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Skippable Video Ads: The Impact of Ad Avoidance Technologies on Online Two-Sided Platforms

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Abstract

This paper analyzes the effect of implementing ad-avoidance technologies, specifically, skippable advertisements in video sharing websites, where the advertisers are given the option of selecting both skippable and non-skippable advertisements. We shed light on the importance of non-skippable advertisements for websites and determine the conditions under which advertisers will switch from non-skippable to skippable advertisements. We also determine a threshold limit of non-skippable advertisements beyond which the revenues of the website may be compromised. The System Dynamics methodology is employed to further probe into the complex interactions and feedback processes of the phenomenon in order to determine the best possible ratio of non-skippable and skippable advertisements for a website. The results of our experiments with the model indicate that the best ratio depends on the content size of the website and the maturity level of the business.

Keywords: Skippable advertisements, ad-avoidance technologies, video sharing websites, two-sided online platforms

1 Introduction

Video sharing websites like Youtube, Dailymotion, Vimeo, etc. are experiencing phenomenal growth over the last few years. Data indicates that together the above-mentioned websites

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are responsible for more than 6 billion video views per month. The unprecedented growth of Youtube is the epitome of the recent developments in this field [10]. Founded in 2005, today Youtube boasts of over 1 billion users, and over 4 billion hours of video are watched each month on the website with 300 hours of video being uploaded to Youtube every minute. Competing websites like Vimeo and Dailymotion are not far behind - Dailymotion has 128 million unique monthly visitors and 2.5 billion video views per month [6], and Vimeo handles web traffic of close to 715 million video views per month, which is an year-on-year growth of 80% [21]. This rapid growth in popularity of video sharing websites is not restricted to a geography, for example, Youtube is localized in 75 countries and across 61 languages [24].

However, the exponential growth has not translated into financial successes for these companies. Although Youtube is expected to be profitable in the near future (see [20] for a good discussion on Youtube), other video sharing websites are struggling to generate sufficient cash flows from their operations. The business model of the majority of these websites is essentially that of a two-sided platform provider [15], i.e., the website acts as a platform to connect advertisers on one side to the users/viewers on the other side. The users are allowed to view and upload content free of cost, and the income from advertisements is the only source of revenue for these websites. Besides the standard problem with an advertisement dependent business model, viz., an unabated increase in advertisements may lead to depletion in the user base, there are two other issues with the business model of video sharing websites. As most of the content in these websites are short duration videos created by amateurs, a majority of it is not very relevant for businesses, and more importantly, users may not be tolerant of intrusive advertisements particularly when the duration of an average video is only a few minutes. In fact, advertisers would be wary of a negative impact on their brands. As a result, the response of the advertisers has been lukewarm at best, although recent reports suggest that online marketers prefer videos over other marketing tools as videos make it easier to connect to the audience. The pressure to perform is mounting as the video sharing space is heating up with Facebook and Twitter developing their own video offerings. These two websites drive an enormous amount of traffic to the video sharing websites, and therefore these developments are going to hurt websites like Youtube. In addition, giants Amazon and Netflix are licensing Hollywood content and investing on creating original programs.

A new initiative from Youtube, *skippable ads*, promises to revolutionize advertising on video sharing websites [2]. Skippable advertisements is a category of in-line video advertisements that are shown before, after or in between the main video. The specialty of skippable advertisements is that the viewers get the option to skip an advertisement after watching it for the first five seconds. If a viewer skips the advertisement, the advertiser would not pay for the view, i.e., the advertiser pays only if a viewer watches the advertisement [1]. The unquestionable advantage of this scheme over non-skippable (or traditional advertisements) is that the advertiser has to pay only when the website has been able to connect him to a user who is interested in the product or service. At the same time, the option to skip advertisements addresses the problem of users who are otherwise compelled to watch irrelevant advertisements and thereby develop a negative perception of the brand. Youtube is now earning advertising revenues at the same rate as cable television, which indicates the success of this model [5, 14].

The advantages of skippable advertisements suggest that it may not be unreasonable to expect advertisers to shun non-skippable advertisements and as a result the video sharing websites may completely switch to skippable advertisements. However, non-skippable advertisements are very much present in video sharing websites. In fact, according to Youtube, 15% of the advertisements in its website are non-skippable. Indeed, we observe that for websites where content is relatively less, for example news channel websites, non-skippable advertisements far outnumber skippable advertisements. It is therefore important for both incumbents and new entrants to be cognizant of the contribution of non-skippable advertisements to the business model. However, once users get exposed to skippable advertisements it may not be prudent to deploy too many non-skippable advertisements as users would be tempted to join a competing website with fewer non-skippable advertisements. This leads to a few questions of great importance to business leaders and managers who are responsible for video sharing websites:

(i) Is there a right mix of skippable and non-skippable advertisements for a video sharing website?

(ii) Does the right mix of skippable and non-skippable advertisement change with time? This is particularly important for fresh entrants in the industry.

[1] For some video sharing websites, the advertiser may have to pay even if the viewer does not watch the entire advertisement, for example, Youtube charges the advertiser if the viewer watches the first 30 seconds of the advertisement or the entire advertisement, whichever is shorter.

(iii) Would it be judicious on the part of managers to let the market decide on the right mix, or is it necessary for them to intervene and keep a check on the ratio of skippable and non-skippable advertisements?

In this paper, we develop an analytical model and a System Dynamics model to answer the above questions. Our work attempts to present a clear understanding of the effect of non-skippable advertisements on the utility derived by advertisers, and the profits earned by the website. The work provides specific guidelines to video sharing websites on how to manage advertisements in their websites.

1.1 Relationship to Existing Literature

Advertisements are generally considered to be utility reducing, i.e., the viewer's utility from watching a video would go down if he is compelled to watch an advertisement before being allowed to watch the video. This is particularly true for television and radio programs [4]. Advertisements that lower utility are less likely to be placed in newspapers as it is easy to ignore. Thus, television and radio advertisements, and by extension of the same logic, in-line video advertisements in video sharing websites would reduce utility. In his study of television networks, Wilbur [23] finds that viewers tend to be averse to advertisements. The model developed in the study predicts a gain of television audience close to 25% for a 10% decrease in advertising time. His other finding of considerable interest is that if the audience has access to an advertisement avoidance technology, then an increase in penetration of this technology leads to an increase in the number of advertisements and a reduction in revenues. We may consider the option to skip as an advertisement avoidance technology; one difference is that the platform itself provides the technology in this case. Industry reports point out that the number of advertisements in video sharing websites have increased after the introduction of skippable advertisements. However, unlike television, the revenues of video sharing websites have increased. Therefore, there is a need to develop an analytical understanding of the dynamics of advertisement avoidance technology in the context of an online video sharing business like Youtube.

In-line video advertisements which do not allow users to skip are a form of interstitials [8], and are intended to capture the involuntary attention of the viewers [12]. Although the idea is to induce viewers to remember the message, such forced exposures may actually lead to a negative perception, and therefore result in avoidance of the advertisement. Advertisers have long struggled with the problem of minimizing the formation of negative perception while ensuring that the advertising message gets across to the users. The primary criticism of advertisements has been the annoyance or irritation that it causes [1], however, research indicates that the cause of this irritation is the tactics that the advertisers employ which makes the processing of information difficult, and not advertising in general [3, 7, 19]. Consumers find it irritating or annoying if the advertisement is too long [1] or if too many ads are shown during a short interval, or if the same advertisement is shown repeatedly [3]. In the case of video sharing websites, an in-line video advertisement of thirty seconds may be perceived to be too long if the video of interest is only two or three minutes long. Indeed, research suggests that the chances of an user abandoning an advertisement is lower if the ad is placed in a video of considerably longer duration, for example, movies or TV episodes [13]. However, a user while browsing a video sharing website is likely to move from one short video to another and as a result the user may be forced to watch the same advertisement more than once during a very short period of time.

It has been observed that an advertisement which causes irritation in one group of users may not have any negative effect on another group of users. The concept which is used to explain this is termed intrusion [11], which is an interruption of editorial content. An in-line video advertisement just before or in the middle of a video clip is a good example of an interruption of editorial content. Such interruptions may interfere with the goals of the users, and the users have to adjust to include advertising in their goals, failure of which may lead to a negative reaction against the brand. Therefore, advertisement by itself is not intrusive, rather we may define intrusiveness as the degree to which a person deems the presentation of information in the advertisement as contrary to his or her goals [8]. Therefore, if an advertisement is relevant and has useful information for the user it will not be as irritating [16]. In spite of the latest targeting technologies that the video sharing websites employ to ensure that advertisements are meaningful to the user, a large number of in-line video advertisements would be irrelevant to most users, and hence may cause annoyance or irritation. The option to skip advertisements gives the user the power to decide which advertisement she would like to watch, in effect, the platform is able to significantly reduce the irritation or annoyance that an advertisement may cause.

Research on the impact of skippable advertisements is limited due to the recency of the phenomenon. In one of the first studies on skippable advertisements, Pashkevich et al. [17] conducted an experiment on Youtube users to determine the effects of allowing users to skip advertisements. They found a dramatic improvement in user experience which makes the website more attractive to users. The authors have developed a metric, viz., Follow-On-Search (FOS) which attempts to measure the degree of user engagement by looking at the relevancy of user searches in Youtube with respect to the advertisements. They found that the FOS for billed skippable advertisements (skippable advertisements which are not skipped) is higher than that for skipped or abandoned ones. The authors have looked at searches in Youtube alone while calculating FOS. If we consider users who watch the entire advertisement and then go to a search engine with the relevant keyword, the user engagement would likely to be much higher. One other notable finding is that skippable advertisements indeed reduce the negative impact of advertisements on users. This is in accordance with the extant literature which suggests that users are not annoyed or irritated with advertising, rather with the intrusive nature of advertisements.

The extant literature on skippable advertisements have not tackled the questions that we have attempted to answer in our work. Also, to the best of our knowledge, there has been no work on developing an analytical framework which quantifies the effect of skippable advertisements on the revenues of a video sharing website, or helps formulate the strategies of a platform in the presence of advertisement avoidance technologies.

1.2 The Dynamics of Skippable and Non-Skippable Advertisements

In this paper, we have considered a video sharing website which allows both skippable and non-skippable in-line video advertisements. The decision to provide the skip option for an advertisement is taken by the advertiser. We assume that an advertiser would always create separate advertisements for skippable and non-skippable options. There are a few reasons for this assumption. First, the length of a skippable advertisement would be longer. If the platform can connect the advertiser to an interested user via a skippable advertisement, then the advertiser would prefer to deliver a longer message. Second, the nature of the advertisement would be different. For a skippable advertisement, the advertiser's focus would be to generate considerable interest in the first few seconds so that the user does not exercise option to skip. Once the interest is generated the advertisement can slowly ease into focusing on information about the product or the service. For non-skippable advertisements which are mostly 15 -20 seconds, the stress would be to give as much information as possible in those few seconds and hold the interest of the user so that she does not abandon the video, particularly because the advertiser has to pay irrespective of whether the user watches the advertisement or abandons the video.

Advertisers have to specify the budget (daily or monthly) and also the price that they are ready to pay for each view of the advertisement. The website then allocates slots for the advertiser, and shows the advertisement until the budget is exhausted. Generally, non-skippable advertisements (less than 30 seconds) are shorter than skippable ones (more than a minute in some cases). As skippable advertisements allow an advertiser to connect to the right audience, and as the advertiser pays only if the advertisement has been able to create an interest in the viewer, it may be expected that advertisers will be ready to pay a higher price for skippable video advertisements.

The higher price of skippable advertisements would force some of the advertisers to opt for non-skippable advertisements. The business, thus stands to gain by offering the option of non-skippable advertisements. However, the downside of increasing the number of non-skippable advertisements is that users would feel annoyed or irritated with the advertisements and may ultimately leave for competing websites where advertisements are perceived to be less intrusive. The advertisers would be interested in the website as long as they can connect to an ample user base. Therefore, if the users depart owing to an excess of intrusive non-skippable advertisements, eventually the advertisers will also abandon the website, resulting in a loss of revenues. Therefore, the video sharing website has to strike the right balance, i.e., the number of non-skippable advertisements may be increased to boost revenues but only to a certain limit; to determine this limit, or in other words , the optimal ratio of skippable and non-skippable advertisements, is a non-trivial problem.

1.3 Methodology and Contributions

We have adopted a two pronged approach to develop an understanding of the dynamics of skippable advertisements and to generate guidelines for practicing managers. We start with a mathematical model to demonstrate that non-skippable advertisements increases profits for a video sharing website and also determine the conditions under which advertisers would shift from non-skippable advertisements to skippable advertisements. We then demonstrate that it is profitable for the video sharing website to start with a higher share of non-skippable advertisements and then slowly increase the number of skippable advertisements as more content (number of videos) is added to the website. In addition, we have shown that advertisers also benefit if they increase their share of skippable advertisements with increasing content. We then extend the model to a multi-period one and determine the threshold value of non-skippable advertisements. This will help the business determine the maximum number of non-skippable advertisements that can be accommodated without causing a decline in profit. If the website crosses the threshold value owing to a lack of knowledge of the dynamics of the process, and continues to have more than the desirable number of non-skippable advertisements for a prolonged period of time, our model predicts that the number of advertisers will start reducing till only a few advertisers remain associated with the website. Moreover, the number of skippable advertisements will reduce as well, and finally, the advertisers will invest on non-skippable advertisements only.

The second prong of our analysis involves a model developed on System Dynamics (SD) methodology. The need for a SD model stems from the fact that the effect of changing the ratio of non-skippable and skippable advertisements is indeed a complex phenomenon, the mechanics of which develop over a period of time. While it might not be difficult to characterize the individual parameters which play a role in this phenomenon, it is certainly difficult to visualize the complex interactions and feedback that is associated with this phenomenon without an integrated view. We discuss the integrated view using the SD model in Section 5. We have been able to generate a few very interesting insights from the SD model. First, the optimal ratio varies with time, i.e., the optimal ratio in the initial stages of the business would be different from the later, matured stages of the business. Second, it is better to leave it to the market forces to arrive at the optimal ratio during the early stages and the later stages of the business; it is during the middle phases where the business needs to force an optimal ratio. Indeed, that is precisely what we observe. Youtube discourages non-skippable advertisements and allows it only after a lengthy approval process. This helps reduce the number of non-skippable advertisements. Third, and in tune with our insights from the mathematical model, we demonstrate that it is better to have more non-skippable advertisements if the video sharing website is not rich in content, and if the users have a tendency to skip advertisements. This is especially true for news channel websites. These websites have hundreds of videos, however, most users would be interested only in the latest content, which implies that the websites cannot be as rich in content as, let us say, Youtube. Moreover, the users of these websites would not mind a non-skippable advertisement if they are compelled to watch one as the content they are interested in watching would not be available in any other website. However, they are likely to skip the advertisement if they are allowed such an option. As a skipped advertisement does not fetch any revenue, these websites may find it difficult to earn revenues from advertisements if they have too many skippable advertisements.

There are several contributions of this paper. For managers who are in charge of video sharing websites, or are responsible for revenues from advertisements, the findings in this paper provide a guideline in determining the right mix of skippable and non-skippable advertisement. It also provides a direction on extent of interventions required to maintain the right mix for different stages of the business. The analytical model developed in this paper does not assume any specific functional form and is therefore, suitable for applying to a variety of business scenarios. The underlying assumptions have also been derived from the understanding of real businesses and hence are not restrictive.

From researchers' point of view, the paper contributes not only in terms of its findings but also in terms of the methodology adopted. First, to the best of our knowledge, this is the first work that studies the dynamics of skippable and non-skippable advertisements for online two-sided platform providers, and looks at its impact on the business of these providers. Second, our work provides an analytical model to quantify the impact of skippable advertisements, which has not been addressed so far in the literature. The system dynamics model developed in this paper also contributes to the methodology used in advertising literature.

The paper is organized as follows. In the next section, Section 2, we present our analytical model, followed by the decision problems of advertisers and of video sharing websites in Section 3. In Section 4 we present the cross-side network effect in video sharing websites. We present a system dynamic model of this problem in Section 5 and conclude the paper in Section 6.

2 Model

In this paper, we consider a video sharing website which allows both skippable and non-skippable in-line video advertisements. We assume that the variable cost of displaying an advertisement is zero for the video sharing website. However, there is a cost of monitoring each advertisement which would be different for skippable and non-skippable advertisements. For non-skippable advertisements it is important to provide the billing details to the advertisers, such as, the number of times the advertisement was shown, the price that was charged, etc. For skippable advertisements, besides the above details, the website must have precise data on whether the advertisement was skipped, and the number of seconds after which it was skipped, and this has to be provided to the advertisers. This leads to a higher monitoring cost for the website. Therefore, we assume that the monitoring costs of skippable advertisements will be higher than the monitoring costs of non-skippable advertisements.

In our model, we characterize the advertisers by their ad sensitivity (λ), where we define ad sensitivity as the change in the advertiser's revenue per unit change in advertisement spending on video sharing websites. The advertisers are heterogeneous, and we index them by their ad sensitivity $\lambda \in \left[\underline{\lambda},\ \overline{\lambda}\right]$. We characterize video sharing websites by their content size (η), defined as the number of video hours uploaded on the website. Websites are also considered heterogeneous and we index them by their content size $\eta \in \left[\underline{\eta},\ \overline{\eta}\right]$.

The utility function of an advertiser with ad sensitivity λ is represented by $U(\mu(\lambda,\eta),\lambda,\eta)$, where μ is the number of times the advertisement is viewed, and η is the content size of the website. We assume μ, i.e., the number of times the advertisement is viewed to be a function of λ and η, as the number of views is dependent on the budget of the advertiser (which depends on λ) and the attractiveness of the website (which depends on η). We denote the utility functions for skippable and non-skippable advertisements as $U^s(\mu^s(\lambda,\eta),\lambda,\eta)$and $U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta)$ respectively.

We now describe the properties of the advertiser's utility function. In this discussion, numbered subscripts to functions denote the partial derivatives with respect to the corresponding arguments. For example, $U_1(\mu(\lambda,\eta),\lambda,\eta)$ is the first order partial derivative of the utility function U with respect to the first argument, μ, while $U_{11}(\mu(\lambda,\eta),\lambda,\eta)$ is the second order partial derivative with respect to μ. $U_{12}(\mu(\lambda,\eta),\lambda,\eta)$ represents the second order cross partial derivative of utility function U with respect to it's first and the second arguments. The advertiser's utility function satisfies each of the following conditions:

(i) The utility of an advertiser is non-zero only if the advertisement is seen by the users, i.e., $U^i(0,\lambda,\eta) = 0$, for $i \in \{s, ns\}$. The utility of a skipped advertisement is zero.

(ii) The utility of an advertiser increases with an increase in the number of advertisement views, i.e., $U^i_1(\mu^i(\lambda,\eta),\lambda,\eta) \geqslant 0$ for $i \in \{s, ns\}$

It also increases with an increase in the advertiser's ad sensitivity, and also with the increase in website's content size, i.e., $U^i_2(\mu^i(\lambda,\eta),\lambda,\eta) > 0$; $U^i_3(\mu^i(\lambda,\eta),\lambda,\eta) > 0$ for $i \in \{s, ns\}$

(iii) The utility of an advertiser increases with the number of times the advertisement is viewed, however, it increases at a decreasing rate, i.e. $U^i_{11}(\mu^i(\lambda,\eta),\lambda,\eta) < 0$ for $i \in \{s, ns\}$

Also, advertisers with higher values of ad sensitivity get higher utility for the same increase in the number of advertisement views, i.e. $U^i_{12}(\mu^i(\lambda,\eta),\lambda,\eta) > 0$ for $i \in \{s, ns\}$

Moreover, increase in utility, due to same number of increase in views, is higher for a website with larger content size, i.e. $U^i_{13}(\mu^i(\lambda,\eta),\lambda,\eta) > 0$ for $i \in \{s, ns\}$

For the same increase in ad sensitivity, the increase in the advertiser's utility is higher for a website with larger content size, i.e. $U^i_{23}(\mu^i(\lambda,\eta),\lambda,\eta) > 0 \ \forall \mu^i > 0$ for $i \in \{s, ns\}$

(iv) The increase in advertiser's utility due to increase in λ or η is greater for skippable advertisements, i.e. $U^s_2(\mu^s(\lambda,\eta),\lambda,\eta) \geqslant U^{ns}_2(\mu^{ns}(\lambda,\eta),\lambda,\eta)$; $U^s_3(\mu^s(\lambda,\eta),\lambda,\eta) \geqslant U^{ns}_3(\mu^{ns}(\lambda,\eta),\lambda,\eta)$ for $i \in \{s, ns\}$

As discussed in Sections 1.1 and 1.2, the utility from a skippable advertisement is more than the utility from an equal number of views of a non-skippable advertisement for all advertisers. Therefore, one can assume that the price per view of a skippable advertisement will be more than that of a non-skippable advertisement. Hence, a given budget will yield more views of a non-skippable advertisement as compared to a skippable advertisement, i.e. $\mu^{ns}(\lambda,\eta) >$

$\mu^s(\lambda, \eta)\ \forall\ \lambda, \eta$. Further, we assume that as the content in the website increases, the number of views of skippable advertisements increases at a rate which is at least as high as the rate of increase of the number of views of non-skippable advertisements, i.e. $\mu_2^s(\lambda, \eta) \geqslant \mu_2^{ns}(\lambda, \eta)$.

To get insights into the advertiser's net utility, we represent the fee, τ (charged by the video sharing website with content size η, from an advertiser with ad sensitivity λ), as a function of λ, η and advertisement type, i.e. skippable, non-skippable and denote it by $\tau^i(\lambda, \eta)$, where $i \in \{s, ns\}$. Please note that the fee charged by the website is equal to the advertisement budget of the advertiser. We do not explicitly include μ, the number of advertisement views, as an independent variable in the definition for τ since for most of the video sharing websites, the price for a particular view is determined through an auction mechanism. Therefore, we also consider the fee charged to be a function of the advertiser's budget (through the proxy variable λ), and the website's attractiveness (through the proxy variable content size η), and do not explicitly include μ. An increase in ad sensitivity will result in an increase in the advertisement budget, and hence in the fee charged by the website, $\tau_1^i(\lambda, \eta) > 0,$ for $i \in \{s, ns\}$

Using the above definitions, we can define the net utility of an advertiser with ad sensitivity λ from skippable and non-skippable advertisements on a website with content size η, as $U^s(\mu(\lambda, \eta), \lambda, \eta) - \tau^s(\lambda, \eta)$, and $U^{ns}(\mu(\lambda, \eta), \lambda, \eta) - \tau^{ns}(\lambda, \eta) - \beta$ respectively. The term β represents the reduction in utility due to the intrusive nature of non-skippable advertisements. We assume that the net utility for any advertiser under consideration satisfies the Individual Rationality condition (Equations 1 and 2) which implies that an advertiser would be willing to advertise only if the net utility is non-negative. We also assume that for an advertiser with ad sensitivity, λ, who is planning to advertise in a website with content size η, there is always a unique fee $\tau(\lambda, \eta)$ that will yield maximum net utility for the advertiser (Equation 3).

$$U^s(\mu^s(\lambda, \eta), \lambda, \eta) - \tau^s(\lambda, \eta) \geqslant 0, \tag{1}$$

$$U^{ns}(\mu^{ns}(\lambda, \eta), \lambda, \eta) - \tau^{ns}(\lambda, \eta) - \beta \geqslant 0, \tag{2}$$

$$U^i(\mu^i(\lambda, \eta), \lambda, \eta) - \tau^i(\lambda, \eta) \geqslant U^i(\mu^i(x, y), \lambda, \eta) - \tau^i(x, y)$$

$$\forall\ x \in \left[\underline{\lambda},\ \overline{\lambda}\right] \text{ and } \forall\ y \in \left[\underline{\eta},\ \overline{\eta}\right], \forall\ \lambda \in \left[\underline{\lambda},\ \overline{\lambda}\right]\ \eta \in \left[\underline{\eta},\ \overline{\eta}\right] \text{ where } i \in \{s, ns\} \tag{3}$$

3 Skippable versus Non-skippable Advertisements: Advertiser's and Website's Decision Problems

In this section we first consider the advertiser's decision problem which is to choose between investing on a skippable or a non-skippable advertisement in a video sharing website, characterized by η. We determine the conditions under which an advertiser will prefer a non-skippable advertisement over a skippable advertisement. We then consider the video sharing website's decision problem which is to segment advertisers based on their ad sensitivities in order to maximize profit. To approach each of these problems, we first establish some properties of the functions considered in our model. For sake of brevity, we only provide proofs of the propositions in the paper; rest of the proofs are presented in the appendix.

Lemma 1. *For an advertiser with ad sensitivity, λ, who advertises in a website with content size η, the following properties hold.*

(a) The number of advertisement views, μ is a non-decreasing function of the advertiser's ad sensitivity λ, and the website's content size η, both for skippable and non-skippable advertisements, i.e., $\mu_1^i(\lambda,\eta)) \geqslant 0$, $\mu_2^i(\lambda,\eta)) \geqslant 0$ for $i \in \{s, ns\}$.

(b) The net utility from skippable advertisements i.e. $U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta)$ and the net utility from non-skippable advertisements i.e. $U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau^{ns}(\lambda,\eta) - \beta$ are both increasing in λ.

(c) The net utility from skippable advertisements i.e. $U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta)$ and the net utility from non-skippable advertisements i.e. $U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau^{ns}(\lambda,\eta) - \beta$ are both increasing in η.

3.1 Skippable versus Non-skippable Advertisements: Advertiser's Decision Problem

In this section we determine the conditions under which an advertiser would prefer skippable over non-skippable advertisements (or vice-versa).

An advertiser will prefer a skippable advertisement over a non-skippable one if

$$U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta) \geqslant U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau^{ns}(\lambda,\eta) - \beta \tag{4}$$

and would prefer a non-skippable advertisement if

$$U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta) < U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau^{ns}(\lambda,\eta) - \beta \tag{5}$$

where it is assumed that an indifferent advertiser will select a skippable advertisement.

Equation 5 is equivalent to

$$[U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau^{ns}(\lambda,\eta)] - [U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta)] > \beta \tag{6}$$

The left hand side of Equation 6 signifies the surplus that the advertiser generates through non-skippable advertisements over skippable ones. If the surplus generated exceeds the negative impact β, the advertiser will opt for non-skippable advertisements. To understand the switching behavior of an advertiser from non-skippable to skippable ones, we define a preference function $\kappa(\lambda,\eta)$ as the difference in net utilities of skippable and non-skippable advertisements.

$$\kappa(\lambda,\eta) = [U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta)] - [U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau_{ns}(\lambda,\eta) - \beta] \tag{7}$$

In the following lemma we show that this preference function is non-decreasing in λ and in η.

Lemma 2. *For a website which offers both skippable and non-skippable advertisements to the advertiser, the preference function*

$$\kappa(\lambda,\eta) = [U^s(\mu^s(\lambda,\eta),\lambda,\eta) - \tau^s(\lambda,\eta)] - [U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau^{ns}(\lambda,\eta) - \beta] \tag{8}$$

is non-decreasing in λ and η.

This lemma leads to the following insights on the decision problem of the advertiser.

Proposition 1. *If a website with content size η_i offers the option of both skippable and non-skippable advertisements, then the choices of the advertisers are governed by the following conditions.*

(a) If $U^s(\mu^s(\lambda_k, \eta_i), \lambda_k, \eta_i) - \tau^s(\lambda_k, \eta_i) \geqslant U^{ns}(\mu^{ns}(\lambda_k, \eta_i), \lambda_k, \eta_i) - \tau^{ns}(\lambda_i, \eta_i) - \beta$ then all advertisers with ad sensitivity $\lambda \geqslant \lambda_k$ will prefer skippable advertisements over non-skippable ones in websites with $\eta \geqslant \eta_i$.

(b) If $U^s(\mu^s(\lambda_l, \eta_i), \lambda_l, \eta_i) - \tau^s(\lambda_l, \eta_i) < U^{ns}(\mu^{ns}(\lambda_l, \eta_i), \lambda_l, \eta_i) - \tau^{ns}(\lambda_l, \eta_i) - \beta$ then all advertisers with ad sensitivity $\lambda \leqslant \lambda_l$ will prefer non-skippable advertisements over skippable ones in websites with $\eta \leqslant \eta_i$.

(c) If $U^s(\mu^s(\underline{\lambda}, \eta_i), \underline{\lambda}, \eta_i) - \tau_s(\underline{\lambda}, \eta_i) < U^{ns}(\mu^{ns}(\underline{\lambda}, \eta_i), \underline{\lambda}, \eta_i) - \tau^{ns}(\underline{\lambda}, \eta_i) - \beta$ and $U^s(\mu^s(\overline{\lambda}, \eta_i), \overline{\lambda}, \eta_i) - \tau_s(\overline{\lambda}, \eta_i) \geqslant U^{ns}(\mu^{ns}(\overline{\lambda}, \eta_i), \overline{\lambda}, \eta_i) - \tau^{ns}(\overline{\lambda}, \eta_i) - \beta$ then advertisers with ad sensitivity $\lambda \in \left[\underline{\lambda}, \lambda^F\right)$ will opt for non-skippable advertisements, and advertisers with ad sensitivity $\lambda \in \left[\lambda^F, \overline{\lambda}\right]$ will opt for the skippable advertisements, where

$$\lambda^F = Min\{\lambda : (U^s(\mu^s(\lambda, \eta_i), \lambda, \eta_i) - \tau_s(\lambda, \eta_i)) - (U^{ns}(\mu^{ns}(\lambda, \eta_i), \lambda, \eta_i) - \tau_{ns}(\lambda, \eta_i) - \beta) = 0\} \tag{9}$$

Proof. Proof of Part (a):
At $\lambda = \lambda_k$ and $\eta = \eta_i$, advertisers prefer skippable advertisements over non-skippable ones because $\kappa(\lambda_k, \eta_i) \geqslant 0$. As $\kappa(\lambda, \eta)$ is non-decreasing in λ and η from Lemma 2, $\kappa(\lambda, \eta) \geqslant 0$ for $\lambda \geqslant \lambda_k$ and $\eta \geqslant \eta_i$. Therefore, all advertisers with ad sensitivity $\lambda \geqslant \lambda_k$ will prefer skippable advertisements over non-skippable ones in websites with $\eta \geqslant \eta_i$.

Proof of Part (b):
At $\lambda = \lambda_l$ and $\eta = \eta_i$, advertisers prefer non-skippable advertisements over skippable ones because $\kappa(\lambda_l, \eta_i) < 0$. As $\kappa(\lambda, \eta)$ is non-decreasing in λ and η from Lemma 2, $\kappa(\lambda, \eta) < 0$ for $\lambda \leqslant \lambda_l$ and $\eta \leqslant \eta_i$. Therefore, all advertisers with ad sensitivity $\lambda \leqslant \lambda_l$ will prefer non-skippable advertisements over skippable ones in websites with $\eta \leqslant \eta_i$.

Proof of Part (c):
If $U^s(\mu^s(\underline{\lambda}, \eta_i), \underline{\lambda}, \eta_i) - \tau_s(\underline{\lambda}, \eta_i) < U^{ns}(\mu^{ns}(\underline{\lambda}, \eta_i), \underline{\lambda}, \eta_i) - \tau^{ns}(\underline{\lambda}, \eta_i) - \beta$ and $U^s(\mu^s(\overline{\lambda}, \eta_i), \overline{\lambda}, \eta_i) - \tau_s(\overline{\lambda}, \eta_i) \geqslant U^{ns}(\mu^{ns}(\overline{\lambda}, \eta_i), \overline{\lambda}, \eta_i) - \tau^{ns}(\overline{\lambda}, \eta_i) - \beta$, there must exist a λ, such that $U^s(\mu^s(\lambda, \eta_i), \lambda, \eta_i) -$

$\tau_s(\lambda, \eta_i) = U^{ns}(\mu^{ns}(\lambda, \eta_i), \lambda, \eta_i) - \tau^{ns}(\lambda, \eta_i) - \beta$ since the preference function is continuous. As λ_F is defined as $\lambda^F = Min\{\lambda : U^s(\mu^s(\lambda, \eta_k), \lambda, \eta_k) - U^{ns}(\mu^{ns}(\lambda, \eta_k), \lambda, \eta_k) - (\tau_s(\lambda, \eta_k) - \tau_{ns}(\lambda, \eta_k)) - \beta = 0\}$, from proofs of Parts (a) and (b), advertisers with ad sensitivity $\lambda \in [\underline{\lambda}, \lambda^F)$ will opt for non-skippable advertisements, and advertisers with ad sensitivity $\lambda \in [\lambda^F, \overline{\lambda}]$ will opt for skippable advertisements. It concludes the proof of Part (c). □

3.2 Skippable versus Non-skippable Advertisements: Video Sharing Website's Decision Problem

In the previous section, we identified the advertiser segments based on the ad sensitivities of the advertisers. In this section, we consider the video sharing website's decision problem of segmenting the advertiser base in order to maximize profits.

Given the negative impact of non-skippable advertisements on the website viewers, a natural question for video sharing websites is whether to allow any non-skippable advertisements at all. In the following lemma we establish that offering non-skippable advertisements to advertisers does not reduce the profit of a video sharing website.

Lemma 3. *Offering non-skippable advertisements besides skippable advertisements does not decrease the profit of a video sharing website.*

We next determine a threshold ad sensitivity (denoted by $\lambda^T(\eta)$) such that for all advertisers with ad sensitivity $\lambda < \lambda^T$, the website would prefer these advertisers to opt for non-skippable advertisements, and opt for skippable advertisements for $\lambda \geqslant \lambda^T$. To determine the threshold ad sensitivity λ^T, we develop an optimization model which maximizes the website's profit subject to constraints on the maximum number of non-skippable and skippable advertisements. We define c^{ns} and c^s to be the unit costs of monitoring a view of non-skippable and skippable advertisements respectively. As discussed in Section 2, we assume $c^s > c^{ns}$. We develop the following optimization problem for the website to identify the optimal value of λ^T for a given η:

$$\max_{\lambda^T} \int_{\underline{\lambda}}^{\lambda^T} [\tau^{ns}(\lambda,\eta) - c^{ns}\mu^{ns}(\lambda,\eta)] f(\lambda,\eta) d\lambda + \int_{\lambda^T}^{\overline{\lambda}} [\tau^{s}(\lambda,\eta) - c^{s}\mu^{s}(\lambda,\eta)] f(\lambda,\eta) d\lambda \quad (10)$$

subject to

$$\int_{\lambda^T}^{\overline{\lambda}} \mu^s(\lambda,\eta) f(\lambda,\eta) d\lambda \leqslant \Delta(\eta) g(\eta) \quad (11)$$

$$\int_{\underline{\lambda}}^{\lambda^T} \mu^{ns}(\lambda,\eta) f(\lambda,\eta) d\lambda \leqslant \Delta(\eta) h(\eta) \quad (12)$$

$$-\lambda^T \leqslant \underline{\lambda} \quad (13)$$

$$\lambda^T \leqslant \overline{\lambda} \quad (14)$$

In this optimization model, $f(\lambda,\eta)$ denotes the joint probability density function of advertiser with ad sensitivity λ advertising on website with content size η. Total potential number of viewers for the website is considered to be a function of its content size, and is denoted by $\Delta(\eta)$. $g(\eta)$ denotes the fraction of total potential viewers who may watch a skippable advertisement. $h(\eta)$ denotes the tolerance level of the website in terms of number of non-skippable advertisements shown. For example, if $h(\eta) = 0.2$, it says that non-skippable advertisements can be shown to 20% of the potential viewers without any loss of revenue. We will restrict the discussion on loss of revenue here as we elaborate on this in the next section. Equations 11 and 12 denote these restrictions in terms of maximum number of non-skippable and skippable advertisements that the website can host.

If we let $u_1, \ldots, u_4$ denote the dual variables associated with the constraints represented in Equations 11 to 14 then an optimal solution (say, λ^{T*}) to this problem must satisfy the following first order necessary conditions (the Karush-Kuhn-Tucker conditions):

$$[\tau^s(\lambda^{T*},\eta) - c^s\mu^s(\lambda^{T*},\eta)] - [\tau^{ns}(\lambda^{T*},\eta) - c^{ns}\mu^{ns}(\lambda^{T*},\eta)] \geqslant$$
$$u_1[\mu^s(\lambda^{T*},\eta)] - u_2[\mu^{ns}(\lambda^{T*},\eta)] + \frac{(u_3 - u_4)}{f(\lambda^{T*},\eta)} \tag{15}$$
$$\lambda^{T*}[[\tau^s(\lambda^{T*},\eta) - c^s\mu^s(\lambda^{T*},\eta)] - [\tau^{ns}(\lambda^{T*},\eta) - c^{ns}\mu^{ns}(\lambda^{T*},\eta)] -$$
$$u_1[\mu^s(\lambda^{T*},\eta)] + u_2[\mu^{ns}(\lambda^{T*},\eta)] - \frac{(u_3 - u_4)}{f(\lambda^{T*},\eta)}] = 0 \tag{16}$$
$$u_1[\int_{\lambda^{T*}}^{\overline{\lambda}} \mu^s(\lambda,\eta)f(\lambda,\eta)d\lambda - \Delta(\eta)g(\eta)] = 0 \tag{17}$$
$$u_2[\int_{\underline{\lambda}}^{\lambda^{T*}} \mu^{ns}(\lambda,\eta)f(\lambda,\eta)d\lambda - \Delta(\eta)h(\eta)] = 0 \tag{18}$$
$$u_3[\lambda^{T*} - \overline{\lambda}] = 0 \tag{19}$$
$$u_4[\underline{\lambda} - \lambda^{T*}] = 0 \tag{20}$$
$$\lambda^{T*}, u_1, u_2, u_3, u_4 \geqslant 0 \tag{21}$$

Using Equations 15 to 21, we determine the value of $\lambda^{T*}(\eta)$ which partitions the set of advertisers into two sets.

3.3 Skippable versus Non-skippable Advertisements: The Impact of Website Content

In the previous two sections, we have determined the cut off value of ad sensitivity $\lambda^F(\eta)$, for a website with content size η, such that advertisers with ad sensitivity less than $\lambda^F(\eta)$ will not opt for skippable advertisements on this website and we have also determined the threshold ad sensitivity $\lambda^{T*}(\eta)$ for the website such that it would prefer advertisers with ad sensitivity greater than $\lambda^{T*}(\eta)$ to provide non-skippable advertisements. We now study the behavior of $\lambda^F(\eta)$ and $\lambda^{T*}(\eta)$ for websites with different content sizes, η.

Lemma 4. *(a) $\lambda^F(\eta)$ is non-increasing in η.*

(b) $\lambda^{T*}(\eta)$ is non-increasing in η.

Lemma 4 shows that with increasing content size, if advertisers opt for skippable advertisements at lower levels of ad sensitivity, it is beneficial for both the advertisers and the website;

while the advertisers would gain more utility, the website would be able to increase its profits. Therefore, it can be inferred from the model that the number of skippable advertisements will necessarily increase with the increase in content in the website. This indicates that the video sharing websites should offer non-skippable advertisements in the initial stages of the business and slowly shift to skippable advertisements as the content gets richer. It would not be wise on the part of the business to shun non-skippable advertisements initially. The above result also explains the relatively higher number of non-skippable advertisements in websites where the content size cannot go up beyond a limit, for example, news channel websites.

The decision problem of managers of video sharing websites on whether they should intervene to keep a check on the number of non-skippable advertisements can be well appreciated from the above lemma. If $\lambda^F(\eta) \leqslant \lambda^{T*}(\eta)$, then the managers can leave it to the market to decide on the ratio of skippable and non-skippable advertisements. However, if $\lambda^F(\eta) > \lambda^{T*}(\eta)$, managers have to intervene to reduce the number of non-skippable advertisements so that there is no resulting loss in profits. In the next section, we develop this idea further to understand its impact on the choice of advertisers and the revenue of the website.

4 Cross-Side Network Effect in Video Sharing Websites

As discussed earlier a video sharing website is a two sided platform business with advertisers on one side and users on the other. Advertisers join such a website because of the opportunity to connect to the users on the other side, which is known as the Cross-Side Network Effect [15]. Therefore, if users leave the website due to an increase in intrusive advertisements, advertisers will also follow, and the website will experience a dip in revenue. However, an increase in non-skippable advertisements does not immediately lead to fewer advertisers or reduced revenues, this phenomenon asserts itself with a delay, i.e., over a period of time. In this section, we first develop a two period model to demonstrate, if the number of non-skippable advertisements goes beyond the threshold limit, there will be a reduction in the number of skippable advertisements and consequently a drop in revenue for the website. We then extend the two-period model to an n-period model, where we show that if the number of non-skippable advertisements stays above the threshold for an extended time period, then the number of skippable advertisements would decrease and after a certain period it would drop to zero. If there is still no intervention to bring it down to the threshold limit, the number of non-skippable advertisements would also start declining.

4.1 The Impact of Cross-Side Network Effect: A Two Period Model

In this section we develop a two period model to demonstrate the debilitating effect of keeping the number of non-skippable advertisements beyond the threshold limit. We quantify the increase in number of non-skippable advertisement by suitably defining total number of non-skippable and skippable advertisements as $NS(\eta)$ and $S(\eta)$ respectively. Using the threshold ad sensitivity (λ^{T*}) we also compute the maximum (threshold) number of non-skippable advertisement, $TS(\eta)$, that the website should allow in order to maximize its profit. The values for $NS(\eta)$, $S(\eta)$ and $TS(\eta)$ can be obtained using the following expressions:

$$NS(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \mu(\lambda, \eta) f(\lambda, \eta) d\lambda \tag{22}$$

$$S(\eta) = \int_{\lambda^F}^{\overline{\lambda}} \mu(\lambda, \eta) f(\lambda, \eta) d\lambda \tag{23}$$

$$TS(\eta) = \int_{\underline{\lambda}}^{\lambda^T} \mu(\lambda, \eta) f(\lambda, \eta) d\lambda \tag{24}$$

The point of contention lies whenever $NS(\eta) > TS(\eta)$ and hence we introduce a difference parameter $DF(\eta) = NS(\eta) - TS(\eta)$ to measure the magnitude of difference in the current time period. In case $NS(\eta) \leqslant TS(\eta)$, $DF(\eta)$ will take a value of 0. In Section 3.2, we have defined potential viewers of the website as $\Delta(\eta)$. To include the effect of threshold number of non-skippable advertisements, we augment the definition of the number of viewers as $\Delta(\eta, DF(\eta))$. Following the definition of $DF(\eta)$, it will only act as an impediment to reduce actual number of viewers and hence $\Delta_2(\eta, DF(\eta)) \leqslant 0$. Because of this reduction in total number of viewers, advertisers get adversely affected which can be quantified as a reduction in their ad sensitivity λ as we anticipate that the effectiveness of the advertisement is also dependent on the utility viewers derive by visiting the website. We illustrate this argument by writing the dependency expression of ad sensitivity λ of an advertiser in period $t + 1$ as $\lambda^{t+1} = \lambda^t - \xi(DF(\eta))$. We incorporate this change by modifying joint probability density function at time period $t + 1$ as $f^{t+1}(\lambda, \eta) = f^t(\lambda + \xi(DF(\eta)), \eta)$. $\xi(DF(\eta)) \geqslant 0 \; \forall DF(\eta) > 0$, defines the extent of shift in the density function depending on $DF(\eta)$ with $\xi(0) = 0$.

We now present our result for the two period model. We refer to the two periods as period 0 and period 1. We denote the total number of skippable advertisements as $S^0(\eta)$ and $S^1(\eta)$ for periods 0 and 1 respectively. Also the total revenue earned by the website in the two periods is represented as $R^0(\eta)$ and $R^1(\eta)$ respectively. Although we do not explicitly show the n-period extension of set of proofs presented in this two period model, we have found very little difference in the proofs or the understanding involved while extending it for n-period.

Lemma 5. *If, for a website with content size η, the total number of non-skippable advertisements in period 0 exceeds the threshold number of non-skippable advertisements for the website, i.e. $DF(\eta) > 0$, then*

(a) the total number of skippable advertisement views reduces from period 0 to period 1, i.e., $S^0(\eta) > S^1(\eta)$.

(b) total revenue reduces from period 0 to period 1, i.e., $R^0(\eta) > R^1(\eta)$.

The lemma can be generalized to periods i and $i + 1$ without any significant change in the basic outline of the proof. Lemma 5 establishes the effect of allowing the number of non-skippable advertisements to remain above the threshold level for the website, and answers the question that has been raised in the beginning of this discourse, viz., whether managers should intervene to keep a check on the number of non-skippable advertisements. The lemma clearly presents the effect of not intervening - a decline in the number of skippable advertisements followed by a drop in revenues. We extend this two-period model to an n-period model in the next section and estimate its effect on skippable advertisements.

4.2 The Impact of Cross-Side Network Effect: An n-Period Model

In this section, we draw on our understanding from the two period model developed in the previous section and extend the model to n periods. An important finding of this section is, if the number of non-skippable advertisements for a website is greater than its threshold level of non-skippable advertisements in period 0 and if the website does not intervene to bring it to a level equal to or below its threshold level, then the number of skippable advertisements will not only decrease from one period to another but will eventually reduce to zero, moreover after a certain time period the number of non-skippable advertisements would also start decreasing. A loss in revenue for the website would follow.

In particular, we consider a website with content size η, such that $\lambda^F(\eta) > \lambda^{T*}(\eta)$. We then consider the scenario in which the website allows the advertisers to choose between non-skippable and skippable advertisements and does not consider its threshold ad sensitivity $\lambda^{T*}(\eta)$ in any time period. That is, for the website $DF^0(\eta) > 0$ and in no period the website forces $DF^0(\eta)$ to be zero. We establish the following result for this scenario.

Proposition 2. *If a website with content size η, such that $\lambda^F(\eta) > \lambda^{T*}(\eta)$, allows the advertisers to choose between non-skippable and skippable advertisements and does not force $DF^k(\eta)$ to be zero in any time period k, then*

(a) the number of skippable advertisements in period k is less than the number of skippable advertisements in period $k-1$, for $k \leqslant i$.

(b) in time period i the number of skippable advertisements reduces to zero, where i is the first time period that satisfies the equation $\lambda^F + \Xi^{i-1}(DF(\eta)) = \overline{\lambda}$.

Proof. The proof of Lemma 5 can be easily extended to prove Part (a) of the proposition, and we do not present it here for the sake of brevity. We now prove Part (b) of the proposition. Total number of skippable advertisements in period i is given by:

$$\begin{aligned} S^i(\eta) &= \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda, \eta) f^i(\lambda, \eta) d\lambda \\ &= \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda, \eta) f^0(\lambda + \Xi^{i-1}(DF(\eta)), \eta) d\lambda \end{aligned} \tag{25}$$

Here $\Xi^{i-1}(DF(\eta)) = \sum_{k=0}^{i-1} \xi(DF^k(\eta))$. Using the transformation $\lambda + \Xi^{i-1}(DF(\eta)) = t$,

$$\int_{\lambda^F + \Xi^{i-1}(DF(\eta))}^{\overline{\lambda}} \mu^s(\lambda - \Xi^{i-1}(DF(\eta)), \eta) f^0(\lambda, \eta) d\lambda \tag{26}$$

For $S^i(\eta) = 0$, the value of i must satisfy the equation $\lambda^F + \Xi^{i-1}(DF(\eta)) = \overline{\lambda}$. □

Corollary 1. *Considering an n-period problem with $DF(\eta) > 0$, the time period i in which the number of skippable advertisements will reduce to zero is expressed by $i = \frac{\overline{\lambda} - \lambda^F}{\xi(DF(\eta))}$ if $f(\lambda, \eta)$ follows a uniform distribution, i.e. $DF(\eta)$ is constant till period $i-1$.*

Proposition 5 and Corollary 1 establish that if for a website $DF(\eta) > 0$ is greater than zero in period zero, and the website does not intervene to make it zero, then the number of skippable advertisements for the website eventually reduces to zero. We now establish the effect of $DF(\eta) > 0$ on the number of non-skippable advertisements for the website.

Lemma 6. *If for a website with content size η the total number of non-skippable advertisements in period zero exceeds the threshold number of non-skippable advertisements for the website, i.e. $DF(\eta) > 0$, and number of skippable advertisements reduces to zero in period i, then*

(a) the total number of non-skippable advertisements for the website will reduce between two consecutive time periods k and $k+1$ for $k > i$, i.e. $NS^k > NS^{k+1}$.

(b) the total revenue will reduce between two consecutive time periods k and $k+1$ for $k > i$, i.e. $R^k > R^{k+1}$.

Proposition 3. *The number of non-skippable advertisements in period j, $NS^j(\eta)$, where $j > i$, and i is the period for which the number of skippable advertisements for the website first becomes zero, can be expressed as:*

$$NS^j(\eta) = \int_{\underline{\lambda}+\Xi^{j-1}(DF(\eta))}^{\lambda^F} \mu^{ns}(\lambda - \Xi^{j-1}(DF(\eta)), \eta) f^0(\lambda, \eta) d\lambda$$

If $NS^j(\eta) = TS(\eta)$, then Period j is defined as the stabilization period after which no change in terms of reduction in non-skippable advertisements will take place.

Proof. There will be no further reduction in the number of non-skippable advertisements once $DF(\eta) = 0$. This implies that the system will be stabilized in the time period j for which the number of non-skippable advertisements is equal to the threshold number of non-skippable advertisements for the website, i.e., $NS^j(\eta) = TS(\eta)$. $NS^j(\eta)$ can be expressed as:

$$\begin{aligned} NS^j(\eta) &= \int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda, \eta) f^j(\lambda, \eta) d\lambda \\ &= \int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda, \eta) f^0(\lambda + \Xi^{j-1}(DF(\eta)), \eta) d\lambda \\ &\text{Using transformation, } \lambda + \Xi^{j-1}(DF(\eta)) = t, \\ &= \int_{\underline{\lambda}+\Xi^{j-1}(DF(\eta))}^{\lambda^F} \mu^{ns}(\lambda - \Xi^{j-1}(DF(\eta)), \eta) f^0(\lambda, \eta) d\lambda \end{aligned} \tag{27}$$

The value of j for which it is equal to $TS(\eta)$ will be the stabilization period. □

5 A System Dynamics Model

In the previous sections we have showed the effects of an increase in intrusive, non-skippable advertisements beyond the threshold limit. In this section, we present a model using the System Dynamics methodology [9, 18] to capture an integrated view of the intricate internal dynamics of this phenomenon; the objective is to generate further insights for practicing managers. In the System Dynamics methodology, the idea is to model the causal structure of the system as a set of differential equations. The model can then be simulated to understand the dynamic behavior of the system. The effect of an increase in the number of non-skippable advertisements is manifested in the revenue and profit figures of the video sharing website, and the mechanics of this complex phenomenon develop over time. Therefore, the System Dynamics methodology can serve as a good tool for a deeper understanding of this phenomenon.

5.1 A Basic Model for Skippable and Non-skippable Advertisements

We have modeled a basic scenario along the lines of the model that we have developed in the previous sections. As already discussed in the previous sections, the model assumes that the advertisers have the option to opt for either a skippable advertisement or a non-skippable advertisement. The skippable advertisement is expected to be preferred by the advertisers because of its unique advantages, and as a result, the number of skippable advertisements would go up at a faster rate. This will in turn lead to a faster increase in the price of the skippable advertisements (the amount that an advertiser would be ready to pay for one view of the advertisement). Finally, the price of skippable advertisements would reach a point where some advertisers whose budget is not as high will not be able to afford skippable advertisements, and will be forced to opt for non-skippable advertisements. Therefore, the option of non-skippable advertisements will help the website cater to advertisers who otherwise would have been priced out of the market. However, as we have seen in the previous sections, an increase in non-skippable advertisements, if unchecked, may eventually lead to an exodus of advertisers.

Figure 1 above gives the stock and flow diagram of the model that we have created. The

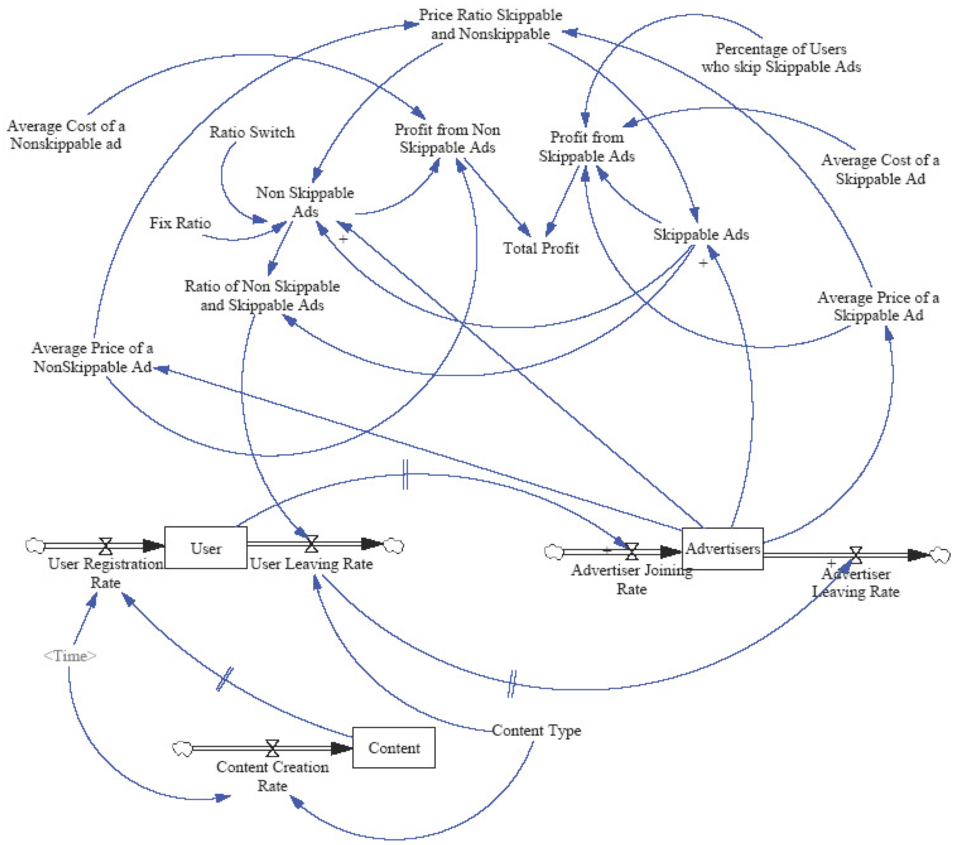

Fig. 1: Stock and Flow Diagram of the Model

best way to understand the model is to start at the bottom with the variable *content type*. We have looked at two types of video sharing websites - a website with large content size, for example, Youtube, Vimeo, Dailymotion, etc., and a website with low content size, for example, news channel websites. The content created is determined by the content creation rate which we have assumed to be constant for low content websites, for example, approximately, the same number of videos would be uploaded each day for a news channel website and the previous day's videos would be rendered stale. For high content website, like Youtube, the content will increase monotonically with time; we have assumed a linear increase in the content creation rate. The user joining rate will increase with the content that is available in the website, and will also increase with time. The number of users would determine the advertising joining rate. Here again, we have assumed a linear dependency of advertiser joining rate with the number of users. We have also assumed a time lag in the relationship, i.e., an increase in the number of users in the website will not lead to an immediate increase in the advertiser joining rate, the effect will be delayed.

The increase in the number of advertisers will increase the prices of the advertisements, and we have assumed that the prices will exhibit a non-linear dependency on the number of advertisers. Moreover, the average price for a skippable advertisement will increase at a faster rate than that of a non-skippable advertisement. Similarly, owing to the better returns from skippable advertisements, more advertisers will opt for skippable advertisements. The revenue from the two different types of advertisements are calculated by taking into consideration the number of advertisements, the average price of each type of advertisement, and the percentage of users who skip a skippable advertisement. According to industry reports, the present figure for skip rate is 70%. We have assumed the cost of skippable advertisements to be higher than that of non-skippable advertisements (discussed in section 2). The profit is calculated by subtracting the cost from the revenue. The increase in the price of skippable advertisements will eventually lead to an increase in the number of non-skippable advertisements. In the model, the parameter, *Price Ratio Skippable and Non-Skippable* is used to represent the relative prices of the two types of advertisements, and this ratio affects the number of each type of advertisements.

The website can keep a check on the number of non-skippable advertisements in a multitude of ways, for example, the website may discourage an advertiser from opting for non-skippable advertisements. Some websites employ this tactic by enforcing a lengthy approval process for users who upload videos (channel partners) and who want to display in-line non-skippable advertisements. It can also be implemented by changes in the policy. For example, the website may announce that only videos longer than a certain duration will be allowed to show a non-skippable advertisement. These websites have very little control over the prices (the price of an advertisement is decided by an auction mechanism), however, it might offer special discounts on skippable advertisements to discourage non-skippable advertisements. To study the effect of intervening measures to force a certain ratio of non-skippable and skippable advertisements, we have considered a variable, *Fix Ratio*. An advantage of the Systems Dynamics model that we have created is that it has allowed us to experiment with the skip rate, i.e., the number of users who would skip a skippable advertisement. The results are discussed in the next section.

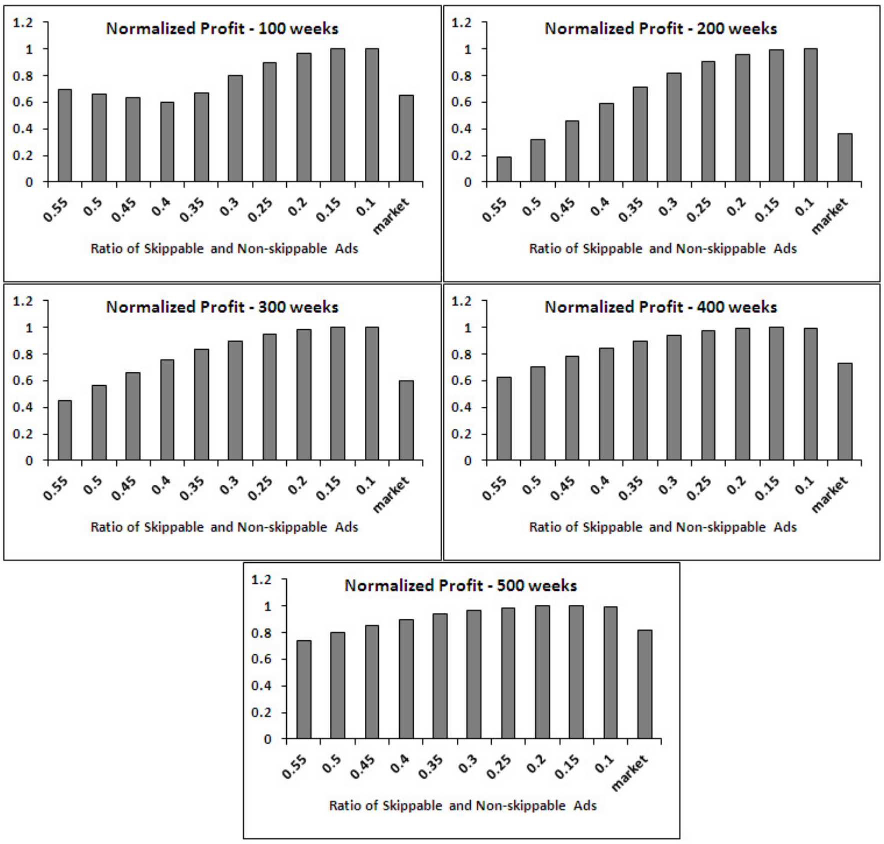

Fig. 2: Normalized Profit for High Content Websites with 70% Skip Rate

5.2 Computational Experiments

The stock and flow diagram depicted in figure 1 was converted to its equation form representation and implemented using the Vensim package. We then simulated the model to understand the dynamics of the phenomenon under different scenarios. We set parameters for three variables in the model - *Content Type*, *Percentage of Users who Skip Skippable Ads* and *Fix Ratio*. We simulated for two content types - High Content Type Websites (1) and Low Content Type Websites (0). We wanted to study the effect of increasing the skip rate for high content websites, and therefore we took two values of *Percentage of Users who Skip Skippable Ads*, 70% which is the current industry skip rate, and 95% which can be considered to be a high skip rate. The simulation duration is set to 500 weeks.

Figure 2 shows the results for the simulation run for high content type websites with 70% skip rate [2]. We have increased *Fix Ratio* from 0.1 to 0.55 with an increment of 0.05. We have shown five plots of profit figures (normalized by dividing with the maximum profit figure)

[2] the market column indicates profits for cases where the website does not intervene to keep a check on non-skippable advertisements

for the following time periods - 100 weeks, 200 weeks, 300 weeks, 400 weeks and 500 weeks. The plots depict that the best option is to keep the ratio of non-skippable and skippable advertisements between 0.1 and 0.15. Interstingly, 85% of the advertisements in Youtube are skippable, which is close to a ratio of 0.17. Another interesting observation is the behavior of the system when the website does not intervene and lets the market decide the ratio. From the figure (the column labelled *market*) it is clear that for high content websites where the tendency of users to skip is not very high, it would not be judicious to let the market decide the ratio, managers would need to intervene. In the initial stages of the business, the best performance of the market is close to 60% of that of the optimal ratio, and even during the later stages, the market does not perform better than 80%. The reason for a better performance towards the later stages is because of the steady growth of content which results in an increase in the number of users and advertisers. Thus, for high content websites, the debilitating effect of non-skippable advertisements becomes less potent with time. It is interesting to note that the optimal ratio varies with time. Therefore, an incumbent business with a high content size, for example, Youtube, has to continuously keep searching for the 'sweet spot'. For new entrants, our results show that it is essential that the managers take necessary steps to stay close to the optimal ratio at each stage of the business.

Figure 3 shows the results for a high content type website with high skip rate (95%). The five plots for the different time periods show that in the case of high skip percentage, it would be wise to leave it to the market to decide on the best ratio of skippable and non-skippable advertisements in the initial stages and the later stages of the business. It is during the middle stages that managers have to be careful about the number of non-skippable advertisements. Our model indicates that the in the middle phases (for the period 100 - 300 weeks, the ratio would lie between 0.25 and 0.3. After 300 weeks, it is seen that it is best for the video sharing website to let the market decide the correct ratio of non-skipapble and skippable advertisements. The key learning here is that the managers must avoid a high proportion of non-skippable advertisements in the phase where the content size is low. Once the website is rich in content, the managers may increase the number of non-skippable advertisements as the negative effect of non-skippable advertisements weaken substantially in the presence of a large content size. Moreover, as the skip rate is high, the contribution of skippable advertisements

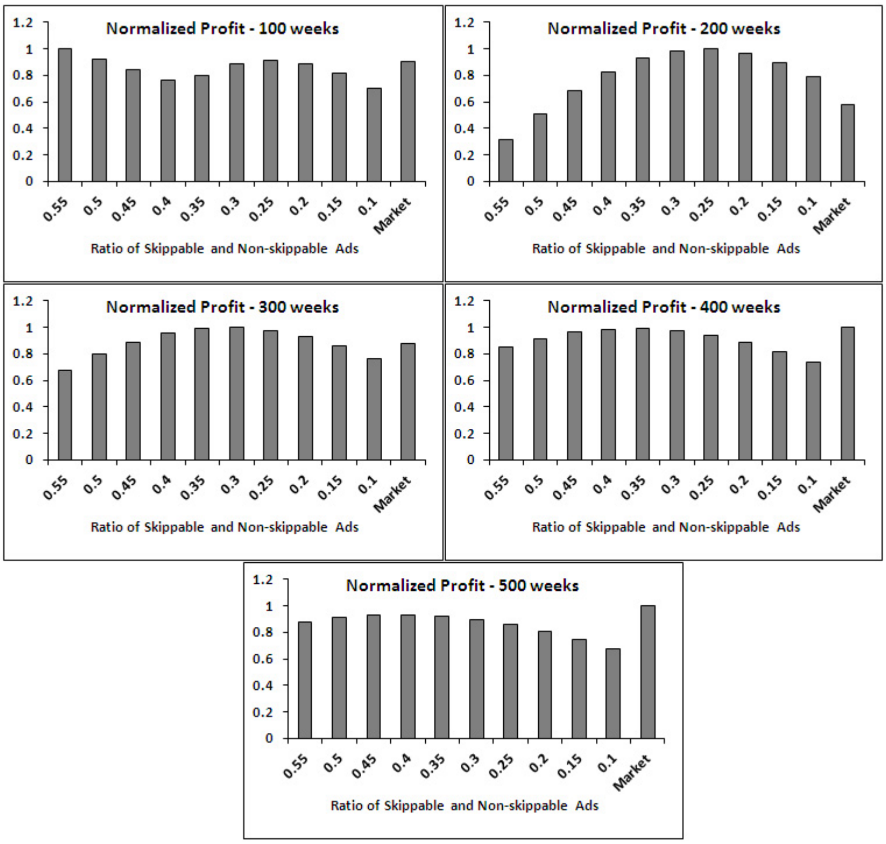

Fig. 3: Normalized Profit for High Content Websites with 95% Skip Rate

to the revenue would be lmiited, and therefore, the increase in non-skippable advertisements has a positive effect on revenue. Thus, for managers of high content type websites with high skip rate, it is important to realize that non-skippable advertisements are unavoidable. So, the focus of the managers should be on building content; faster the website adds content, better it is for the business.

Our last analysis of the System Dynamics model involves the optimal ratio for cases where the content type is low and the skip percentage is high, for example, news websites. We have considered the skip percentage to be 95%. We started our analysis with a ratio of 0.1, we then increased it to 1, and then from 1 to 10 with an increment of 1. The results are shown in Figure 4. Our results indicate that profit increases monotonically with the increase in the ratio, i.e., with an increase in the number of non-skippable advertisements. Ideally, the focus of managers of such websites should be to alleviate the intrusive effects of non-skippable advertisements. The best way of achieving this would be to collect data on the users and use a powerful analytical engine to ensure that users are exposed to relevant advertisements only.

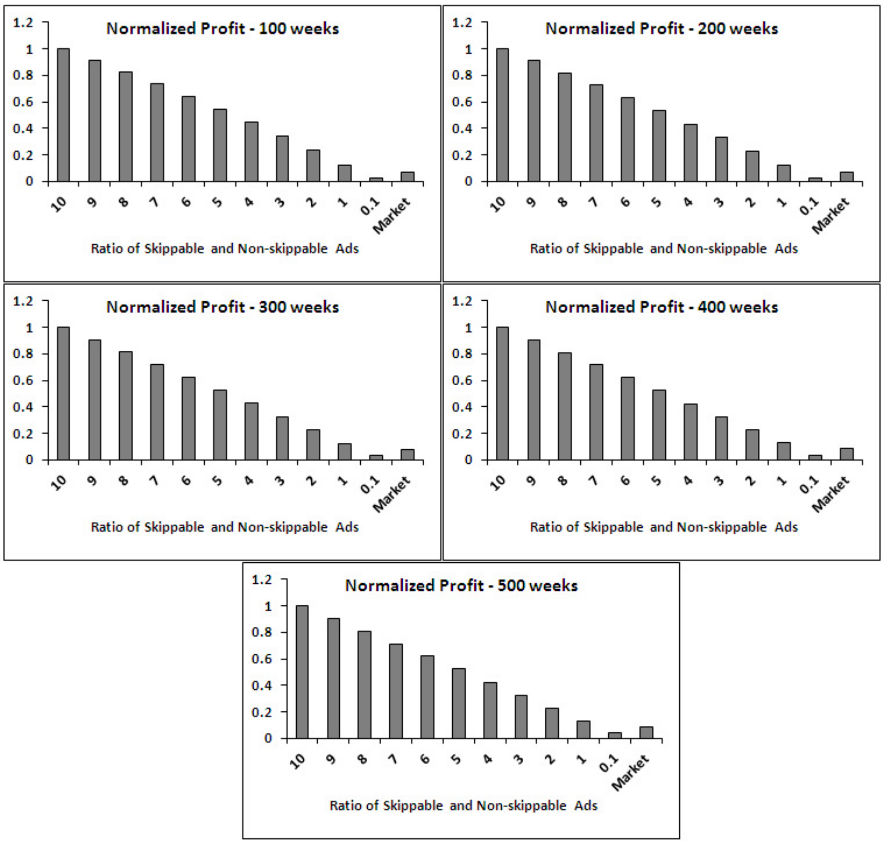

Fig. 4: Normalized Profit for Low Content Websites with 95% Skip Rate

6 Concluding Discussions

Skippable advertisements seem to be an answer to two questions that have plagued advertisers for long - how to reach the viewer who is interested in our product or service, and how to ensure that our advertisement does not create a negative impression on the user who does not find it relevant at present. Skippable advertisements not only resolves the two problems but also ensures that the advertiser does not pay if the video sharing website has not been able to connect him to an interested user. Moreover, it has been found that users of video sharing websites respond favorably to skippable advertisements.

It is therefore, hardly surprising that the introduction of skippable advertisements for in-line videos has resulted in improvements in the advertising revenues of video sharing websites. This innovation is being looked at as the key which will help unlock the business potential of video sharing websites. Although the advantages may suggest a slow demise of traditional advertisements for in-line videos, reality indicates an entirely different scenario. Non-skippable advertisements, although much fewer now, is indeed very much present in in-line videos. This article looks at the role of non-skippable advertisements as a means of effective segmentation of the market for advertisements in in-line videos.

We have modeled the utility function of advertisers characterized by their ad sensitivity, and the content size of the website, and demonstrated that the advertisers with low ad sensitivity would opt for non-skippable advertisements, and would switch to skippable advertisements if there is an increase in content size, or if the ad sensitivity of the advertiser goes up. Furthermore, we have determined the value of ad sensitivity at which the advertisers would switch from non-skippable to skippable advertisements. We have also demonstrated that the profit of the video sharing website increases with the introduction of non-skippable advertisements. The model also allows us to show that for low content websites, it is important for the website to start with non-skippable advertisements, and then slowly shift to skippable advertisements. Video sharing websites can draw on these results to reflect on the content size of their websites and design their strategies accordingly. For example, it would not be wise for a low content website to follow the trend, and discourage non-skippable advertisements; this would have a negative impact on its revenues.

We then consider the effect of the number of non-skippable advertisements increasing beyond a threshold limit. We first develop a two-period model and then extend it to an n-period one to demonstrate that skippable advertisement will reduce in such a case, which in turn will reduce the revenues of the website. Indeed, we demonstrate that skippable advertisements will eventually go down to zero, and finally will result in a reduction of non-skippable advertisements as well. These results indicate that the managers have to be very careful about the number of non-skippable advertisements they allow in their websites. An increase beyond the threshold limit, if continued over a period of time can cause substantial financial losses to the company. We next tackle the question of how we can help the website managers form an idea of what would be the ideal ratio of non-skippable advertisements to skippable advertisements.

To answer the question, we adopted the System Dynamics (SD) methodology to model a two sided platform, like Youtube, and the interaction effects of skippable and non-skippable advertisements. With the help of the SD model we have been able to demonstrate that the correct ratio of non-skippable and skippable advertisements depends on the content size of the website, the stage of the business, and most importantly it varies with time. These findings would serve as useful guidelines in formulating the strategies for managing advertisements for online platforms. The findings imply that managers would have to be aware of the nature of content of the website, i.e., whether the website would qualify as a high content or a low content website, and also have to be alert to the number of non-skippable advertisements in the website.

In this work, we have not considered the indirect effect of non-skippable advertisements on other advertisements, for example, an increase in the number of non-skippable advertisements might lead to a reduction in the impact of a skippable advertisement. That is, it is quite possible that the tendency of users to skip may go up simply because of a high number of non-skippable advertisements. Therefore, the debilitating effect of an increase in the number of non-skippable advertisements can be stronger than what we have considered in this work. In fact, it would be interesting to look at the externality imposed by a non-skippable in-line advertisement on other in-line skippable advertisements in the same video - it might lead to a higher skip rate of the skippable advertisements. Audience externalities has been studied for television audience [22], it may be interesting to extend our work to include externalities for video sharing websites.

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Appendix

Proof of Lemma 1. Part (a). By contradiction, let us assume $\mu^i_1(\lambda,\eta) < 0$, where i $\in \{s, ns\}$. This implies, $\mu^i(\lambda,\eta) > \mu^i(\lambda+\epsilon,\eta)$ for $\epsilon > 0$. From incentive compatibility condition, Equation 3 in the article, for $i \in \{s, ns\}$ we can write

$$U^i(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i(\lambda,\eta) \geqslant U^i(\mu^i(\lambda+\epsilon,\eta),\lambda,\eta) - \tau^i(\lambda+\epsilon,\eta) \tag{1}$$

$$U^i(\mu^i(\lambda+\epsilon,\eta),\lambda+\epsilon,\eta) - \tau^i(\lambda+\epsilon,\eta) \geqslant U^i(\mu^i(\lambda,\eta),\lambda+\epsilon,\eta) - \tau^i(\lambda,\eta) \tag{2}$$

Adding Inequalities 1 and 2, we get

$U^i(\mu^i(\lambda,\eta),\lambda,\eta) + U^i(\mu^i(\lambda+\epsilon,\eta),\lambda+\epsilon,\eta) \geqslant U^i(\mu^i(\lambda+\epsilon,\eta),\lambda,\eta) + U^i(\mu^i(\lambda,\eta),\lambda+\epsilon,\eta)$, which leads to

$$U^i(\mu^i(\lambda,\eta),\lambda,\eta) - U^i(\mu^i(\lambda+\epsilon,\eta),\lambda,\eta) \geqslant U^i(\mu^i(\lambda,\eta),\lambda+\epsilon,\eta) - U^i(\mu^i(\lambda+\epsilon,\eta),\lambda+\epsilon,\eta) \tag{3}$$

From Property (ii) of the advertiser's utility function considered in this paper, $U^i_1(\mu^i(\lambda,\eta),\lambda,\eta) \geqslant 0$ for $i \in \{s, ns\}$. Therefore, Inequality 3 and in turn $\mu^i_1(\lambda,\eta) < 0$, implies

$$U^i_1(\mu^i(\lambda,\eta),\lambda,\eta)\Big|_{\lambda=\lambda} \geqslant U^i_1(\mu^i(\lambda,\eta),\lambda,\eta)\Big|_{\lambda=\lambda+\epsilon} \tag{4}$$

which in turn implies,

$$U^i_{12}(\mu^i(\lambda,\eta),\lambda,\eta) \leqslant 0 \tag{5}$$

which contradicts Property (iii) of the utility function. Hence, $\mu^i_1(\lambda,\eta) \geqslant 0$. □

The proof of $\mu^i_2(\lambda,\eta) \geqslant 0\ \forall i \in \{s, ns\}$ is similar to the above proof and is omitted for the sake of brevity.

Proof of part(b). From incentive compatibility condition, Equation 3 in the article, the first order condition leads to

$$U^i_1(\mu^i(\lambda,\eta),\lambda,\eta).\mu^i_1(\lambda,\eta) - \tau^i_1(\lambda,\eta) = 0\ \forall\ i \in \{s, ns\} \tag{6}$$

$$U^i_1(\mu^i(\lambda,\eta),\lambda,\eta).\mu^i_2(\lambda,\eta) - \tau^i_2(\lambda,\eta) = 0\ \forall\ i \in \{s, ns\} \tag{7}$$

Differentiating the expression, $U^i(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i(\lambda,\eta)$ w.r.t. λ, yields,

$$U^i_1(\mu^i(\lambda,\eta),\lambda,\eta).\mu^i_1(\lambda,\eta) + U^i_2(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i_1(\lambda,\eta) \tag{8}$$

$$= U^i_2(\mu^i(\lambda,\eta),\lambda,\eta) \quad \text{[From Equation 6]} \tag{9}$$

From the Property (ii) of utility function considered in this article, $U^i_2(\mu^i(\lambda,\eta),\lambda,\eta) > 0$ for $i \in \{s, ns\}$. Hence, $U^i(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i(\lambda,\eta)$ is increasing in $\lambda\ \forall\ i \in \{s, ns\}$. □

Proof of part(c). Differentiating the expression, $U^i(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i(\lambda,\eta)$ w.r.t. η, yields,

$$U^i_1(\mu^i(\lambda,\eta),\lambda,\eta).\mu^i_2(\lambda,\eta) + U^i_3(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i_2(\lambda,\eta) \tag{10}$$

$$= U^i_2(\mu^i(\lambda,\eta),\lambda,\eta) \quad \text{[From Equation 7]} \tag{11}$$

From the Property (ii) of utility function considered in this article, $U^i_3(\mu^i(\lambda,\eta),\lambda,\eta) > 0$ for $i \in \{s, ns\}$. Hence, $U^i(\mu^i(\lambda,\eta),\lambda,\eta) - \tau^i(\lambda,\eta)$ is increasing in $\eta\ \forall\ i \in \{s, ns\}$. □

Proof of Lemma 2. Differentiating $\kappa(\lambda, \eta)$ w.r.t. λ yields,

$$[U_1^s(\mu^s(\lambda,\eta),\lambda,\eta).\mu_1^s(\lambda,\eta) + U_2^s(\mu^i(\lambda,\eta),\lambda,\eta) - \tau_1^s(\lambda,\eta)] - [U_1^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta).\mu_1^{ns}(\lambda,\eta)$$
$$+ U_2^{ns}(\mu^i(\lambda,\eta),\lambda,\eta) - \tau_1^{ns}(\lambda,\eta)]$$
$$= U_2^s(\mu^i(\lambda,\eta),\lambda,\eta) - U_2^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) \text{ [From Equation 6]} \tag{12}$$

From Property (iv) of the utility function considered in this article, $U_2^s(\mu^s(\lambda,\eta),\lambda,\eta) \geqslant U_2^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta)$, hence $\kappa_1(\lambda,\eta) \geqslant 0$, i.e. $\kappa(\lambda,\eta)$ is non-decreasing in λ. □

Using similar arguments and Equation 7, it can be easily proved that $\kappa_2(\lambda,\eta) \geqslant 0$, i.e. $\kappa(\lambda,\eta)$ is non-decreasing in η. We omit this proof for the sake of brevity.

Proof of Lemma 3. We prove the statement by contradiction. The complement of this statement given in Lemma 3 is that it is never profit improving for the video sharing website to host non-skippable advertisements or it is always profit improving for the video sharing website to host only skippable advertisements. Now we consider two cases.

Case 1: Consider a situation where $U^s(\mu^s(\overline{\lambda},\eta),\overline{\lambda},\eta) \leqslant 0$ and $U^{ns}(\mu^{ns}(\overline{\lambda},\eta),\overline{\lambda},\eta) > 0$. If the website offers only skippable ad, revenue is zero as no advertiser will go for skippable advertisement following individual rationality.

Now following the net utility derived from non-skippable advertisements by advertiser with ad sensitivity $\overline{\lambda}$, there will be some advertisers $[\lambda^R, \overline{\lambda}]$ who will for non-skippable ads if available where

$$\lambda^R = Min\{\lambda : U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) - \tau_{ns}(\lambda,\eta) - \beta = 0\}$$

The total profit expression of the advertiser is written as:

$$\Pi(\lambda^R,\eta) = \int_{\lambda^R}^{\overline{\lambda}} (\tau^{ns}(\lambda,\eta) - c^{ns}\mu^{ns}(\lambda,\eta)) f(\lambda,\eta) d\lambda$$

If $\tau^{ns}(\lambda,\eta) > c^{ns}\mu^{ns}(\lambda,\eta)$, then $\Pi(\lambda^R,\eta) > 0$ and it justifies the inclusion of non-skippable advertisement.

Case 2: Consider a situation where $U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) > U^s(\mu^s(\lambda,\eta),\lambda,\eta) > 0 \ \forall \lambda > \underline{\lambda}$.

Based on net utility, advertisers with ad sensitivity λ will opt for non-skippable advertisements. Now it will be more profit making for the advertiser if:

$$(\tau^{ns}(\lambda,\eta) - c^{ns}\mu^{ns}(\lambda,\eta)) - (\tau^s(\lambda,\eta) - c^s\mu^s(\lambda,\eta)) \ \forall \lambda > \underline{\lambda}$$
$$(\tau^s(\lambda,\eta) - \tau^{ns}(\lambda,\eta)) < (c^s\mu^s(\lambda,\eta) - c^{ns}\mu^{ns}(\lambda,\eta))$$

These two cases contradicts the statement of keeping only skippable ads to maximize profit and hence the lemma is proved. □

Proof of Lemma 4. Part(a). Consider two websites with two different content sizes: η_1 and η_2, where $\eta_2 > \eta_1$. Let the value of λ at which advertisers intend to switch from non-skippable to skippable advertisements be λ_1^F for website with content size η_1, and λ_2^F for website with content size η_2.

By contradiction, let us assume that $\lambda_1^F < \lambda_2^F$.

From the definition of λ_1^F,

$$U^s(\mu^s(\lambda_1^F,\eta_1),\lambda_1^F,\eta_1) - \tau^s(\lambda_1^F,\eta_1) \geqslant U^{ns}(\mu^{ns}(\lambda_1^F,\eta_1),\lambda_1^F,\eta_1) - \tau^{ns}(\lambda_1^F,\eta_1) - \beta \tag{13}$$

As the preference function $\kappa(\lambda,\eta)$ is non-decreasing in η from Lemma 2 and $\lambda_1^F < \lambda_2^F$ by our assumption above, we can write,

$$U^s(\mu^s(\lambda_1^F,\eta_2),\lambda_1^F,\eta_2) - \tau^s(\lambda_1^F,\eta_2) \geqslant U^{ns}(\mu^{ns}(\lambda_1^F,\eta_2),\lambda_1^F,\eta_2) - \tau^{ns}(\lambda_1^F,\eta_2) - \beta \tag{14}$$

However, from the definition of λ_2^F, it is the minimum value of λ for which the condition written in Equation 14 holds. As $\lambda_1^F < \lambda_2^F$ by our assumption, Equation 14 contradicts the definition of λ_2^F. □

Proof of Lemma 4. Part(b). From the KKT conditions used for determining λ^T, $\lambda^{T*} \neq \underline{\lambda}$ or $\overline{\lambda}$ and hence $u_3, u_4 = 0$. Now based on constraints given in the optimization problem, following possible cases emerge.

(1) Constraint related to maximum number of skippable ad is non-binding and constraint related to maximum number of non-skippable ad is binding, i.e. $u_1 = 0$ and $u_2 \neq 0$.

(2) Constraint related to maximum number of skippable ad is binding and constraint related to maximum number of non-skippable ad is non-binding, i.e. $u_1 \neq 0$ and $u_2 = 0$.

(3) Constraints related to maximum number of skippable and non-skippable ads are binding, i.e. $u_1 \neq 0, u_2 \neq 0$

(4) Constraints related to maximum number of skippable and non-skippable ads are non-binding, i.e. $u_1 = 0, u_2 = 0$

In all the four cases, we prove by contradiction, i.e. $\lambda^{T\prime} > \lambda^T$, where $\lambda^{T\prime}$ and λ^T are respective values of ad sensitivities to satisfy the optimality conditions of the profit maximization problem of video sharing website with content size $\eta + \epsilon$ and η respectively.

Case 1:

Consider the following optimality condition to maximize total profit of video sharing website with λ^T and η,

$$[\tau^s(\lambda^T, \eta) - c^s\mu^s(\lambda^T, \eta)] - [\tau^{ns}(\lambda^T, \eta) - c^{ns}\mu^{ns}(\lambda^T, \eta)] - u_1\mu^s(\lambda^T, \eta) + u_2\mu^{ns}(\lambda^T, \eta) = 0 \tag{15}$$

Writing similar optimality conditions to maximize total profit of video sharing website with $\lambda^{T\prime}$ and $\eta + \epsilon$ and taking the difference with Equation 15 yield,

$$\begin{aligned}&[\tau^s(\lambda^{T\prime}, \eta + \epsilon) - \tau^{ns}(\lambda^{T\prime}, \eta + \epsilon)] - [\tau^s(\lambda^T, \eta) - \tau^{ns}(\lambda^T, \eta)] \\ &= [c^s\mu^s(\lambda^{T\prime}, \eta + \epsilon) - c^{ns}\mu^{ns}(\lambda^{T\prime}, \eta + \epsilon)] - [c^s\mu^s(\lambda^T, \eta) - c^{ns}\mu^{ns}(\lambda^T, \eta)] + [u_2\mu^{ns}(\lambda^T, \eta) \\ &- u_2'\mu^{ns}(\lambda^{T\prime}, \eta + \epsilon)]\end{aligned} \tag{16}$$

From Equation 16, the right hand side value will be positive if $u_2 > u_2'$ and because $c^s > c^{ns}$ and $\mu_2^s(\lambda, \eta) > \mu_2^{ns}(\lambda, \eta)$. Beyond this finding, it is difficult to conclusively contradict the assumption $\lambda^{T\prime} > \lambda^T$.

From the next optimality condition with $u_2 \neq 0$,

$$\int_{\underline{\lambda}}^{\lambda^T} \mu^{ns}(\lambda, \eta)f(\lambda, \eta)d\lambda = \Delta(\eta)h(\eta) \tag{17}$$

RHS of Equation 17 gives the maximum number of non-skippable advertisements allowed to the video sharing website. Change in the maximum limit with respect to the change in content size η yields $\Delta'(\eta)h(\eta) + \Delta(\eta)h'(\eta)$. The resultant change in upper limit of non-skippable advertisements is decided by total change in number of views, i.e. $\Delta(\eta) > 0$, and change in fraction of total views suitable for non-skippable advertisement $h'(\eta) < 0$. Now consider a situation where $\Delta'(\eta)h(\eta) < \Delta(\eta)h'(\eta)$. For that particular scenario,

$$\int_{\underline{\lambda}}^{\lambda^{T\prime}} \mu^{ns}(\lambda, \eta + \epsilon)f(\lambda, \eta + \epsilon)d\lambda < \int_{\underline{\lambda}}^{\lambda^T} \mu^{ns}(\lambda, \eta)f(\lambda, \eta)d\lambda \tag{18}$$

If $\lambda^{T\prime} > \lambda^T$, Equation 18 does not hold if $f_2(\lambda, \eta) \geqslant 0$ with $\mu_2^{ns}(\lambda, \eta) > 0$.

Case 2:

Writing the first optimality conditions to maximize total profit of video sharing website with ad sensitivities λ^T, $\lambda^{T\prime}$ and content sizes η, $\eta+\epsilon$ yields,

$$[\tau^s(\lambda^{T\prime},\eta+\epsilon)-\tau^{ns}(\lambda^{T\prime},\eta+\epsilon)]-[\tau^s(\lambda^T,\eta)-\tau^{ns}(\lambda^T,\eta)]$$
$$=[c^s\mu^s(\lambda^{T\prime},\eta+\epsilon)-c^{ns}\mu^{ns}(\lambda^{T\prime},\eta+\epsilon)]-[c^s\mu^s(\lambda^T,\eta)-c^{ns}\mu^{ns}(\lambda^T,\eta)]-[u_1\mu^{ns}(\lambda^T,\eta)$$
$$-u_1'\mu^{ns}(\lambda^{T\prime},\eta+\epsilon)] \tag{19}$$

Similar to Case 1, Equation 19 cannot be disproved conclusively following the initial assumption of $\lambda^{T\prime} > \lambda^T$. As $u_1 \neq 0$, the next optimality condition describes,

$$\int_{\lambda^T}^{\overline{\lambda}} \mu^s(\lambda,\eta)f(\lambda,\eta)d\lambda = \Delta(\eta)g(\eta) \tag{20}$$

RHS of Equation 20 gives the maximum number of skippable advertisements which can be shown by the video sharing website. Change in the maximum limit with respect to the change in content size η yields $\Delta'(\eta)g(\eta)+\Delta(\eta)g'(\eta)$. The resultant change in upper limit of skippable advertisements is decided by total change in number of views, i.e. $\Delta(\eta) > 0$, and change in fraction of total views suitable for skippable advertisement $g'(\eta) > 0$. Using this finding, following expression can be written,

$$\int_{T\prime}^{\overline{\lambda}} \mu^s(\lambda,\eta+\epsilon)f(\lambda,\eta+\epsilon)d\lambda < \int_{T\prime}^{\overline{\lambda}} \mu^s(\lambda,\eta)f(\lambda,\eta)d\lambda \tag{21}$$

It is evident that Equation 21 does not always hold if $\lambda^{T\prime} > \lambda^T$ and hence it is proved by contradiction.

Case 3:
In this case, we consider both $u_1 \neq 0$ and $u_2 \neq 0$. using teh finding derived from case 1 and case 2,

$$\int_{\underline{\lambda}}^{\lambda^{T\prime}} \mu^{ns}(\lambda,\eta+\epsilon)f(\lambda,\eta+\epsilon)d\lambda < \int_{\underline{\lambda}}^{\lambda^T} \mu^{ns}(\lambda,\eta)f(\lambda,\eta)d\lambda \tag{22}$$

$$\int_{T\prime}^{\overline{\lambda}} \mu^s(\lambda,\eta+\epsilon)f(\lambda,\eta+\epsilon)d\lambda < \int_{T\prime}^{\overline{\lambda}} \mu^s(\lambda,\eta)f(\lambda,\eta)d\lambda \tag{23}$$

Conditions given in Equations 22 and reflem-lamba-T-case-3-2 are contradicted while discussing respective cases with the assumption $\lambda^{T\prime} > \lambda^T$.

Case 4:
In this case, we consider $u_1, u_2 = 0$. Writing the first optimality conditions to maximize total profit of video sharing website with ad sensitivities λ^T, $\lambda^{T\prime}$ and content sizes η, $\eta+\epsilon$ yields,

$$[\tau^s(\lambda^{T\prime},\eta+\epsilon)-\tau^{ns}(\lambda^{T\prime},\eta+\epsilon)]-[\tau^s(\lambda^T,\eta)-\tau^{ns}(\lambda^T,\eta)]$$
$$=[c^s\mu^s(\lambda^{T\prime},\eta+\epsilon)-c^{ns}\mu^{ns}(\lambda^{T\prime},\eta+\epsilon)]-[c^s\mu^s(\lambda^T,\eta)-c^{ns}\mu^{ns}(\lambda^T,\eta)] \tag{24}$$

With $\lambda^{T\prime} > \lambda^T$, this expression can be contradicted for certain restrictive conditions. We do not pursue much for this particular case as it will increase the number of mathematical conditions imposed without substantial increase in practical understanding. □

Proof of Lemma 5. Part(a). The total number of skippable advertisements in Periods 0 and 1, i.e. $S^0(\eta)$ and $S^1(\eta)$ respectively, can be written as,

$$S^0(\eta) = \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda,\eta)f^0(\lambda,\eta)d\lambda \tag{25}$$

$$S^1(\eta) = \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda,\eta)f^1(\lambda,\eta)d\lambda \tag{26}$$

where $f^1(\lambda,\eta) = f^0(\lambda+\xi(DF(\eta)))$, represents the horizontal shift of the probability density function from Period 0 to Period 1 as discussed in the article, and the amount of shifting depends on

$DF(\eta)$.
As $f^0(\lambda,\eta) = 0 \; \forall \; \lambda > \overline{\lambda}$, $f^1(\lambda,\eta) = 0 \; \forall \; \lambda > \overline{\lambda} - \xi(DF(\eta))$. Therefore, we can write

$$S^0(\eta) - S^1(\eta) = \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\lambda^F}^{\overline{\lambda}-\xi(DF(\eta))} \mu^s(\lambda,\eta) f^0(\lambda + \xi(DF(\eta)),\eta) d\lambda \tag{27}$$

Using transformation,
$\lambda + \xi(DF(\eta)) =$ t, $d\lambda =$ dt, in the second integral in Equation 27, it can be written as

$$\begin{aligned} & S^0(\eta) - S^1(\eta) \\ &= \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\lambda^F+\xi(DF(\eta))}^{\overline{\lambda}} \mu^s(t - \xi(DF(\eta)),\eta) f^0(t,\eta) dt && (28) \\ &= \int_{\lambda^F}^{\lambda^F+\xi(DF(\eta))} \mu^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda + \int_{\lambda^F+\xi(DF(\eta))}^{\overline{\lambda}} f^0(\lambda,\eta)[\mu^s(\lambda,\eta) - \mu^s(\lambda - \xi(DF(\eta)),\eta)] d\lambda && (29) \end{aligned}$$

Now, $\mu^s(\lambda,\eta) > 0 \; \forall \; \lambda \in \left[\lambda^F, \lambda^F + \xi(DF(\eta))\right]$,
$f^0(\lambda,\eta) > 0 \; \forall \; \lambda \in \left[\lambda^F, \overline{\lambda}\right]$, and
$\mu^s(\lambda,\eta) > \mu^s(\lambda - \xi(DF(\eta)),\eta)$ as $\mu^s_1(\lambda,\eta) > 0 \; \forall \; \lambda \in \left[\lambda^F, \overline{\lambda}\right]$
Combining the conditions above, gives, $S^0(\eta) - S^1(\eta) > 0$
Therefore, $S^0(\eta) > S^1(\eta)$. □

Proof of Lemma 5, part(b). Revenues in Periods 0 and 1, i.e. $R^0(\eta)$ and $R^1(\eta)$ respectively can be expressed as:

$$R^0(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^0(\lambda,\eta) d\lambda + \int_{\lambda^F}^{\overline{\lambda}} \tau^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda \tag{30}$$

$$R^1(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^1(\lambda,\eta) d\lambda + \int_{\lambda^F}^{\overline{\lambda}} \tau^s(\lambda,\eta) f^1(\lambda,\eta) d\lambda \tag{31}$$

Considering $f^1(\lambda,\eta) = f^0(\lambda + \xi(DF(\eta)))$, the difference between the revenues in Periods 0 and 1 can be expressed as:

$$\begin{aligned} & R^0(\eta) - R^1(\eta) \\ &= \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^0(\lambda + \xi(DF(\eta)),\eta) d\lambda + \\ & \int_{\lambda^F}^{\overline{\lambda}} (\tau^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\lambda^F}^{\overline{\lambda}} (\tau^s(\lambda,\eta) f^0(\lambda + \xi(DF(\eta)),\eta) d\lambda && (32) \end{aligned}$$

Using substitution $\lambda + \xi(DF(\eta)) = t$ in the second and fourth integrals above, we obtain,

$$
\begin{aligned}
&R^0(\eta) - R^1(\eta)\\
&= \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\underline{\lambda}+\xi(DF(\eta))}^{\lambda^F+\xi(DF(\eta))} \tau^{ns}(\lambda - \xi(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda +\\
&\int_{\lambda^F}^{\overline{\lambda}} (\tau^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\lambda^F+\xi(DF(\eta))}^{\overline{\lambda}} (\tau^s(\lambda - \xi(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \qquad (33)\\
&= \int_{\underline{\lambda}}^{\underline{\lambda}+DF(\eta))} \tau^{ns}(\lambda,\eta) f^0(\lambda,\eta) d\lambda + \int_{\underline{\lambda}+\xi(DF(\eta))}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^0(\lambda,\eta) d\lambda -\\
&\int_{\underline{\lambda}+\xi(DF(\eta))}^{\lambda^F} (\tau^{ns}(\lambda - \xi(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda - \int_{\lambda^F}^{\lambda^F+\xi(DF(\eta))} (\tau^{ns}(\lambda - \xi(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda +\\
&\int_{\lambda^F}^{\lambda^F+\xi(DF(\eta))} \tau^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda + \int_{\lambda^F+\xi(DF(\eta))}^{\overline{\lambda}} (\tau^s(\lambda,\eta) f^0(\lambda,\eta) d\lambda - \int_{\lambda^F+\xi(DF(\eta))}^{\overline{\lambda}} (\tau^s(\lambda - DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \qquad (34)\\
&= \int_{\underline{\lambda}}^{\underline{\lambda}+DF(\eta))} \tau^{ns}(\lambda,\eta) f^0(\lambda,\eta) d\lambda + \int_{\underline{\lambda}+\xi(DF(\eta))}^{\lambda^F} (\tau^{ns}(\lambda,\eta) - \tau^{ns}(\lambda - \xi(DF(\eta)),\eta)) f^0(\lambda,\eta) d\lambda +\\
&\int_{\lambda^F}^{\lambda^F+\xi(DF(\eta))} (\tau^s(\lambda,\eta) - \tau^{ns}(\lambda - \xi(DF(\eta)),\eta)) f^0(\lambda,\eta) d\lambda +\\
&\int_{\lambda^F+\xi(DF(\eta))}^{\overline{\lambda}} ((\tau^s(\lambda,\eta) - \tau^s(\lambda - \xi(DF(\eta)),\eta)) f^0(\lambda,\eta) d\lambda \qquad (35)
\end{aligned}
$$

Following the properties of fee function $\tau(\lambda,\eta)$ considered in the paper $\tau_1^{ns}(\lambda,\eta) > 0$, $\tau_1^s(\lambda,\eta) > 0$, and $\tau^s(\lambda,\eta) > \tau^{ns}(\lambda - \xi(DF(\eta)))\forall\ \lambda \in \left[\lambda^F, \lambda^F + \xi(DF(\eta))\right]$,

$R^0(\eta) - R^1(\eta) > 0$ or $R^0(\eta) > R^1(\eta)$. □

Proof of Corollary 1. Number of total skippable advertisements is given as,

$$
\begin{aligned}
S^i(\eta) &= \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda,\eta) f^i(\lambda,\eta) d\lambda\\
&= \int_{\lambda^F}^{\overline{\lambda}} \mu^s(\lambda,\eta) f^0(\lambda + i.\xi(DF(\eta)),\eta) d\lambda \qquad (36)\\
&= \int_{\lambda^F+i.\xi(DF(\eta))}^{\overline{\lambda}} \mu^s(\lambda - i.\xi(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \text{ [Using transformation } \lambda + i.\xi(DF(\eta),\eta) = t] \qquad (37)
\end{aligned}
$$

For $S^i(\eta) = 0$, $\lambda^F + i.\xi(DF(\eta)) = \overline{\lambda}$ and hence $i = \frac{\overline{\lambda}-\lambda^F}{\xi(DF(\eta))}$. □

Proof of Lemma 6. Part(a). For a website with content size η, let $NS^k(\eta)$ denote the number of non-skippable advertisements in period k, $\forall\ k \geqslant 0$. The number of non-skippable advertisements in any period k, i.e. $NS^k(\eta)$ is given as

$$NS^k(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda,\eta) f^k(\lambda,\eta) d\lambda$$

$$
\begin{aligned}
\text{where } f^k(\lambda,\eta) &= f^0(\lambda + \xi(DF^0(\eta)) + \xi(DF^1(\eta)) + \ldots + \xi(DF^{k-1}(\eta)),\eta)\\
&= f^0(\lambda + \Xi^{k-1}(DF(\eta)),\eta)
\end{aligned}
$$

$$\text{Defining } \Xi^{k-1}(DF(\eta)) = \xi(DF^0(\eta)) + \xi(DF^1(\eta)) + \ldots + \xi(DF^{k-1}(\eta))$$
$$\text{then } \Xi^{k}(DF(\eta)) = \Xi^{k-1}(DF(\eta)) + \xi(DF^k(\eta))$$

Following the above notations,

$$NS^k(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda,\eta) f^0(\lambda + \Xi^{k-1}(DF(\eta)),\eta) d\lambda \tag{38}$$
$$NS^{k+1}(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda,\eta) f^0(\lambda + \Xi^{k}(DF(\eta)),\eta) d\lambda \tag{39}$$
$$\tag{40}$$

The difference in number of non-skippable advertisements in two successive time periods k and $k+1$, i.e. $NS^k(\eta) - NS^{k+1}(\eta)$, can be expressed as:

$$NS^k(\eta) - NS^{k+1}(\eta) =$$
$$\int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda,\eta) f^0(\lambda + \Xi^{k-1}(DF(\eta)),\eta) d\lambda - \int_{\underline{\lambda}}^{\lambda^F} \mu^{ns}(\lambda,\eta) f^0(\lambda + \Xi^{k}(DF(\eta)),\eta) d\lambda \tag{41}$$

Using transformation $\lambda + \Xi^{k-1}(DF(\eta)) = t1$ in the first integral and $\lambda + \Xi^{k}(DF(\eta)) = t2$ in the second integral, we get

$$\begin{aligned} NS^k(\eta) - NS^{k+1}(\eta) &= \\ &= \int_{\underline{\lambda}+\Xi^{k-1}(DF(\eta))}^{\lambda^F+\Xi^{k-1}(DF(\eta))} \mu^{ns}(t1 - \Xi^{k-1}(DF(\eta)),\eta) f^0(t1,\eta) dt1 \\ &- \int_{\underline{\lambda}+\Xi^{k}(DF(\eta))}^{\lambda^F+\Xi^{k}(DF(\eta))} \mu^{ns}(t2 - \Xi^{k}(DF(\eta)),\eta) f^0(t2,\eta) dt2 \end{aligned} \tag{42}$$

Substituting $\lambda = t1$ in the first integral and $\lambda = t2$ in the second integral, we get

$$\begin{aligned} NS^k(\eta) - NS^{k+1}(\eta) &= \\ &= \int_{\underline{\lambda}+\Xi^{k-1}(DF(\eta))}^{\lambda^F+\Xi^{k-1}(DF(\eta))} \mu^{ns}(\lambda - \Xi^{k-1}(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \\ &- \int_{\underline{\lambda}+\Xi^{k}(DF(\eta))}^{\lambda^F+\Xi^{k}(DF(\eta))} \mu^{ns}(\lambda - \Xi^{k}(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \end{aligned} \tag{43}$$

$$\begin{aligned} &= \int_{\underline{\lambda}+\Xi^{k-1}(DF(\eta))}^{\underline{\lambda}+\Xi^{k}(DF(\eta))} \mu^{ns}(\lambda - \Xi^{k-1}(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \\ &+ \int_{\underline{\lambda}+\Xi^{k}(DF(\eta))}^{\lambda^F+\Xi^{i}(DF(\eta))} (\mu^{ns}(\lambda - \Xi^{k-1}(DF(\eta)),\eta) - \mu^{ns}(\lambda - \Xi^{k}(DF(\eta)),\eta)) f^0(\lambda,\eta) d\lambda \\ &- \int_{\lambda^F+\Xi^{i-1}(DF(\eta))}^{\lambda^F+\Xi^{k}(DF(\eta))} \mu^{ns}(\lambda - \Xi^{k}(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \end{aligned} \tag{44}$$

In Equation 44, the first two integrals are positive as $\Xi^k(DF(\eta)) > \Xi^{k-1}(DF(\eta))$ and $\mu_1^{ns}(\lambda,\eta) > 0$, and the third one is zero by definition of period i from Corollary 1, therefore, $NS^k - NS^{k+1} > 0$. □

——-

Proof of Lemma 6. Part(b). Note that the revenue for a video sharing website in period k, $R^k(\eta)$ where $k > i$, and i is the first period in which number of skippable advertisements becomes zero, is equal to the revenue from non-skippable advertisements. The revenue in period k therefore can be given as

$$R^k(\eta) = \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^k(\lambda,\eta) d\lambda$$

$$\begin{aligned} \text{where } f^k(\lambda,\eta) &= f^0(\lambda + \xi(DF^0(\eta)) + \xi(DF^1(\eta)) + \ldots + \xi(DF^{k-1}(\eta)),\eta) \\ &= f^0(\lambda + \Xi^{k-1}(DF(\eta)),\eta) \end{aligned}$$

$$\text{and } \Xi^k(DF(\eta)) = \Xi^{k-1}(DF(\eta)) + \xi(DF^k(\eta))$$

The difference in the revenue from non-skippable advertisements in two successive time periods k and $k+1$, $R^k - R^{k+1}$, can be expressed as :

$$R^k - R^{k+1} = \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^k(\lambda,\eta) d\lambda - \int_{\underline{\lambda}}^{\lambda^F} \tau^{ns}(\lambda,\eta) f^{k+1}(\lambda,\eta) d\lambda$$

Using transformations similar to the ones used in Proof of Part (a) above, and definition of Period i

$$\begin{aligned} &= \int_{\underline{\lambda}+\Xi^{k-1}(DF(\eta))}^{\underline{\lambda}+\Xi^k(DF(\eta))} \tau^{ns}(\lambda - \Xi^{k-1}(DF(\eta)),\eta) f^0(\lambda,\eta) d\lambda \\ &+ \int_{\underline{\lambda}+\Xi^k(DF(\eta))}^{\lambda^F+\Xi^{i-1}(DF(\eta))} (\tau^{ns}(\lambda - \Xi^{k-1}(DF(\eta)),\eta) - \tau^{ns}(\lambda - \Xi^k(DF(\eta)),\eta)) f^0(\lambda,\eta) d\lambda \end{aligned} \quad (45)$$

(46)

As $\Xi^k(DF(\eta)) > \Xi^{k-1}(DF(\eta))$ and $\tau_1^{ns}(\lambda,\eta) > 0$, $R^k(\eta) > R^{k+1}(\eta)$. $\square$

Example

1 Functional Forms of Expressions Used

Functional forms of skippable and non-skippable utility functions are expressed as:

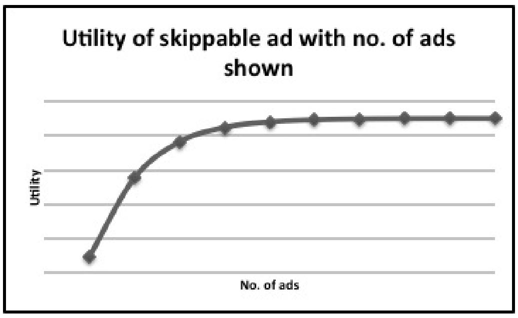
$$U^s(\mu^s(\lambda,\eta),\lambda,\eta) = C(\lambda\eta)\frac{e^{p^s+q^s\mu^s}}{1+e^{p^s+q^s\mu^s}}\ \forall\lambda \in \left[\underline{\lambda},\overline{\lambda}\right],\eta \in \left[\underline{\eta},\overline{\eta}\right]$$

$$U^{ns}(\mu^{ns}(\lambda,\eta),\lambda,\eta) = C(\lambda\eta)\frac{e^{p^{ns}+q^{ns}\mu^{ns}}}{1+e^{p^{ns}+q^{ns}\mu^{ns}}}\forall\lambda \in \left[\underline{\lambda},\overline{\lambda}\right],\eta \in \left[\underline{\eta},\overline{\eta}\right]$$

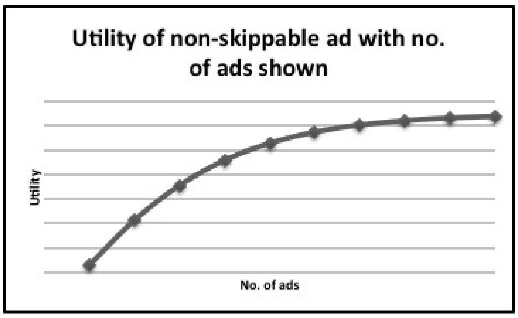
with $p^s > p^{ns}$ and $q^s > q^{ns}$

Figure 1 depicts the change in utility with respect to the change in number of views for skippable and non-skippable advertisements. It shows the increase in utility with increase in number of views but with a decreasing rate. Also the parameter values p^s, q^s, p^{ns} and q^{ns} ensure that the utility derived is more from a skippable advertisement than a non-skippable one given the same number of views.

Figures 2 and 3 show the change in skippable and non-skippable utility with changes in

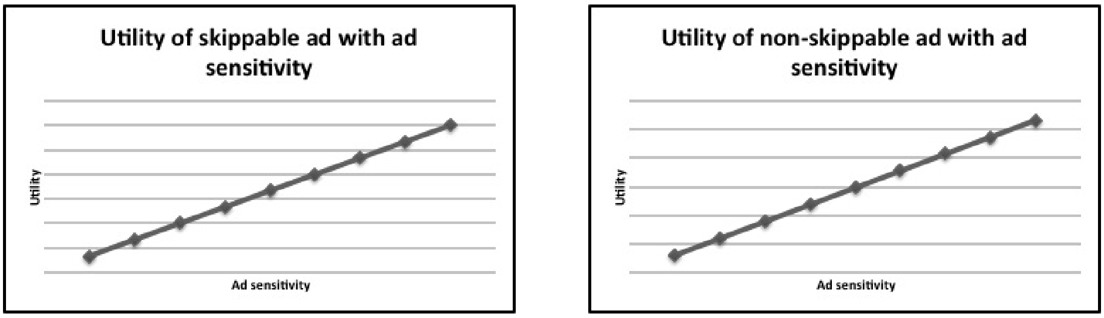


(a) Relation between skippable utility and number of ads shown



(b) Relation between non-skippable utility and number of ads shown

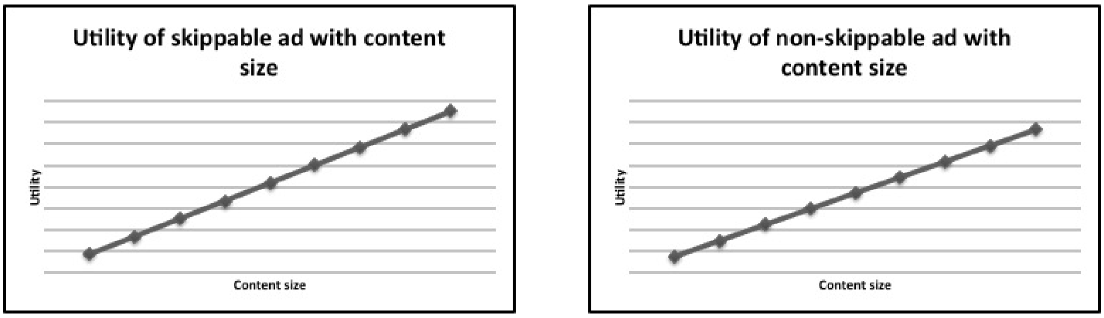
Fig. 1: Change in skippable and non-skippable utility with number of advertisements shown



(a) Relation between skippable utility and λ

(b) Relation between non-skippable utility and λ

Fig. 2: Change in skippable and non-skippable utility with respect to ad sensitivity



(a) Relation between skippable utility function and η

(b) Relation between Non-skippable utility function and η

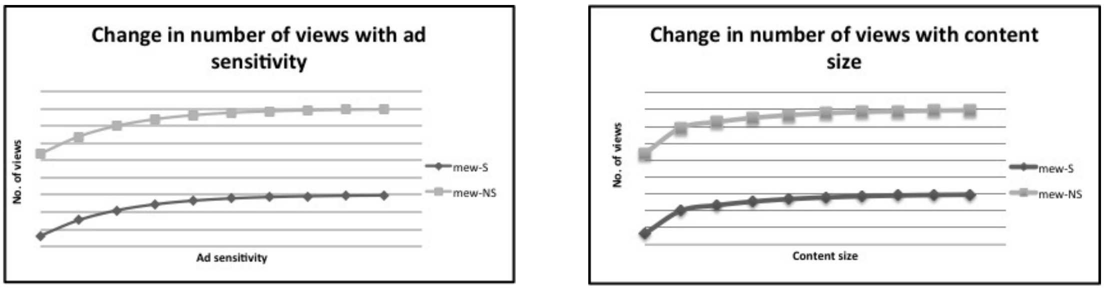
Fig. 3: Change in skippable and non-skippable utility with respect to content size

λ and η. In both cases, utility increases with increasing ad sensitivity λ and content size η as defined in the properties of utility function.

The functional forms of number of views for the non-skippable and skippable advertisements are as follows:

$$\mu^{ns}(\lambda, \eta) = K^{ns} \frac{e^{f^{ns} + g^{ns}\eta + h^{ns}\lambda}}{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}\lambda}}$$
$$\mu^{s}(\lambda, \eta) = K^{s} \frac{e^{f^{s} + g^{s}\eta + h^{s}\lambda}}{1 + e^{f^{s} + g^{s}\eta + h^{s}\lambda}}$$

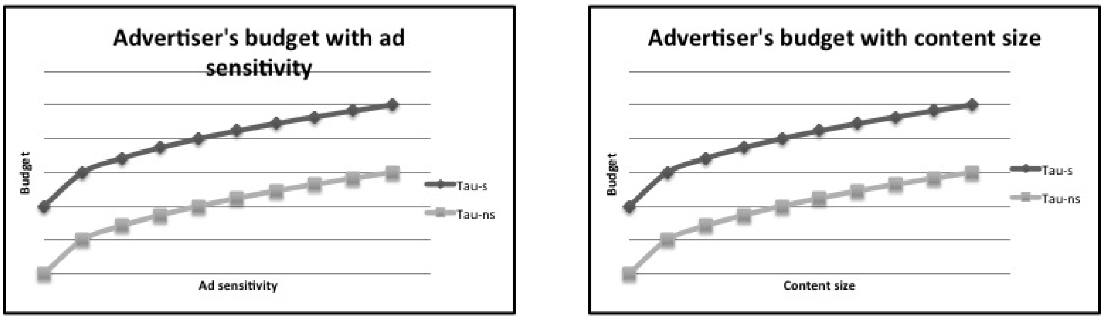
To ensure $\mu^{ns}(\lambda, \eta) \geqslant \mu^{s}(\lambda, \eta)$, we impose the conditions on parameters such as $f^{ns} > f^{s}$, $g^{ns} > g^{s}$, $h^{ns} > h^{s}$ and $K^{ns} > K^{s}$. Figure 4 shows the change in numbers of skippable and non-skippable advertisement views with respect to change in ad sensitivity λ and content size λ. As per the property defined, number of view is increasing with both the parameters.



(a) Relation between number of ad views and λ

(b) Relation between No. of ad views and η

Fig. 4: Change in number of skippable and non-skippable ad views with respect to ad sensitivity and content size



(a) Relation between budget and λ

(b) Relation between budget and η

Fig. 5: Change in budget for skippable and non-skippable ads with respect to ad sensitivity and content size

The budget function for non-skippable and skippable advertisements are given below:

$$\tau^{ns}(\lambda,\eta) = B\sqrt{(\lambda\eta)}$$
$$\tau^{s}(\lambda,\eta) = D + B\sqrt{(\lambda\eta)}$$
$$\text{where } D = \alpha C[\frac{e^{p^s+q^s\mu^s}}{1+e^{p^s+q^s\mu^s}} - \frac{e^{p^{ns}+q^{ns}\mu^{ns}}}{1+e^{p^{ns}+q^{ns}\mu^{ns}}}]$$

Following the expressions, number of views is more for non-skippable advertisements within the same budget because of lower per unit price. Also the budget assigned is more if skippable advertisement is chosen. Figure 5 depicts the change in budget for both skippable and non-skippable advertisements with respect to ad sensitivity λ and content size η.

Net utility expressions for non-skippable and skippable advertisements are written as:

$$(NU)^{ns} = C(\lambda\eta)\frac{e^{p^{ns}+q^{ns}\mu^{ns}}}{1+e^{p^{ns}+q^{ns}\mu^{ns}}} - [B\sqrt{(\lambda\eta)}]$$
$$(NU)^{s} = C(\lambda\eta)\frac{e^{p^s+q^s\mu^s}}{1+e^{p^s+q^s\mu^s}} - [D + B\sqrt{(\lambda\eta)}]$$

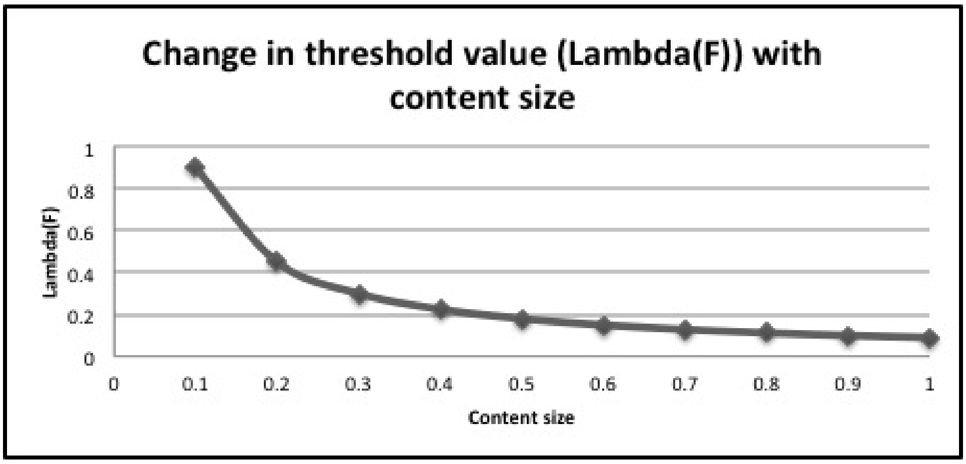


Fig. 6: Change in Threshold ad sensitivity with change in content size (η)

The preference function $\kappa(\lambda, \eta)$ is defined as the difference between net utilities of skippable and non-skippable utility functions:

$$(NU)^s - (NU)^{ns} = C(\lambda\eta)[\frac{e^{p^s+q^s\mu^s}}{1+e^{p^s+q^s\mu^s}} - \frac{e^{p^{ns}+q^{ns}\mu^{ns}}}{1+e^{p^{ns}+q^{ns}\mu^{ns}}}] - D$$

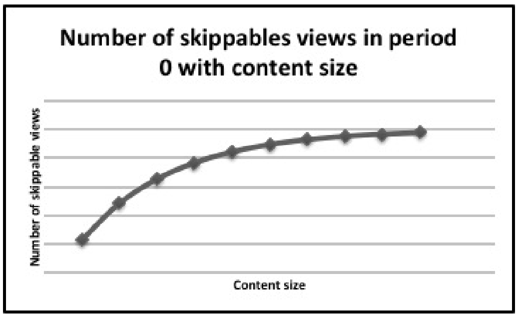
We calculate the threshold value $\lambda^F(\eta) = \alpha/\eta$ by equating net utilities derived from non-skippable and skippable advertisements. Figure 6 shows the change in λ^F with change on content size η.

We consider the joint probability density function $f(\lambda, \eta)$ to follow uniform distribution. Hence it is represented as,

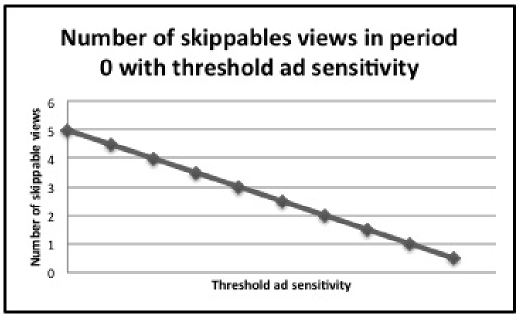
$$f(\lambda, \eta) = 1 \;\forall \lambda \in \left[\underline{\lambda}, \overline{\lambda}\right], \eta \in \left[\underline{\eta}, \overline{\eta}\right]$$

In this context, we have attempted with other distribution functions, e.g. exponential distributions, but obtained similar results with more complex results. Hence we chose to continue with uniform distribution for convenience in representation without loosing on inferences derived.

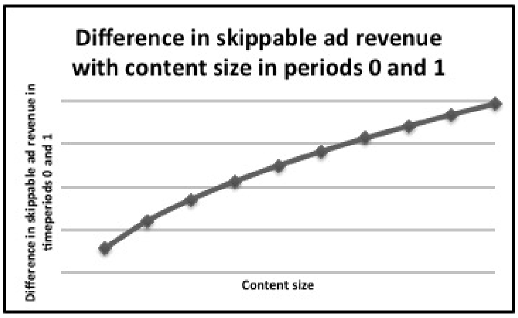
Total number of non-skippable and skippable advertisements at the end of time period 0 is



(a) Number of skippable ad views in period 0 with η



(b) Number of skippable ad views in period 0 with threshold λ^F



(c) Difference in skippable ad views in periods 0 and 1 with η

Fig. 7: Change in number of skippable ads over period with change in content size and threshold ad sensitivity λ^F

expressed as:

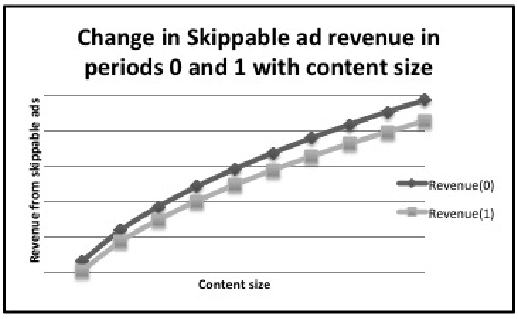
$$S^0(\eta) = \frac{K^s}{C^s} \ln \frac{1 + e^{f^s + g^s\eta + h^s\overline{\lambda}}}{1 + e^{f^s + g^s\eta + h^s\lambda^F}}$$

$$S^1(\eta) = \frac{K^s}{C^s} \ln \frac{1 + e^{f^s + g^s\eta + h^s(\overline{\lambda} - k)}}{1 + e^{f^s + g^s\eta + h^s\lambda^F}}, \text{ where k is the parameter by which ad sensitivity shifts in period 1}$$

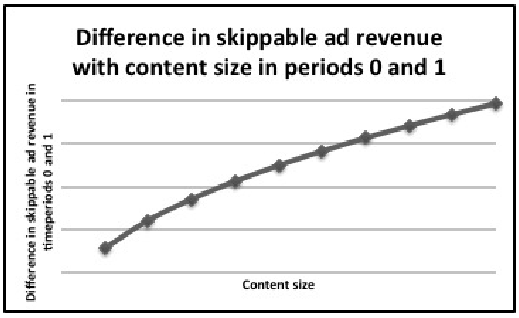
$$S^0(\eta) - S^1(\eta) = \frac{K^s}{C^s} \ln \frac{1 + e^{f^s + g^s\eta + h^s\overline{\lambda}}}{1 + e^{f^s + g^s\eta + h^s(\overline{\lambda} - k)}}$$

Figure 7a shows the change in actual number of skippable advertisements with content size η. We view the change in difference between the number of skippable advertisements with changing content size in Figure 8b.

The revenues from skippable ad views and corresponding difference in periods 0 and 1 are

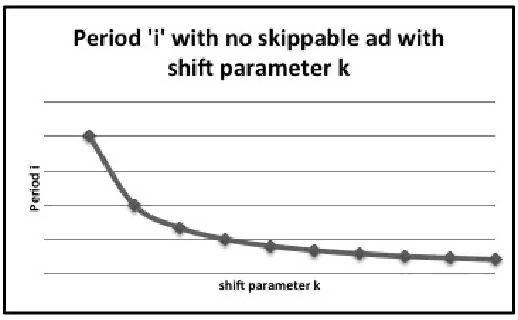


(a) Skippable ad revenues in periods 0 and 1 with change in η

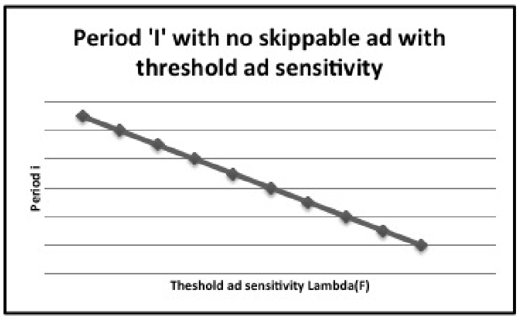


(b) Difference of skippable ad revenue in periods 0 and 1 with change in η

Fig. 8: Change in skippable ad revenue in two consecutive time periods 0 and 1 with change in content size



(a) Change in period i with no skippable ad with change in shifting parameter



(b) Change in period i with no skippable ad with change in λ^F

Fig. 9: Change in period i with no skippable ad with respect to k and λ^F

expressed as follows:

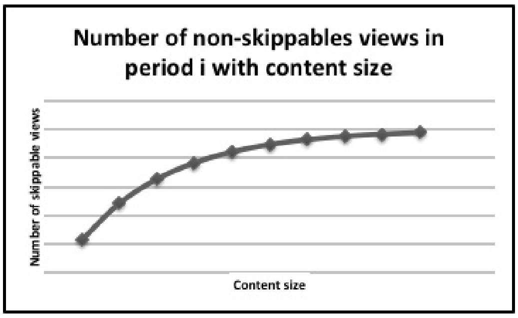
$$R^0(\eta) = \frac{2}{3}B\sqrt(\eta)(\overline{\lambda}^{\frac{3}{2}} - \underline{\lambda}^{\frac{3}{2}}) + D(\overline{\lambda} - \lambda^F)$$

$$R^1(\eta) = \frac{2}{3}B\sqrt(\eta)((\overline{\lambda} - k)^{\frac{3}{2}} - \underline{\lambda}^{\frac{3}{2}}) + D((\overline{\lambda} - k) - \lambda^F), \text{ with k being shifting parameter}$$

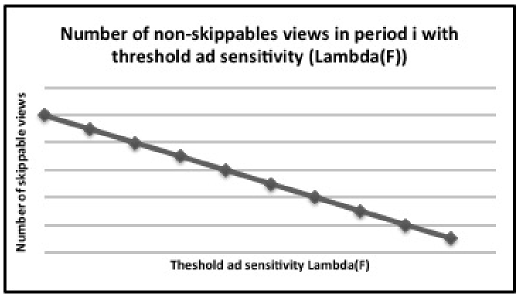
$$R^0(\eta) - R^1(\eta) = Dk + \frac{2}{3}B\sqrt{\eta}(\overline{\lambda}^{\frac{3}{2}} - (\overline{\lambda} - k)^{\frac{3}{2}})$$

Period $i = \frac{\overline{\lambda} - \lambda^F}{k}$ is defined as the minimum period in which skippable ad becomes zero. Change in period i with respect to changes in shift parameter k and threshold ad sensitivity λ^F is depicted in Figure 9.

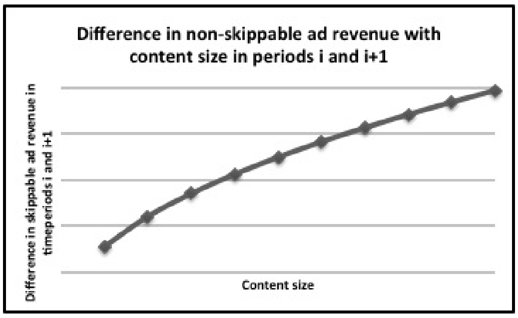
After period i, number of non-skippable ads will start decreasing as still $DF(\eta) > 0$, i.e. total number of non-skippable ad remains more than the threshold value $TS(\eta)$. The number of total non-skippable advertisements in time periods i and $i+1$ and the difference between



(a) Number of non-skippable ad views in period i with η



(b) Number of non-skippable ad views in period i with threshold λ^F



(c) Difference in non-skippable ad views in periods i and $i+1$ with change in η

Fig. 10: Change in numbers of non-skippable ads over period with change in η and threshold λ^F

these two are given as per the following expressions:

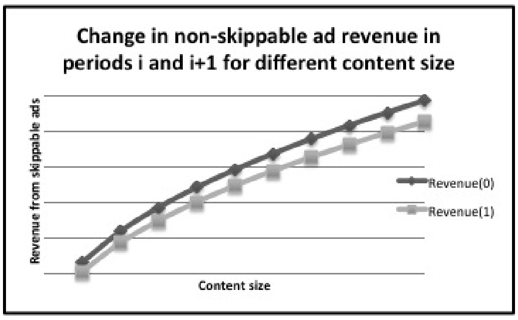
$$NS^i(\eta) = \frac{K^{ns}}{c^{ns}} \ln \frac{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}\lambda^F}}{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}\underline{\lambda}}}$$

$$NS^{i+1}(\eta) = \frac{K^{ns}}{C^{ns}} \ln \frac{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}(\lambda^F - k)}}{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}\underline{\lambda}}}, \text{ where k is the shifting parameter of ad sensitivity}$$

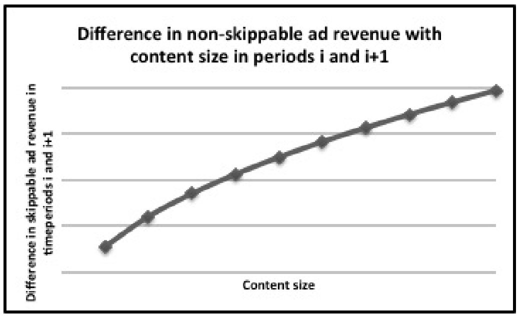
$$NS^i(\eta) - NS^{i+1}(\eta) = \frac{K^{ns}}{C^{ns}} \ln \frac{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}\lambda^F}}{1 + e^{f^{ns} + g^{ns}\eta + h^{ns}(\lambda^F - k)}}$$

Figures 10a and 10b understand the behaviour of views of non-skippable advertisements in period i with change in content size η and threshold ad sensitivity λ^F. The difference between number of non-skippable ad views between two consecutive periods i and $i+1$ with change in content size η is depicted in Figure 11b.

At period i with no skippable ads, the revenues generated from non-skippable advertise-



(a) Non-skippable ad revenues in periods i and $i+1$ with change in η



(b) Difference of non-skippable ad revenue in periods i and $i+1$ with change in η

Fig. 11: Change in non-skippable ad revenue in two consecutive time periods i and $i+1$ with change in content size

ments and the difference between two successive time periods i and $i+1$ are given as follows:

$$R^i(\eta) = \frac{2}{3}B\sqrt(\eta)(\lambda^{F\frac{3}{2}} - \underline{\lambda}^{\frac{3}{2}})$$
$$R^{i+1}(\eta) = \frac{2}{3}B\sqrt(\eta)((\lambda^F - k)^{\frac{3}{2}} - \underline{\lambda}^{\frac{3}{2}}),\ \text{with k being shifting parameter}$$
$$R^i(\eta) - R^{i+1}(\eta) = \frac{2}{3}B\sqrt{\eta}(\lambda^{F\frac{3}{2}} - (\lambda^F - k)^{\frac{3}{2}})$$

Figure 11a shows that the revenue from non-skippable advertisement increase with increase in content size η in periods i and $i+1$ but revenue in period $i+1$ is less because of reduction in number of non-skippable ad views. That difference is indicated in Figure 11b with change in content size η.