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End-of-Season Demand**

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Extension to the Newsvendor Problem with an Exogenous and Stochastic End-of-Season Demand

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Abstract

Newsvendor problems, which have attracted the attention of researchers since 1950's, have wide applications in various industries. There have been many extensions to the standard single-period newsvendor problem. In this paper, we consider the single-period, single-item and single-stage newsvendor problem under random end-of-season demand, and develop a model to determine the optimal order quantity and expected profit. We prove that the optimal order quantity and expected profit thus obtained are lower than their respective values obtained from the standard newsvendor formulation. We also provide numerical examples and perform sensitivity analyses to compute the extent of deviations of the 'true' optimal solutions from the newsvendor solutions. We observe that the deviations are most sensitive to the ratio of the means of the demand distributions. The deviations are also found sensitive to the contribution margin, salvage price, coefficients of variation of the demand distributions and correlation between seasonal and end-of-season demands. We provide broad guidelines for managers as to when the model developed in this paper should be used and when the standard newsvendor formulation would suffice to determine the order quantity. Finally, we present the concluding remarks and directions for future research.

Keywords: Newsvendor problem; end-of-season demand distribution; optimization; stochastic programming

1. Introduction

Newsvendor models, which have been studied for many decades since Whitin (1955), have wide applications in various industries. For example, for inventory control for seasonal and perishable items, and capacity planning for revenue/yield management in services such as hotels and airlines, the application of newsvendor models is observed. There are many variants of the standard newsvendor problem. For a review of the various extensions of the problem, readers are referred to Silver et al (1998), Khouja (1999), Petruzzi and Dada (1999), and Qin et al (2011). In this paper, we focus on the single-period, single-item and single-stage newsvendor problem, such as the sale of a seasonal item by a retailer, which we call the standard newsvendor problem. In this problem, the order for the seasonal item is placed

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before demand is realized and any item left over at the end of the selling season is disposed of at a pre-determined salvage price. The objective of the problem is to maximize the expected profit (or minimize the expected loss) to determine the optimal order quantity. Suppose X is the random variable representing seasonal demand with $f_x(x)$ and $F_x(x)$ as the density and distribution functions, respectively. Also, suppose p , c and v ($v < c < p$) represent the selling price per unit, purchase cost per unit and salvage price per unit, respectively, of the item. Then it is well-known that the optimal order quantity (S) can be determined from the first-order condition: $F_x(S) = \frac{k_u}{k_u + k_o}$ where $k_u (= p - c)$ and $k_o (= c - v)$ represent the understocking and overstocking costs, respectively, which leads to $F_x(S) = \frac{p - c}{p - v}$.

One critical assumption in the standard newsvendor problem is that the end-of-season demand is unlimited and any item left over at the end of the season can be disposed of at a pre-determined salvage price. Therefore, as such there is no need to estimate the end-of-season demand. Such models would work well in situations where one can ensure that the realized end-of-season demand would always exceed the leftover inventory at the end of the season. However, as Gallego (1995) noted, if the probability of the end-of-season demand being lower than the leftover inventory is non-negligible, the solution provided by the standard newsvendor formulation may be sub-optimal. In such a situation, it would be more realistic and appropriate to estimate the end-of-season demand distribution and incorporate it in the model formulation to determine the optimal order quantity and expected profit. In the era of quick obsolescence of products and rapidly changing customer preferences, the assumption that the excess inventory can anyway be disposed of at the pre-determined salvage price may no longer hold, especially for items, such as fashion-wear, that customers perceive to define their unique identity. This is evidenced in the increasing leftover inventory and end-of-season discount sale as reported in the newspapers (See, for example, Cohn, 2016). Also, the sales figures in the normal season may not pick up for various reasons, thereby increasing the leftover inventory at the end of the selling season. For example, after an unusual warm winter in 2015-2016 in India, retailers of winterwear were disappointed by end-of-season sales, which were drastically lower than the corresponding figures for last year (The Economic Times, 2016).

In this paper, we have extended the standard newsvendor problem with random end-of-season demand. We have derived the expression for the expected profit and from it determined the expression for the optimal order quantity. Next, we have performed a sensitivity analysis to assess the extent of deviations of the optimal expected profit and order quantity under random end-of-season demand from the corresponding solutions given by the standard newsvendor formulation. The sensitivity analysis provides us with many key managerial insights with respect to deciding on the appropriate order quantity, i.e. whether the end-of-season demand should be taken into account while deciding on the order quantity or the standard newsvendor order quantity would suffice. The following, we feel, are the major contributions and findings of this paper:

1. There has been research on the estimation of the demand distribution and its parameters (Alwan et al, 2016), and decision-making in the absence of information on demand distribution, i.e. the so-called distribution-free newsvendor problem with only the estimated mean and variance (Gallego and Moon, 1993; Moon and Choi, 1995). Other research includes studies on the risk-averse/loss-averse behaviour of the newsvendor (Wang et al, 2009; Ma et al, 2016; Arıkan and Fichtinger, 2017), experimental studies on the order quantity decision-making process (Schweitzer and Cachon, 2000; Benzion et al, 2008), sensitivity of the order quantity decisions to the parameter estimates (Khanra et al, 2014), and scope of having an emergency supply option (Khouja, 1996). However, to the best of the authors' knowledge, newsvendor problems under a general continuous end-of-season demand distribution have not been considered in the literature so far (Karakul (2008) considered a general discrete end-of-season demand distribution for a newsvendor problem where the seasonal demand distribution is endogenous and stochastic, i.e. the newsvendor is a monopolist and a price-setter. However, in this paper, we have assumed that the newsvendor operates in a perfectly competitive market and, therefore, is a price-taker. The seasonal demand distribution, as such, is exogenous and stochastic). When the probability of the end-of-season demand being lower than the leftover inventory is non-negligible, using the order quantity given by the standard newsvendor formulation without specifically taking into account the end-of-season demand distribution may lead to significant deviations from the 'true' optimal solutions in terms of the expected profit and order quantity. Therefore, the order quantity decisions taken by managers may be flawed and the expected profits may not be realized. In this paper, we have performed a sensitivity analysis, which can provide managers with broad guidance as to when the end-of-season demand distribution should be accounted for and when the standard newsvendor formulation would suffice for order quantity decision-making.
2. It has been proved that the optimal order quantity and expected profit under random end-of-season demand are always lower than the respective quantities for the standard newsvendor problem.
3. Sensitivity analyses show that the deviations are most sensitive to the ratio of the mean of the end-of-season demand distribution to the mean of the seasonal demand distribution. As the ratio falls below one, the deviations increase rapidly while the deviations become negligible when the ratio exceeds one. Therefore, depending on the ratio, managers may make an informed decision as to which model to use for determining the order quantity.
4. Sensitivity analyses also reveal that when the contribution margins and salvage prices are high, the per cent deviations increase. Hence, the type of product and the customer profile would dictate whether the model developed in this paper should be used or the standard newsvendor model would suffice to determine the order quantity.
5. It has been observed that as the coefficients of variation of the seasonal and end-of-season demand distributions increase, the deviations also increase. Therefore, when the

coefficients of variation are low, it would suffice to use the standard newsvendor formulation.

6. Finally, the per cent deviations are seen to increase with the correlation between seasonal and end-of-season demands. When the correlation is negative, the per cent deviations are lower than when the correlation is positive. Therefore, for highly negatively correlated demands, it might suffice to use the standard newsvendor solutions whereas for weakly negative to positive correlations, the decision-maker may have to use the model developed in this paper.

The rest of the paper is organized as follows. Section 2 provides the problem description. The theoretical model for independent seasonal and end-of-season demands is derived in Section 3. Section 4 provides numerical examples and sensitivity analyses for the model developed in Section 3. An extension to the case when seasonal and end-of-season demands are correlated is presented in Section 5. Section 6 provides managerial implications. Finally, concluding remarks and directions for future research are presented in Section 7.

2. Problem description

Consider a retailer selling a seasonal item for which she can place the purchase order only once. Demand is unknown before placing the purchase order and is realized only after the order is fulfilled. Although demand is unknown and is known only after it is realized, the retailer has full information on the demand distribution. Now, depending on the order quantity and realized demand, the retailer may have leftover inventory at the end of the selling season which she has to dispose of at a pre-determined salvage price. We assume that the salvage price is market-driven and the retailer is a price-taker, or there is a company policy to pre-fix the salvage price at the beginning of the season irrespective of the leftover inventory or end-of-season demand. In case there is no inventory left over at the end of the selling season, the retailer will take no further action. So far, the problem is exactly the same as the standard newsvendor problem where the objective of the retailer would be to determine the optimal order quantity such that her expected profit is maximized. However, suppose besides the selling season demand distribution, the end-of-season demand distribution is also known to the retailer. In this scenario, if the number of items left over at the end of the selling season is higher than the end-of-season realized demand, the retailer can only sell the leftover items up to the end-of-season realized demand at the pre-determined salvage price and the rest has to be disposed of with no salvage value. Unlike the standard newsvendor model that assumes that all the leftover items can be disposed of at the pre-determined salvage price, in this case the retailer has to incorporate the end-of-season demand distribution into her model to determine the optimal order quantity. For this purpose, she needs to derive the expected profit function which must be maximized to determine the optimal order quantity and expected profit.

3. Model development for independent seasonal and end-of-season demands

In this section, we develop the model for the standard newsvendor problem under independent seasonal and end-of-season demands. In addition to the notations already mentioned, the following notations are also used in developing the model:

Y	Random variable representing the end-of-season demand
$f_y(y)$	Density function of the end-of-season demand distribution
$F_y(y)$	Distribution function of the end-of-season demand distribution
$\mu_x(\mu_y)$	Mean of the seasonal (end-of-season) demand distribution
$\sigma_x(\sigma_y)$	Standard deviation of the seasonal (end-of-season) demand distribution

The profit function can be written in the following way:

$$\Pi = \begin{cases} pX + vY - cS; & X \leq S, Y \leq S - X \\ pX + v(S - X) - cS; & X \leq S, Y > S - X \\ (p - c)S; & X > S \end{cases} \quad (1)$$

Hence the expected profit function is the following:

$$E\Pi = \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (px + vy - cS) f_y(y) dy \right\} f_x(x) dx + \int_{x=0}^S \left\{ \int_{y=S-x}^{\infty} (px + v(S-x) - cS) f_y(y) dy \right\} f_x(x) dx + \int_{x=S}^{\infty} (p-c)S f_x(x) dx \quad (2)$$

It is shown in Appendix A.1 that Expression (2) simplifies to the following:

$$E\Pi = (p-c)S - (p-v) \int_{x=0}^S F_x(x) dx - v \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx \quad (3)$$

To maximize $E\Pi$, the first-order necessary condition is to be derived from $\frac{d(E\Pi)}{dS} = 0$. It is

shown in Appendix A.2 that $\frac{d}{dS} \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx = \int_{x=0}^S F_y(S-x) f_x(x) dx$.

Therefore, the following first-order condition can be written:

$$(p-c) - (p-v)F_x(S) - v \int_{x=0}^S F_y(S-x) f_x(x) dx = 0 \quad (4)$$

From Equation (4), the following proposition may be derived.

Proposition 1: There exists a solution for S and the solution is unique iff the cdf of X or the cdf of $X + Y$ is strictly increasing.

Proof: Let $h(S) = (p - c) - (p - v)F_x(S) - v \int_{x=0}^S F_y(S - x)f_x(x)dx$ which can be rewritten as follows: $h(S) = (p - c) - (p - v)\Pr(X \leq S) - v\Pr(X + Y \leq S)$. Therefore, $h(S)$ is a decreasing function of S since $0 < v < c < p$. Then $\lim_{S \rightarrow 0^+} h(S) = (p - c) - 0 - 0 = p - c > 0$ since X and Y are both non-negative. Also, $\lim_{S \rightarrow \infty} h(S) = (p - c) - (p - v) - v = -c < 0$. Therefore, there exists a solution for S and the solution is unique iff $\Pr(X \leq S)$ or $\Pr(X + Y \leq S)$ is strictly increasing in S .

Next we prove the sufficiency condition that $E\Pi$ is concave in S which means the value of S obtained by solving Equation (4) would maximize $E\Pi$.

Proposition 2: $E\Pi$ is a concave function of S .

Proof: It is shown in Appendix A.2 that $\frac{d}{dS} \int_{x=0}^S F_y(S - x)f_x(x)dx = \int_{x=0}^S f_y(S - x)f_x(x)dx$.

Therefore, $\frac{d^2(E\Pi)}{dS^2} = -(p - v)f_x(S) - v \int_{x=0}^S f_y(S - x)f_x(x)dx < 0$ which means $E\Pi$ is concave in S .

Now we show in Propositions 3 and 4 that the optimal order quantity and expected profit in this problem would be lower than their respective values in the standard newsvendor problem.

Proposition 3: If the cdf of X or the cdf of $X + Y$ is strictly increasing, then the optimal value of S in this problem would be lower than the optimal value of S in the standard newsvendor problem.

Proof: In the standard newsvendor problem, the optimal value of S is obtained from the following first-order condition: $(p - c) - (p - v)\Pr(X \leq S) = 0$. Since $\Pr(X \leq S)$ or $\Pr(X + Y \leq S)$ is strictly increasing in S and $v > 0$, it is obvious from the first-order condition of this problem, i.e. $h(S) = (p - c) - (p - v)\Pr(X \leq S) - v\Pr(X + Y \leq S) = 0$ that only a lower value of S than the optimal value of S in the standard newsvendor problem would satisfy the condition.

Proposition 4: The optimal expected profit in this problem is lower than that for the standard newsvendor problem.

Proof: The first two terms in Expression (3), i.e. $(p - c)S - (p - v) \int_{x=0}^S F_x(x)dx$ represent the expected profit function for the standard newsvendor problem. Since it is known that the above function is concave in S and also the optimal value of S in this problem is lower than that for the standard newsvendor problem, the corresponding terms in the $E\Pi$ function in Expression (3) at optimality would be lower in value than the optimal expected profit for the

standard newsvendor problem. It remains to show that the third term in Expression (3), i.e.

$$\int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx > 0, \text{ which is always positive by definition.}$$

4. Numerical examples

In this section, we provide numerical examples and perform sensitivity analyses. For illustration, we have assumed that both the seasonal demand and the end-of-season demand are normally distributed. For i.i.d. customer demands, the aggregate demand faced by the retailer during the planning horizon can be reasonably assumed to be normally distributed based on the Central Limit Theorem (Gallego, 1995). We are aware that the normal distribution has a negative tail which might introduce errors in the analysis. However, we have made sure that the negative tail probability is negligible and almost 100% of the density lies on the positive side of the distribution. From the literature, we have found that this assumption is very practical. The following parameter values have been considered for numerical illustration:

$$p = 5; c = 1, 2, 3, 4$$

$$v = 0.5 \text{ when } c = 1; v = 0.5, 1 \text{ when } c = 2; v = 0.5, 1, 2 \text{ when } c = 3; v = 0.5, 1, 2, 3 \text{ when } c = 4$$

$$\mu_x = 500, 1000, 2000, 5000; \frac{\sigma_x}{\mu_x} = 0.05, 0.1, 0.2, 0.3$$

$$\mu_y = 0.25\mu_x, 0.5\mu_x, \mu_x, 2\mu_x; \frac{\sigma_y}{\mu_y} = 0.05, 0.1, 0.2, 0.3$$

We have fixed p and varied the other parameters. It may be noticed that the maximum value of the coefficient of variation taken for the examples is 0.3 for both the distributions which ensures that the negative tail probability of the normal distribution is negligible. As the coefficient of variation exceeds 0.3, the negative tail of the normal distribution can no more be ignored (Lau, 1997). There are altogether 2560 instances for each of which the optimal order quantities and expected profits were computed by using the standard newsvendor formulation and the formulation derived in this paper with the end-of-season demand distribution. We call the latter the ‘true’ optimal solutions and compute per cent deviations vis-à-vis the optimal solutions obtained by the standard newsvendor formulation for each problem instance. It is to be noted that to compute the optimal order quantity and expected profit by the standard newsvendor formulation, the end-of-season demand distribution and its parameters are not required.

To illustrate the computations, the following instance is considered:

$$p = 5; c = 4; v = 3; \mu_x = 2000; \frac{\sigma_x}{\mu_x} = 0.3; \mu_y = 0.25 \mu_x; \frac{\sigma_y}{\mu_y} = 0.3$$

The optimal order quantity and expected profit given by the standard newsvendor formulation are 2005.806 and 1522.557, respectively whereas the ‘true’ optimal solutions given by the formulation derived in this paper are 1738.423 and 1372.192, respectively. Therefore, the per cent deviations of the ‘true’ optimal solutions turn out to be 13.33% and 9.88%, respectively.

Following are the major observations from the numerical results and sensitivity analyses:

1. As c increases, other parameters remaining constant, the per cent deviations decrease.

Figure 1 shows the per cent deviations for different values of c , given $v = 0.5$, $\frac{\mu_y}{\mu_x} = 0.25$

and $\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$. As c increases, the under-stocking or opportunity cost decreases,

which in turn reduces the optimal order quantity and the expected profit in both the solutions and also the per cent deviations decrease. As v increases, other parameters remaining constant, the per cent deviations increase. Figure 2 shows the per cent

deviations for different values of v , given $c = 4$, $\frac{\mu_y}{\mu_x} = 0.25$ and $\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$. As v

increases, the overstocking cost decreases, as a result of which the order quantity and the expected profit increase in both the solutions. However, as we can see from Expression (3) that v is a multiplier of the third negative term of the expression, the per cent deviations increase with v . Also, the per cent deviations increase as c approaches p and v approaches c . Figure 3 shows the sensitivity of per cent deviations with c and v . For each

value of c , we have taken the highest value of v . Also, we have taken $\frac{\mu_y}{\mu_x} = 0.25$ and

$$\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3.$$

Insert Fig. 1, Fig. 2 and Fig. 3 here

2. Figure 4 shows the sensitivity of per cent deviations with respect to the ratio of the means of the end-of-season and seasonal demand distributions, given $c = 4$, $v = 3$ and

$\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$. It is observed that as the ratio of the mean of the end-of-season demand

distribution to the mean of the seasonal demand distribution decreases, the per cent deviations increase. This phenomenon is implicit since as the mean of the end-of-season demand distribution declines with respect to the mean of the seasonal demand distribution, the chances that all of the residual inventory at the end of the normal season could be sold off at the salvage price v would decrease, and therefore, the differences in the optimal solutions would increase. In other words, when the mean of the end-of-season demand distribution is a multiple of the mean of the seasonal demand distribution, the standard newsvendor formulation can provide a ‘close-to-true-optimal’ solution to the problem.

Insert Fig. 4 here

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3. Figure 5 and Figure 6 show the per cent deviations against the coefficients of variation of the seasonal and end-of-season demand distributions, respectively, for the given parameter values as shown in the figures. It is observed from the numerical results that as the coefficient of variation of the seasonal demand distribution increases, the optimal order quantities in the solutions either increase or decrease or remain at the same levels depending on the parameter values while the optimal expected profits always decrease. Also, the per cent deviations increase with the coefficient of variation. On the other hand, when the coefficient of variation of the end-of-season demand distribution increases, both the order quantity and the expected profit in the ‘true’ optimal solution decrease, and the per cent deviations increase. However, as observed from the figures, changes in the per cent deviations with changes in the coefficient of variation in this case are not as sensitive as those for the former case.
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Insert Fig. 5 and Fig. 6 here

Detailed results for a subset of 2560 instances are shown in Table 1 for the following parameter values:

$$p = 5; c = 4; v = 3; \mu_x = 2000; \frac{\sigma_x}{\mu_x} = 0.05, 0.1, 0.2, 0.3$$

$$\mu_y = 0.25\mu_x, 0.5\mu_x, \mu_x, 2\mu_x; \frac{\sigma_y}{\mu_y} = 0.05, 0.1, 0.2, 0.3$$

Insert Table 1 here

It may be noted that in Table 1, some of the per cent deviations are shown as zero since in these instances the probability of the end-of-season demand being lower than the leftover inventory is negligible and as such there is no appreciable difference between the standard newsvendor solution and the solution given by the formulation derived in this paper.

It is expected that as the coefficient of variation increases, the per cent deviations would also increase. However, since in this paper we have assumed normal distributions for both the seasonal and end-of-season demands, the coefficients of variation could not be increased beyond a certain value to ensure that the probability of generating negative demands was negligible. Instead, we could have also assumed other distributions having positive density only for which the coefficients of variation could be set at a high level without any issue. For example, if we assume the exponential distribution for both the seasonal and end-of-season demands, it would have only positive density and also the coefficient of variation is equal to one. Lau (1997), Halkos and Kevork (2013), and Rossi et al (2014) have considered the newsvendor problem under exponential demand. Lau (1997) and Halkos and Kevork (2013) have especially mentioned that demands for certain seasonal fashion items are highly uncertain, and hence in these cases demands can be modelled as exponentially distributed.

Let $f_x(x) = \lambda_x e^{-\lambda_x x}$ ($x \geq 0; \lambda_x > 0$) and $f_y(y) = \lambda_y e^{-\lambda_y y}$ ($y \geq 0; \lambda_y > 0$) be the density functions of the seasonal and end-of-season demands, respectively. Therefore, $\mu_x = \frac{1}{\lambda_x}$, $\mu_y = \frac{1}{\lambda_y}$ and

$\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 1$. We worked out the standard newsvendor solution and the ‘true’ optimal solution for the following instance:

$$p = 5; c = 4; v = 3; \lambda_x = \frac{1}{500}; \lambda_y = \frac{1}{125}$$

It may be noted that we have maintained $\mu_y = 0.25 \mu_x$.

The optimal order quantity and expected profit given by the standard newsvendor formulation are 346.57 and 153.43, respectively, while the same quantities given by the ‘true’ optimal solutions are 170 and 96.27, respectively. Therefore, we observe that for this particular instance under exponential demand distributions, the per cent deviations of the order quantity and expected profit for the ‘true’ optimal solutions are as high as 50.95% and 37.25%, respectively. The above instance is a testimony to the fact that under different distributional assumptions and parameter settings, the ‘true’ optimal solutions can deviate significantly from the optimal solutions given by the standard newsvendor formulation.

5. Extension to the case when seasonal and end-of-season demands are correlated

We now extend the model developed in Section 3 to the case when seasonal and end-of-season demands are correlated. Let ρ be the correlation coefficient between seasonal and end-of-season demands and $f(x, y)$ be the joint density function. Then the expected profit function can be written as follows:

$$\begin{aligned} E\Pi &= \int_{x=0}^S \int_{y=0}^{S-x} (px + vy - cS) f(x, y) dx dy + \int_{x=0}^S \int_{y=S-x}^{\infty} (px + v(S-x) - cS) f(x, y) dx dy \\ &\quad + \int_{x=S}^{\infty} (p-c) S f_x(x) dx \\ &= \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (px + vy - cS) f_{y/x}(y/x) dy \right\} f_x(x) dx + \int_{x=0}^S \left\{ \int_{y=S-x}^{\infty} (px + v(S-x) - cS) f_{y/x}(y/x) dy \right\} f_x(x) dx \\ &\quad + \int_{x=S}^{\infty} (p-c) S f_x(x) dx \end{aligned} \quad (5)$$

where $f_{y/x}(y/x)$ denotes the conditional density function of Y given $X = x$.

As in Section 3, it can be easily shown that $E\Pi$ reduces to the following:

$$E\Pi = (p-c)S - (p-v) \int_{x=0}^S F_x(x) dx - v \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_{y/x}(y/x) dy \right\} f_x(x) dx \quad (6)$$

Also, the first-order condition is the following:

$$(p - c) - (p - v)F_x(S) - v \int_{x=0}^S F_{y/x}(S - x/x)f_x(x)dx = 0 \quad (7)$$

where $F_{y/x}(y/x)$ denotes the conditional distribution function of Y given $X = x$.

Propositions 1, 2, 3 and 4 will also hold for the case when seasonal and end-of-season demands are correlated and the proofs are analogous with those presented in Section 3.

In this section, we observe the effect of the correlation coefficient on the optimal order quantity and expected profit. We assume that the joint distribution of seasonal and end-of-season demands is bivariate normal, and conduct a numerical study with the following parameter values:

$$p = 5; c = 4; v = 3; \mu_x = 2000; \frac{\sigma_x}{\mu_x} = 0.3; \mu_y = 0.25 \mu_x; \frac{\sigma_y}{\mu_y} = 0.3; \rho = 0, \pm 0.25, \pm 0.50, \pm 0.75$$

Table 2 shows the optimal order quantities, expected profits and per cent deviations from the standard newsvendor solutions for different values of the correlation coefficient.

Insert Table 2 here

It may be observed from Table 2 that as the correlation coefficient increases, optimal order quantities and expected profits decrease, and as such the per cent deviations from the standard newsvendor solutions increase. Figure 7 shows the sensitivity of per cent deviations for different values of the correlation coefficient.

Insert Fig. 7 here

It is to be mentioned here that the above results hold for specific problem instances and may be difficult to generalize. For example, if the values of c and v are changed to 1 and 0.5, respectively, keeping the other parameter values the same, numerical results show that while optimal order quantities now increase with the correlation coefficient (although marginally), optimal expected profits still decrease with the correlation coefficient. The change in optimal order quantities with the correlation coefficient may be explained from the fact that the third term in Equation (7), i.e. $\Pr(X + Y \leq S)$ will increase with the correlation coefficient when $S < \mu_x + \mu_y$ and decrease with the correlation coefficient when $S > \mu_x + \mu_y$. However, the change in optimal expected profits with the correlation coefficient is difficult to explain from Expression (6). The behaviour of optimal expected profits with change in the correlation coefficient could possibly be explained as follows. When the realized seasonal demand is high, the retailer would be able to sell most of the item and there will be a very few units of the item left over at the end of the season for end-of-season sale. Therefore, the correlation between seasonal and end-of-season demands would be of lesser significance in this case. However, when the realized seasonal demand is low, the retailer would still be able to sell most of the leftover units at the end of the season if the correlation between seasonal and end-of-season demands is negative. Hence, as the correlation coefficient increases, optimal

expected profits decrease (This was also observed by Gupta et al (2006) in the context of clearance pricing) and per cent deviations from the standard newsvendor optimal expected profit increase.

6. Managerial implications

The model developed in this paper and the analysis presented are expected to facilitate informed decision-making by practising managers in connection with placing orders for seasonal items. If it is assured that the end-of-season demand will always exceed the residual inventory at the end of the season such that all of it can be disposed of at the pre-determined salvage price, the standard newsvendor solution would work just fine. However, when it is not guaranteed that the end-of-season demand will always exceed the leftover inventory, the order quantity given by the standard newsvendor formulation may lead to sub-optimal results, and the per cent deviation of the optimal expected profit given by the standard newsvendor formulation from the 'true' optimal expected profit given by the formulation developed in this paper may be substantial depending on the demand distributions and parameter values, as already demonstrated by the numerical examples and sensitivity analyses. In such a scenario, managers will benefit if they compute the order quantity based on the model developed in this paper.

From the sensitivity analyses, it has been observed that the per cent deviations are most sensitive to the ratio of the mean of the end-of-season demand distribution to the mean of the seasonal demand distribution. When the former is a multiple of the latter, the per cent deviations are negligible irrespective of the parameter values. As the ratio decreases below one, the per cent deviations increase. The per cent deviations are also sensitive to the contribution margin and salvage price. The higher the contribution margin, the higher the per cent deviations. Also, the higher the salvage price, the higher the per cent deviations. However, their combined effect is the lower the contribution margin and the higher the salvage price, the higher the per cent deviations. Therefore, for decision-making managers should not consider the contribution margin and salvage price in isolation; rather they should be considered together to determine their combined effect on the order quantity and expected profit. It has also been observed that the per cent deviations increase with the variability of both the seasonal and end-of-season demands, although the sensitivity of per cent deviations to the variability of the seasonal demand is more than that of the end-of-season demand. Also, optimal expected profits decrease with the correlation between seasonal and end-of-season demands and the per cent deviations from the standard newsvendor solutions increase with the correlation coefficient. Finally, the per cent deviations vary depending on the underlying demand distributions.

Whether managers should use the model developed in this paper or the standard newsvendor solutions would suffice would depend on the type of product and the customer profile. Seasonal products can be broadly classified into two types – high-end and low-end. Similarly, customers can be broadly classified into two categories – (i) affluent, quality- or style-conscious and not so price-sensitive and (ii) lower-to-middle class, quality-conscious to some extent and very price-sensitive. High-end products are expensive for which value-added is

high and these products are extremely seasonal in nature in the sense that products in high demand in one season may lose their relevance completely into the next season. For example, expensive fashion-wear designed by renowned fashion designers showcased as summer, fall, winter and spring collections for one year become out-of-fashion the following year. High-end products are generally bought by the first category of customers, who would not buy once the season is nearing end even if discounted prices are offered and would rather wait for the next season when new designs/styles would become fashionable, upgrades of products would be available and/or the quality of products would improve. High-end products are characterized by high contribution margins, high salvage prices (Since the price of a product is associated with its brand value, it cannot be lowered beyond a certain level to protect the brand value), very low (in fact a tiny fraction) ratio of the mean of the end-of-season demand distribution to the mean of the seasonal demand distribution and high variability of demands. Therefore, it is expected that the model developed in this paper would be more applicable for high-end products. On the hand, low-end products are less expensive for which contribution margins are tight because of more competition, and the scope of upgradation and product quality improvement is rather limited in the short run such that these products retain their relevance for a longer period. For example, durable products such as air conditioners or low-end winterwear bought at the end of the season at discounted prices would retain their utility to the buyer in the long run. Low-end products are generally bought by the second category of customers, who would perceive value-for-money for products offered for sale at discounted prices at the end of the season more important than the style or uniqueness of products. Low-end products are characterized by low contribution margins, low salvage prices, medium-to-high ratio of the mean of the end-of-season demand distribution to the mean of the seasonal demand distribution and low variability of demands. In most of the low-end products, it would suffice to use the standard newsvendor solutions. However, as mentioned earlier, since among all the parameters the per cent deviations are most sensitive to the ratio of the mean of the end-of-season demand distribution to the mean of the seasonal demand distribution, managers need to very closely monitor the past sales records and project future trends to estimate this ratio so that an informed decision may be taken as to which of the two models would be appropriate.

As far as the effect of correlation between seasonal and end-of-season demands is concerned, there may be situations when the said correlation is positive or negative. An example of negative correlation would be price-dependent demands. If the in-season price is high, in-season demand may be low, and customers may wait until the end-of-season when discounts are offered by the retailer and end-of-season sales pick up. On the contrary, if the in-season price is low, customers will buy in-season and end-of-season demand will be low. Another example of negative correlation would be that if the chill in winter is not enough to boost winterwear sales, the stock may still be cleared at the end-of-season sale while for a chilly winter, in-season demand would be high and end-of-season demand could be low anticipating quality issues with unsold units that also quickly go out-of-fashion next winter. With respect to positive correlation, an example could be a prolonged winter season that leads to high in-season and end-of-season sales while a short season may lead to low-to-moderate in-season and end-of-season sales. Also, if the item under consideration is perceived by customers to be

of very high quality, both seasonal and end-of-season demands would be high. On the other hand, if the quality is perceived to be low, both demands would be low. Therefore, depending on the item, customer perception and various other factors, the correlation between seasonal and end-of-season demands could be positive or negative and could be high, medium or low (See, for example, Cachon and Kok (2007)). It is shown in the paper that if the correlation coefficient is strongly negative, the per cent deviations are on the lower side and probably it would suffice to use the standard newsvendor solutions. On the other hand, for weakly negative to positive correlation, the per cent deviations are significantly high to warrant the use of the model developed in this paper.

As discussed so far, the ratio of the means of the two demand distributions is the most important parameter. Had the ratio been known *a priori*, managers would have no issue in using the formulation derived in this paper. However, if no information on the distribution and its parameters of the end-of-season demand is available in advance, the model developed in this paper may not be applicable. As discussed in the previous paragraph, based on the type of products and customer profile, managers may make a qualitative assessment of the model parameters and based on their judgement may decide whether using the standard newsvendor solutions would suffice or using the model developed in this paper would be appropriate. If they decide to use the standard newsvendor solutions, no information on the end-of-season demand distribution would be required. However, if they decide to use the model developed in this paper, they would need the end-of-season demand distribution, the information on which may not be available to them. In this case, managers may make a subjective assessment *a priori* as to to what extent the expected profit would increase if they used the model developed in this paper instead of the standard newsvendor model. For example, for

the following instance: $p = 5$; $c = 4$; $v = 3$; $\mu_x = 2000$; $\frac{\sigma_x}{\mu_x} = 0.3$; $\mu_y = 0.25 \mu_x$; $\frac{\sigma_y}{\mu_y} = 0.3$, the

optimal order quantity given by the standard newsvendor formulation is 2005.806 while the ‘true’ optimal order quantity given by the model developed in this paper is 1738.423. Now, if managers use the order quantity of 2005.806, the expected profit turns out to be 1280.723 from Expression (3) while the ‘true’ optimal expected profit of 1372.192 is given by the ‘true’ optimal order quantity of 1738.423. Therefore, if managers use the optimal order quantity given by the model developed in this paper instead of the optimal order quantity given by the standard newsvendor formulation, they will be able to increase the expected profit by 7.14%. In general, depending on the extent of increase, managers may decide whether to invest money, and to what extent, in collecting data and gathering information so as to estimate the end-of-season demand distribution and its parameters. Models developed for the censored newsvendor problem (See, for example, Ding et al, 2002) would be appropriate for such a situation. Therefore, based on the assessment of the extent of increase in the expected profit, investment required for collecting data for estimation of the end-of-season demand distribution and quality of the collected data for the estimation process, managers may make an informed decision whether the use of the model developed in this paper would be appropriate.

7. Conclusions and directions for future research

In this paper, we have extended the standard newsvendor problem with random end-of-season demand. We have presented numerical examples and performed sensitivity analyses to check to what extent the standard newsvendor solutions deviate from the ‘true’ optimal solutions given by the formulation developed in this paper. We have also provided broad guidelines for managers as to under what parameter settings it would be appropriate to use the formulation developed in this paper and when the application of the standard newsvendor formulation would suffice to determine the order quantity. We have highlighted the fact that the estimation of the end-of-season demand distribution at the beginning of the selling season may not be straightforward and therefore managers will be tempted to apply the standard newsvendor formulation that does not require estimation of the end-of-season demand distribution. However, as demonstrated in this paper, the decision-making process may be flawed and the actual profit may significantly deviate from the ‘true’ optimal profit. One possible direction for future research may be the estimation of the end-of-season demand distribution based on past trends and future projections. As mentioned, there is rich literature on censored and distribution-free newsvendor problems, which might be appropriate in the case of *unobservable* sales data and lack of adequate information on demand distributions. Another possible direction for future research could be to allow the salvage price to be a decision variable. In this paper, we have taken a pre-determined value of the salvage price, either market-driven or fixed by the company policy; however, in practice, retailers do offer clearance or marked-down prices to boost end-of-season sales. In such cases, the salvage price would be an additional decision variable besides the order quantity. Cachon and Kok (2007) and Wang and Webster (2009) did consider endogenous end-of-season demand, which was a function of the salvage price, but the functional relationship between demand and price was taken to be deterministic. Future research may consider endogenous and stochastic end-of-season demand, and extend the work of Cachon and Kok (2007).

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Appendix A.1

$$\begin{aligned}
E\Pi &= \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (px + vy - cS) f_y(y) dy \right\} f_x(x) dx + \int_{x=0}^S \left\{ \int_{y=S-x}^{\infty} (px + v(S-x) - cS) f_y(y) dy \right\} f_x(x) dx \\
&\quad + \int_{x=S}^{\infty} (p-c) S f_x(x) dx \\
&= \int_{x=0}^S \left\{ \int_{y=0}^{S-x} [(p-c)x + vy - c(S-x)] f_y(y) dy \right\} f_x(x) dx \\
&\quad + \int_{x=0}^S \left\{ \int_{y=S-x}^{\infty} [(p-c)x - (c-v)(S-x)] f_y(y) dy \right\} f_x(x) dx + \int_{x=S}^{\infty} (p-c) S f_x(x) dx \\
&= \int_{x=0}^S \left\{ \int_{y=0}^{S-x} [(p-c)x + vy - c(S-x)] f_y(y) dy \right\} f_x(x) dx \\
&\quad + \int_{x=0}^S \left\{ \int_{y=0}^{\infty} [(p-c)x - (c-v)(S-x)] f_y(y) dy \right\} f_x(x) dx \\
&\quad - \int_{x=0}^S \left\{ \int_{y=0}^{S-x} [(p-c)x - (c-v)(S-x)] f_y(y) dy \right\} f_x(x) dx + \int_{x=S}^{\infty} (p-c) S f_x(x) dx \\
&= (p-c)\mu_x - (c-v) \int_{x=0}^S (S-x) f_x(x) dx - (p-c) \int_{x=S}^{\infty} (x-S) f_x(x) dx \\
&\quad - v \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx \\
&= (p-c)S - (p-v) \int_{x=0}^S F_x(x) dx - v \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx
\end{aligned}$$

Appendix A.2

- To show $\frac{d}{dS} \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx = \int_{x=0}^S F_y(S-x) f_x(x) dx$

$$\begin{aligned}
 & \frac{d}{dS} \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx \\
 &= \lim_{h \rightarrow 0} \frac{\int_{x=0}^{S+h} \left\{ \int_{y=0}^{S-x+h} (S-x-y+h) f_y(y) dy \right\} f_x(x) dx - \int_{x=0}^S \left\{ \int_{y=0}^{S-x} (S-x-y) f_y(y) dy \right\} f_x(x) dx}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h \int_{x=0}^S \left\{ \int_{y=0}^{S-x} f_y(y) dy \right\} f_x(x) dx + \int_{x=0}^S \left\{ \int_{y=S-x}^{S-x+h} (S-x-y+h) f_y(y) dy \right\} f_x(x) dx}{h} \\
 & \quad + \int_{x=S}^{S+h} \left\{ \int_{y=0}^{S-x+h} (S-x-y+h) f_y(y) dy \right\} f_x(x) dx \\
 &= \lim_{h \rightarrow 0} \frac{h \int_{x=0}^S F_y(S-x) f_x(x) dx + h^2 \int_{x=0}^S f_y(S-x) f_x(x) dx + \int_{x=S}^{S+h} \left\{ \int_{y=S-x}^{S-x+h} (S-x-y+h) f_y(y) dy \right\} f_x(x) dx}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h \int_{x=0}^S F_y(S-x) f_x(x) dx + h^2 \int_{x=0}^S f_y(S-x) f_x(x) dx + h^2 \int_{x=S}^{S+h} f_y(S-x) f_x(x) dx}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h \int_{x=0}^S F_y(S-x) f_x(x) dx + h^2 \int_{x=0}^S f_y(S-x) f_x(x) dx + h^3 f_y(0) f_x(S)}{h} \\
 &= \int_{x=0}^S F_y(S-x) f_x(x) dx
 \end{aligned}$$

- To show $\frac{d}{dS} \int_{x=0}^S F_y(S-x) f_x(x) dx = \int_{x=0}^S f_y(S-x) f_x(x) dx$

$$\begin{aligned}
 & \frac{d}{dS} \int_{x=0}^S F_y(S-x) f_x(x) dx = \lim_{h \rightarrow 0} \frac{\int_{x=0}^{S+h} F_y(S-x+h) f_x(x) dx - \int_{x=0}^S F_y(S-x) f_x(x) dx}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\int_{x=0}^S \{F_y(S-x+h) - F_y(S-x)\} f_x(x) dx + \int_{x=S}^{S+h} F_y(S-x+h) f_x(x) dx}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h \int_{x=0}^S f_y(S-x) f_x(x) dx + h F_y(h) f_x(S)}{h} \\
 &= \int_{x=0}^S f_y(S-x) f_x(x) dx \quad \left[\because \lim_{h \rightarrow 0} F_y(h) = 0 \right]
 \end{aligned}$$

Tables and Figures

Table 1: Standard newsvendor solutions, true optimal solutions and per cent deviations for instances with different parameter values, given $p = 5$, $c = 4$, $v = 3$ and $\mu_x = 2000$ (Correlation coefficient, $\rho = 0$)

$\frac{\mu_y}{\mu_x}$	$\frac{\sigma_x}{\mu_x}$	$\frac{\sigma_y}{\mu_y}$	Standard newsvendor solution		True optimal solution		Deviation (%)	
			Exp. profit	Order qty.	Exp. profit	Order qty.	Exp. profit	Order qty.
0.25	0.05	0.05	1920.426	2000.968	1920.426	2000.968	0	0
		0.1			1920.426	2000.968	0	0
		0.2			1920.426	2000.968	0	0
		0.3			1919.762	1997.748	0.034563	0.16088
	0.1	0.05	1840.852	2001.935	1840.852	2001.935	0	0
		0.1			1840.061	1992.878	0.042978	0.452418
		0.2			1834.65	1985.246	0.336925	0.833651
		0.3			1637.878	1886.453	2.606089	5.859533
	0.2	0.05	1681.705	2003.871	1635.605	1886.453	2.741221	5.859533
		0.1			1628.131	1877.882	3.185669	6.287247
		0.2			1619.527	1878.24	3.697314	6.269398
		0.3			1389.579	1755.58	8.733841	12.4751
0.3	0.05	1522.557	2005.806	1386.562	1751.598	8.932	12.6736	
	0.1			1380.438	1748.129	9.334224	12.84654	
	0.2			1372.192	1738.423	9.875845	13.33044	
	0.3							
0.50	0.05	0.05	1920.426	2000.968	1920.426	2000.968	0	0
		0.1			1920.426	2000.968	0	0
		0.2			1920.426	2000.968	0	0
		0.3			1920.426	2000.968	0	0
	0.1	0.05	1840.852	2001.935	1840.852	2001.935	0	0
		0.1			1840.852	2001.935	0	0
		0.2			1840.852	2001.935	0	0
		0.3			1839.525	1995.497	0.072115	0.321604
	0.2	0.05	1681.705	2003.871	1681.705	2003.871	0	0
		0.1			1681.705	2003.871	0	0
		0.2			1680.122	1985.756	0.09409	0.903962
		0.3			1669.3	1970.492	0.737621	1.665692
0.3	0.05	1522.557	2005.806	1495.44	1916.587	1.781007	4.448027	
	0.1			1494.608	1899.377	1.835628	5.306048	
	0.2			1484.219	1866.82	2.517992	6.929182	
	0.3			1468.369	1851.142	3.558988	7.710827	
1	0.05	0.05	1920.426	2000.968	1920.426	2000.968	0	0
		0.1			1920.426	2000.968	0	0
		0.2			1920.426	2000.968	0	0
		0.3			1920.426	2000.968	0	0
	0.1	0.05	1840.852	2001.935	1840.852	2001.935	0	0
		0.1			1840.852	2001.935	0	0
		0.2			1840.852	2001.935	0	0
		0.3			1840.852	2001.935	0	0
	0.2	0.05	1681.705	2003.871	1681.705	2003.871	0	0
		0.1			1681.705	2003.871	0	0
		0.2			1681.705	2003.871	0	0
		0.3			1679.05	1990.994	0.157879	0.642587
0.3	0.05	1522.557	2005.806	1522.557	2005.806	0	0	
	0.1			1522.557	2005.806	0	0	
	0.2			1522.557	2005.806	0	0	
	0.3			1513.915	1974.083	0.567577	1.581552	
2	0.05	0.05	1920.426	2000.968	1920.426	2000.968	0	0
		0.1			1920.426	2000.968	0	0
		0.2			1920.426	2000.968	0	0
		0.3			1920.426	2000.968	0	0
	0.1	0.05	1840.852	2001.935	1840.852	2001.935	0	0
		0.1			1840.852	2001.935	0	0
		0.2			1840.852	2001.935	0	0
		0.3			1840.852	2001.935	0	0
	0.2	0.05	1681.705	2003.871	1681.705	2003.871	0	0
		0.1			1681.705	2003.871	0	0
		0.2			1681.705	2003.871	0	0
		0.3			1681.705	2003.871	0	0
0.3	0.05	1522.557	2005.806	1522.557	2005.806	0	0	
	0.1			1522.557	2005.806	0	0	
	0.2			1522.557	2005.806	0	0	
	0.3			1522.557	2001.772	0	0.201101	

Table 2: Standard newsvendor solutions, true optimal solutions and per cent deviations for instances with different correlation coefficients, given $p = 5$, $c = 4$, $v = 3$, $\mu_x = 2000$, $\mu_y = 0.25 \mu_x$ and $\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$

Correlation coefficient (ρ)	Standard newsvendor solution		True optimal solution		Deviation (%)	
	Exp. profit	Order qty.	Exp. profit	Order qty.	Exp. profit	Order qty.
-0.75	1522.557	2005.806	1439.326	1826.749	5.466539	8.926932
-0.50			1417.839	1786.699	6.877783	10.92361
-0.25			1396.427	1759.27	8.284107	12.29114
0			1372.192	1738.423	9.875845	13.33044
0.25			1352.93	1697.4	11.14092	15.37565
0.50			1332.139	1695.675	12.50643	15.46165
0.75			1313.741	1687.979	13.7148	15.84533

Figures

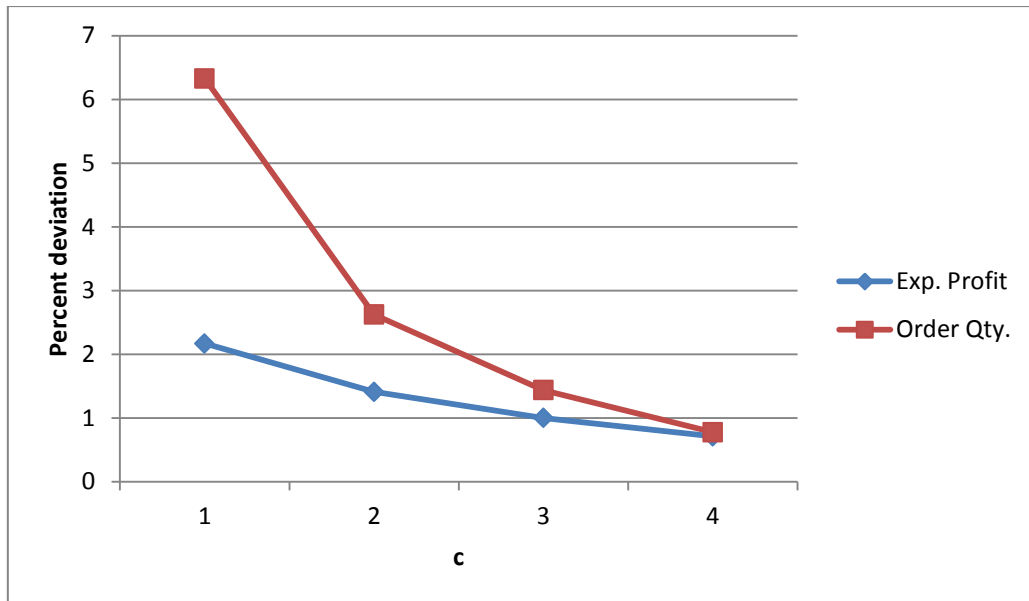


Fig. 1: Per cent deviations of expected profit and order quantity for different values of c

$$\left(\nu = 0.5; \frac{\mu_y}{\mu_x} = 0.25; \frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3 \right)$$

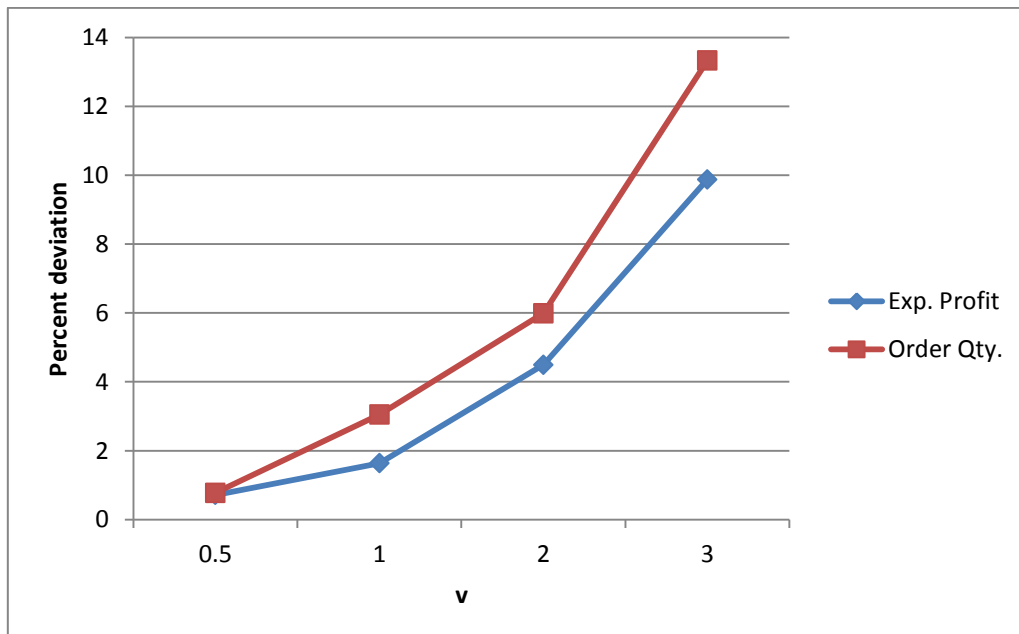


Fig. 2: Per cent deviations of expected profit and order quantity for different values of ν

$$\left(c = 4; \frac{\mu_y}{\mu_x} = 0.25; \frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3 \right)$$

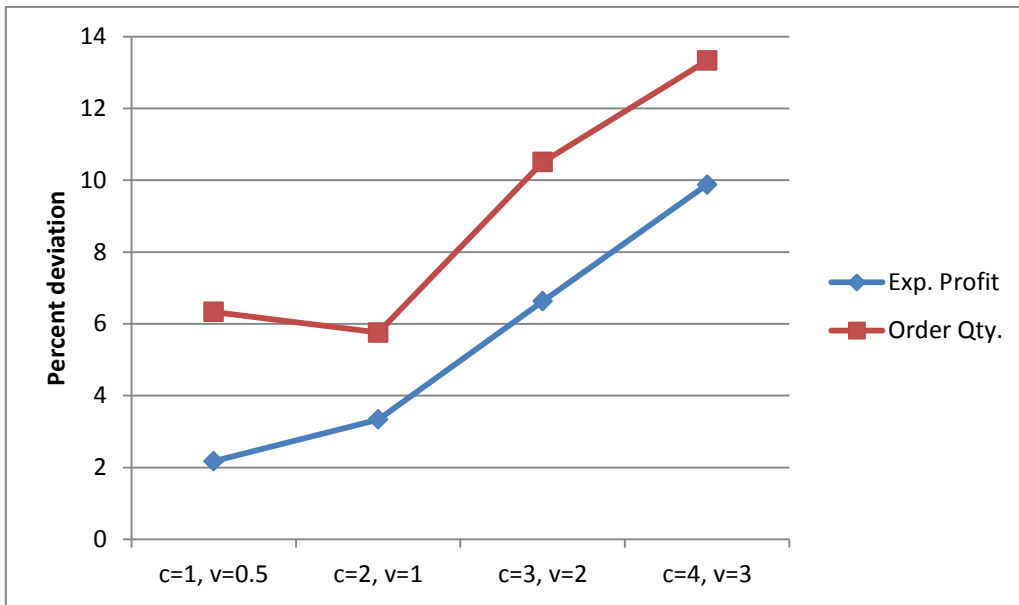


Fig. 3: Per cent deviations of expected profit and order quantity for different values of c

and v ($\frac{\mu_y}{\mu_x} = 0.25$; $\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$)

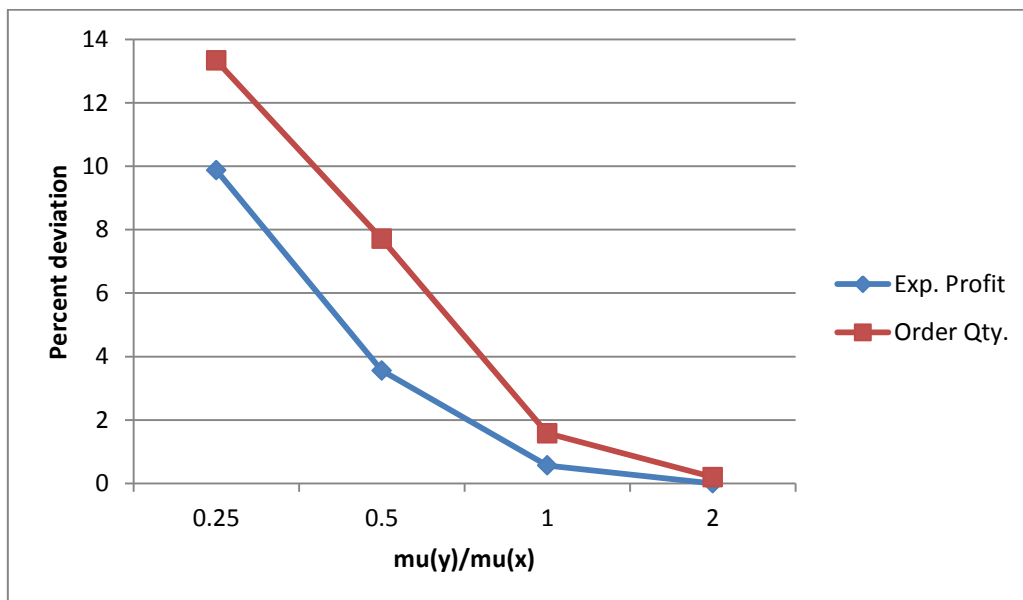


Fig. 4: Per cent deviations of expected profit and order quantity for different values of

$\frac{\mu_y}{\mu_x}$ ($c = 4$; $v = 3$; $\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$)

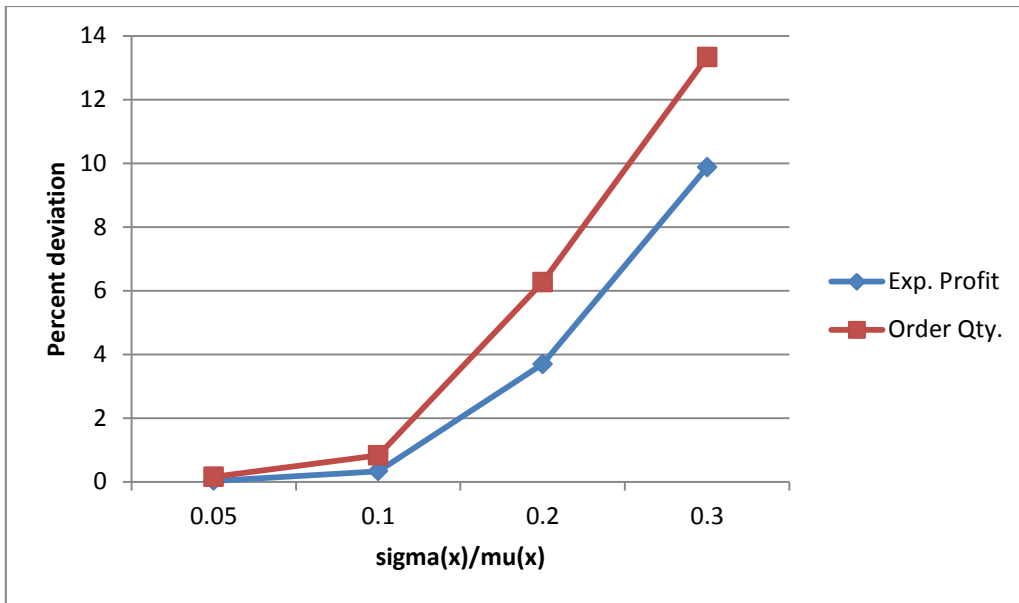


Fig. 5: Per cent deviations of expected profit and order quantity for different values of

$$\frac{\sigma_x}{\mu_x} \quad (c = 4; \nu = 3; \frac{\mu_y}{\mu_x} = 0.25; \frac{\sigma_y}{\mu_y} = 0.3)$$

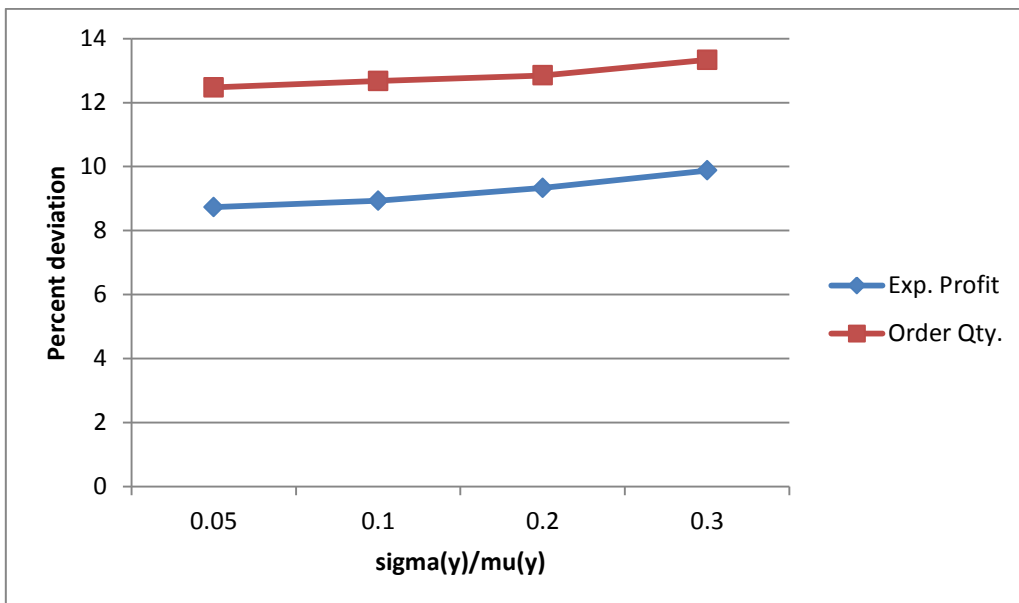


Fig. 6: Per cent deviations of expected profit and order quantity for different values of

$$\frac{\sigma_y}{\mu_y} \quad (c = 4; \nu = 3; \frac{\mu_y}{\mu_x} = 0.25; \frac{\sigma_x}{\mu_x} = 0.3)$$

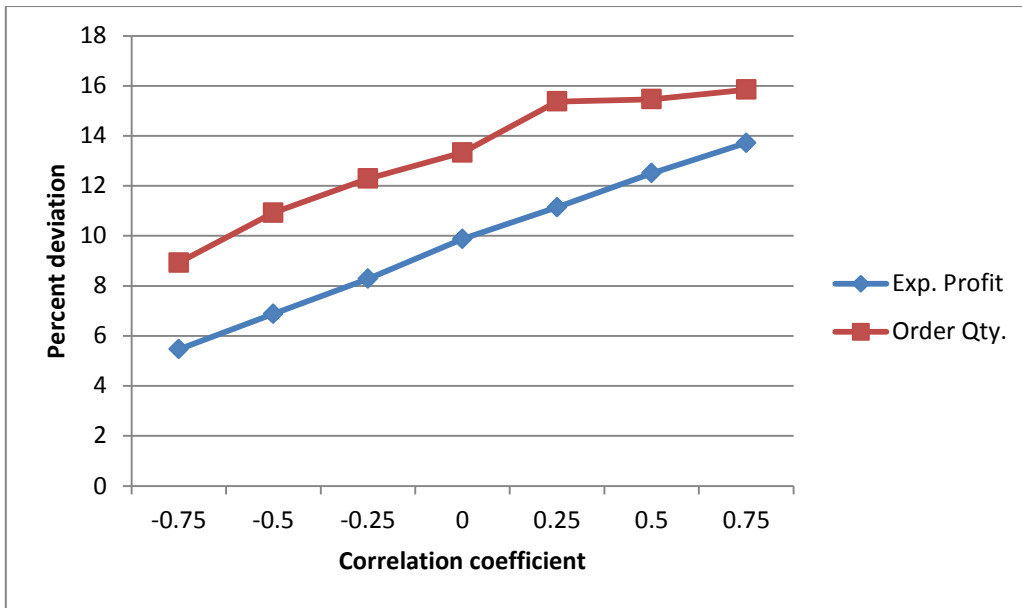


Fig. 7: Per cent deviations of expected profit and order quantity for different values of the correlation coefficient (ρ) ($c = 4$; $v = 3$; $\frac{\mu_y}{\mu_x} = 0.25$; $\frac{\sigma_x}{\mu_x} = \frac{\sigma_y}{\mu_y} = 0.3$)