especially, designing a robust performance assessment methodology remains a matter of concern for developing countries such as India. Motivated by these examples, we devise an integrated VIKOR-TOPSIS-Regression based framework to evaluate the input-level performance of 82930 primary and secondary schools of West Bengal, a state of India, and to explore the impact of this performance along with the medium of instruction and location of the school on the school output level performance, i.e., student outcome. To evaluate the performance of the schools, we select two prevalent MCDM methods, viz., VIKOR and TOPSIS due to their intrinsic advantages in the presence of conflicting decision-making criteria. For each of the methods, first, we determine the score of a school in each of the input parameters, i.e., infrastructure, school management, and quality of education. The weights of the criteria under a parameter are determined using Shannon entropy-based approach. Then, we aggregate these parameter scores into a single score for each of the methods using the same entropy concept. After obtaining the scores of a school as per VIKOR and TOPSIS method, we integrate them into a final score following the same approach mentioned earlier. For the sake of holistic performance measurement, we design both conservative as well as optimistic integrated MCDM methodology. We perform a rigorous comparative analysis on district-level as well as state-level performance across the methods. Finally, we investigate the impact of the input-level performance of the schools, medium of instruction, and location of the school on the student pass rate separately for boys and girls using multiple linear regression analysis. Through the hypotheses development, we conclude that all the factors exhibit a significant impact on boys' pass rate whereas only input-level performance of the school and location of the school have a significant influence on the girls' pass rate.

From the perspective of future research avenues, department of primary and secondary education of West Bengal provides the data only for the year 2014-15. If researchers use the dataset for multiple years, it can be useful to determine inequality of school scores through Gini index and to investigate improvement or deterioration of the school-level performance of the districts. Also, incorporation of multiple outputs apart from student outcome can hint at the other policy-level recommendations.

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