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Implied Volatility and Predictability of GARCH Models

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ABSTRACT: We have examined the predictive power of GARCH model to forecast return volatility for Nifty 50 index. Realized volatility, which is the sum of intraday squared returns, is used as the proxy for the true volatility. Three models of the GARCH family have been used to forecast return volatility i.e., GARCH, GJR-GARCH and EGARCH along with their implied volatility (IV) augmented counterparts i.e., GARCH IV, GJR-GARCH IV and EGARCH IV. Implied Volatility forecasting has been done using AR, MA, ARMA, ARIMA and Random Walk. But GARCH model augmented with implied volatility perform better than GARCH models without augmentation or implied volatility alone. Forecasting performance of the competing models is judged by using mean absolute error (MAE) and root mean squared error (RMSE). MAE and RMSE show that GARCH IV model is best suited for the volatility forecasting in the context of Nifty 50 index.

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INTRODUCTION

Estimation and forecast of volatility plays a very important role in finance as it is used as a proxy for the risk. It is used as an input for the option pricing, margin setting and risk management techniques. Forecasting the movement of stock market indices is of prime importance for investors and several models have been developed to achieve the same. Return volatility is defined as the fluctuation or variations associated with the stock returns. A number of factors are responsible for these fluctuations and arrival of relevant information is the cause of fluctuations in the prices of the assets. Uncertainty about the relevant information makes the investments riskier. Risk is an integral part of any financial investment and to gauge the risks effectively, we need to accurately estimate the volatility in returns from assets.

LITERATURE REVIEW

To estimate the return volatility, several models has been discussed in the finance literature. The autoregressive conditional heteroskedastic (ARCH) models developed by Engels (1982) and the generalized ARCH (GARCH) models by Bollerslev (1986) have proven to be quite effective in predicting volatility. Major studies (Akgiray, 1989; West et al., 1993) have shown that volatility predictions by GARCH-type models are more accurate than Moving Averages or Exponentially Weighted Moving Averages (EWMA). Christensen and Prabhala (1998) shows that future volatility can be accurately forecasted by implied volatility using S&P 100 index options. Blair, Poon, and Taylor (2001) show that VIX (implied volatility index) provides better forecasts than GARCH models as the time span to forecast increases.

Andersen, Bollerslev, Diebold and Labys (2003) established a link between realized volatility and conditional covariance matrix. GARCH models are successful in capturing the characteristics of the return distribution. However, for forecasting return volatility, these models are not very successful. The information contained VIX may be used to improve the predictability of GARCH models. In this paper, we have tested the predictability of the GARCH models after augmenting VIX. Our findings show that predictability of the GARCH model improves after the augmentation of VIX.

DATA

In this paper, we have taken daily and intraday data of the Nifty 50 index and VIX index Jan 2011 to December 2015. We used data from 1st January 2011 to 31st December 2014 as in-sample period and 1st January, 2015 to 31st December 2015 as out-of-sample period. Both the Nifty index and VIX have been obtained from NSE India, while the realized variance data has been taken from Oxford-Man Institute's Realized Library. Descriptive statistics given in table below show that daily returns are close to 0 with a negative skewness and kurtosis greater than 3, indicating a left skewed leptokurtic distribution. Jarque-Bera test shows that daily returns are and normal. Descriptive statistics shows the presence of fat tails and non-normality in returns which is a similar to the findings in financial literature. Augmented Dickey Fuller test reject the null hypothesis of the presence of unit-root in daily returns and VIX. Thus, the daily return series is not normal and return distribution is stationary.

Table 1 : Descriptive Statistics

	Daily Returns	VIX
Mean	0.000545	0.183444
Median	0.000741	0.170400
SD	0.009875	0.044519
Max	0.037380	0.377050
Min	-0.060973	0.115650
Skewness	-0.346732	1.313351
Kurtosis	5.425860	4.667420
JB Test	262.8499343	399.697243
JB Test p-value	0.0000	0.0000
ADF Test	-29.27994	-4.13119
ADF Test p-value	0.0000	0.0009

VOLATILITY FORECASTING MODELS

GARCH

The GARCH model developed by Bollerslev (1986) and Engle (1982) involves simultaneous estimation of the mean and variance equations as follows:

The return process follows the mean equation:

$r_t = \mu + \varepsilon_t$

where μ is the constant mean and $\varepsilon_t = h_t z_t$ is the innovation with $z_t \sim N(0,1)$

The variance equation: $h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1}$ (1)

GARCH model has been very successful in estimating and forecasting return volatility and capturing the stylized facts, such as long memory, of return volatility.

With IV augmented, the equation becomes: $h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \theta\, IV^2_{t-1}$ (2)

GJR-GARCH

GJR-GARCH model, proposed by Glosten, Jagannathan, and Runkle (1993), takes into account the leverage effect along with long memory.

The variance equation:

$h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \gamma\, \varepsilon^2_{t-1} I_{t-1}$ (3)

With IV augmented, the variance equation becomes

$h_t^2 = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \beta_1 h^2_{t-1} + \gamma\, \varepsilon^2_{t-1} I_{t-1} + \theta\, IV^2_{t-1}$ (4)

Here, the leverage effect is captured by γ, such that, $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $I_{t-1} = 0$ if $\varepsilon_{t-1} > 0$.

EGARCH

EGARCH model, proposed by Nelson (1991) captures the leverage effect as well with the long memory property of the return volatility.

The variance equation:

$\ln(h_t^2) = \alpha_0 + \alpha_1 \mid \varepsilon_{t-1} \mid / h_{t-1} + \gamma\, \varepsilon_{t-1} / h_{t-1} + \beta_1 \ln(h^2_{t-1})$ (5)

With IV augmented, the equation becomes

$\ln(h_t^2) = \alpha_0 + \alpha_1 \mid \varepsilon_{t-1} \mid / h_{t-1} + \gamma\, \varepsilon_{t-1} / h_{t-1} + \beta_1 \ln(h^2_{t-1}) + \theta\, IV^2_{t-1}$ (6)

Where the coefficient γ captures the presence of the leverage effects if $\gamma < 0$.

Implied Volatility and Realized Volatility

Here we want to forecast the actual volatility using implied volatility and realized volatility. In order to investigate whether the IV index model forecast or RV forecast will be more accurate than the GARCH type models, AR, MA, ARMA, ARIMA and Random Walk models are going to be used.

India VIX is a volatility index computed by NSE based on the order book of NIFTY Options. For this, the best bid-ask quotes of near and next-month NIFTY options contracts which are traded on the F&O segment of NSE are used. India VIX indicates the investor's perception of the market's volatility in the near term i.e. it depicts the expected market volatility over the next 30 calendar days.

Daily implied volatility is obtained from VIX index using the formula VIX/100/sqrt(250).

Realized Variance is the sum of 5-minute intraday squared returns. It is calculated using the formula $\sigma_t^2 = \Sigma\, r^2_{t,j}$ where $r_{t,j}$ is the return in interval j on day t

Correlogram Test for Implied Volatility

```
        Autocorrelation                 Partial Correlation
=====================================================================
      |*******                        |*******
      |*******                        |      |
      |*******                        |*     |
      |******|                        |      |
      |******|                        |      |
      |******|                        |      |
      |******|                        |      |
      |******|                        |      |
      |***** |                        |      |
      |***** |                        |      |
```

Correlogram Test for Realized Volatility

Autocorrelation	Partial Correlation
|***		|***	
|**		|*	
|**		|*	
|**		|*	
|**		|*	
|**		|*	
|**				
|**				
|**				
|**				

From the correlogram test, we observe that both implied volatility and realized volatility series have a large partial correlation at AR(1).

So, for each IV and RV series, AR(1), MA(1), ARMA(1,1) and ARIMA(1,1,1) models are used.

The mean equations of the 5 models are as follows:

AR Model

$$IV_t = c_0 + \phi_1 IV_{t-1} + \varepsilon_t \quad (7)$$

$$RV_t = c_0 + \phi_1 RV_{t-1} + \varepsilon_t \quad (8)$$

MA Model

$$IV_t = c_0 + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (9)$$

$$RV_t = c_0 + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (10)$$

ARMA Model

$$IV_t = c_0 + \phi_1 IV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (11)$$

$$RV_t = c_0 + \phi_1 RV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (12)$$

ARIMA Model

A generalization of the ARMA models is the autoregressive integrated moving average (ARIMA) model. It is usually denoted as ARIMA (p, d, q) and is employed to capture the possible presence of short memory features in the dynamics of implied volatility. The ARIMA (1,1,1) specification is given by

$$\Delta IV_t = c_0 + \phi_1 \Delta IV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (13)$$

$$\Delta RV_t = c_0 + \phi_1 \Delta RV_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (14)$$

Random Walk

$$IV_t = IV_{t-1} + \varepsilon_t \quad (15)$$

$$RV_t = RV_{t-1} + \varepsilon_t \quad (16)$$

IN-SAMPLE RESULTS

As mentioned, the in-sample period is from 1st January 2012 to 31st December 2014.

Table 2: Estimation Output of GARCH models

	GARCH	GJR GARCH	EGARCH
α_0	1.40E-06 (0.0485)	1.94E-06 (0.0046)	-0.203765 (0.0017)
α_1	0.038811 (0.0011)	-0.004671 (0.6763)	0.069476 (0.0011)
β_1	0.944393 (0.0000)	0.942231 (0.0000)	0.983968 (0.0000)
γ		0.080098 (0.0001)	-0.066159 (0.0000)
Log-Likelihood	2422.352	2430.868	2428.474

(Values in brackets indicate the p-values)

The constant term α_0 is statistically significant at the 5% level for all the three GARCH specifications. α_1, β_1 and γ are statistically significant at the 1% level except for α_1 in GJR GARCH.

Table 3: Estimation Output of GARCH IV models

	GARCH IV	GJR GARCH IV	EGARCH IV
α_0	1.20E-06 (0.2509)	9.64E-07 (0.2197)	-0.395493 (0.0003)
α_1	0.043593 (0.0068)	-0.033805 (0.0067)	0.048884 (0.0403)
β_1	0.907242 (0.0000)	0.927462 (0.0000)	0.964916 (0.0000)
γ		0.121604 (0.0000)	-0.099616 (0.0000)
θ	0.022255 (0.0744)	0.022445 (0.0162)	196.0586 (0.0004)
Log-Likelihood	2425.053	2438.24	2435.457

(Values in brackets indicate the p-values)

The constant term α_0 is not statistically significant except for EGARCH IV where it is significant at the 1% level. α_1, β_1 and γ are statistically significant at the 1% level for all the GARCH specifications, except for α_1 in EGARCH IV, where it is significant at the 5% level. All the IV augmented GARCH specifications have a higher log-likelihood than their restrictive counterparts, indicating that lagged IV terms contain some extra information useful for forecasting conditional variance.

Table 4: Estimation Output of Implied Volatility

	AR	MA	ARMA	ARIMA
c_0	0.011366 (0.0000)	0.011751 (0.0000)	0.011379 (0.0000)	-9.82E-06 (0.7053)
ϕ_1	0.973101 (0.0000)		0.972083 (0.0000)	-0.645865 (0.0059)
θ_1		0.866007 (0.0000)	0.020426 (0.5878)	0.706264 (0.0012)

The constant term c_0 is statistically significant at the 1% except for ARIMA where it is insignificant. ϕ_1 and θ_1 are statistically significant at the 1% except for θ_1 in ARMA.

Table 5: Estimation Output of Realized Volatility

	AR	MA	ARMA	ARIMA
c_0	0.00742 (0.0000)	0.007427 (0.0000)	0.007331 (0.0000)	-3.23E-06 (0.8102)
ϕ_1	0.427376 (0.0000)		0.961977 (0.0000)	0.13047 (0.0026)
θ_1		0.311523 (0.0000)	-0.794037 (0.5878)	-0.88091 (0.0000)

Like implied volatility, here also the constant term c_0 is statistically significant at the 1% except for ARIMA where it is insignificant. ϕ_1 and θ_1 are statistically significant at the 1% except for θ_1 in ARMA.

FORECAST EVALUATION

The out-of-sample period is from 1st January 2015 to 31st December 2015. We have used static forecasting method for all the models. Since volatility is latent, realized volatility has been assumed to be actual volatility. In static forecasting, the estimated parameters remain fixed throughout. It takes into account actual values to make one-step ahead forecast. For example, if we are standing t=T, we will use the actual value at t=T to forecast for t=T+1. Then, we will use the actual value at t=T+1 to forecast for t=T+2 and so on. We have calculated MAE and RMSE for all the forecasting outputs. MAE and RMSE have been calculated as follows:

$$\text{MAE} = \frac{1}{\tau} \sum_{t=T+1}^{T+\tau} | h_t - \sigma_t |$$

$$\text{RMSE} = \sqrt{\left\{\frac{1}{\tau} \sum_{t=T+1}^{T+\tau} (h_t - \sigma_t)^2\right\}}$$

Here, h_t is GARCH volatility, implied volatility or realized volatility forecast. The benchmark σ_t is always the realized volatility. τ is the number of out-of-sample observations.

Table 6: GARCH Models

	MAE	RMSE
GARCH	0.003048636	0.003503677
GARCH IV	0.002790281	0.003240706
GJR GARCH	0.003693603	0.004198322
GJR GARCH IV	0.003511671	0.004059830
EGARCH	0.003463845	0.003909024
EGARCH IV	0.003513606	0.004062078

Among the restrictive specifications, GARCH outperforms GJR-GARCH AND EGARCH. Among the IV augmented specifications, GARCH IV has the lowest MAE and RMSE. Overall, GARCH IV is the best predictor.

Table 7: Implied Volatility

	MAE	RMSE
AR	0.004108810	0.004508168
MA	0.004384109	0.004728894
ARMA	0.004107242	0.004507025
ARIMA	0.002244576	0.003046050
Random Walk	0.007396625	0.007867259

Here ARIMA model is significantly better than the others when we are using implied volatility to predict actual volatility. The Random Walk performs poorly in this case.

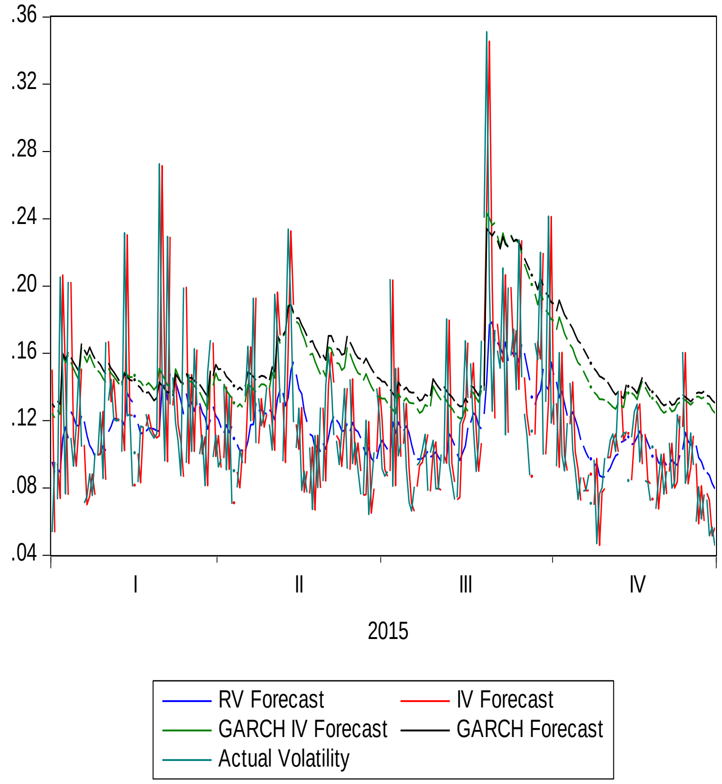
Table 8: Realized Volatility

	MAE	RMSE
AR	0.001813105	0.002507961
MA	0.001867198	0.002562000
ARMA	0.001777585	0.002439231
ARIMA	0.001789978	0.002455826
Random Walk	0.002251863	0.003066360

As depicted in table above ARMA model is seems to be best predictor when we are using realized volatility to forecast.

Overall, among GARCH volatility, implied volatility and realized volatility, realized volatility, the best predictor is realized volatility using ARMA Model.

In the following diagram, we have chosen the best method from each of GARCH volatility, implied volatility and realized volatility and compared the forecasts in the out-of-sample period with the actual volatility.



CONCLUSION

This paper provides a comparative evaluation of the ability of a range of GARCH, IV and RV models to forecast the Nifty 50 return volatility. A total of six GARCH models have been considered, i.e., GARCH, GJR GARCH, EGARCH, GARCH IV, GJR GARCH IV, EGARCH IV. Additionally, AR, MA, ARMA, ARMA and Random Walk Models have been used for forecasting with implied volatility and realized volatility.

ARIMA performs the best when we are analysing the forecasting ability of IV. In case of RV, ARMA performs the best. As for the GARCH models, the inclusion of IV in the GARCH variance equations improves the out-of-sample performance of the GARCH models.

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