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**Channel coordination using Option Contract under simultaneous price
and inventory competition**

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# Channel coordination using Option Contract under simultaneous price and inventory competition

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**Abstract**

Supply chain network consisting of a common supplier and multiple downstream retailers faces channel conflict due to both price and inventory competition. In this context wholesale price contract fails to coordinate the channel and leads to conflicts of interest between supply chain agents. In this article we establish that such a supply chain network can be coordinated by option contract mechanism. We show that in the presence of option contract a pure strategy unique Nash equilibrium exists for the retailers' game; the supplier can coordinate the entire supply chain system to achieve the best performance, even in the presence of retailer competition in both price and inventory. We analytically demonstrate that option contract provides the supplier with better flexibility in terms of profit allocation compared to other channel coordinating contract like buyback. We also calculate the limitation of this contract form and show that this contract form can only achieve coordination with a limited number of retailers. We conclude by discussing the managerial implications of the results and the directions of future research.

*Keywords:* Option contract, coordination, supply chain management, equilibrium analysis, pricing, inventory

## 1. Introduction

In a supply chain, buyers prefer to have ordering flexibility due to demand uncertainty and it also helps her to avoid high inventory cost. On the other hand, suppliers prefer to have full orders in place so that she can avoid situations of either over production or under production. This results in conflict between the supplier and the buyer, and often leads to inefficient performance of the

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supply chain due to sub-optimal decisions taken by the individual stakeholders of the chain. From apparel retailers[1] to toy makers (Mattel Inc), various companies have faced situations of over-ordering or unexpected change in market demand. In today's world of globalization, supply chains are increasingly turning into supply networks and designing coordination mechanism for such network is also becoming more challenging. In order to achieve coordination, supplier-buyer relationship has dramatically changed in recent years. Partnerships have become prevalent among stakeholders. Channel coordinating contracts like, buy-back contract (Pasternack, 1985), quantity-discount contract (Weng, 1995), revenue-sharing contract (Cachon and Lariviere, 2005) etc. have been designed to align supply chain members' incentives to adopt the optimal action throughout the supply chain network. These feasible coordinating supply contracts vary in terms of profit allocation among the supply chain agents. Moreover, these contract forms cannot necessarily enforce retailers to place full orders upfront, as they prefer to have ordering flexibility in order to accommodate fluctuating market demand. Therefore suppliers fail to hedge against the risks of over- or under-production. These conflicts between the retailer and the supplier lead to sub-optimal actions by supply chain agents and eventually results in an inefficient supply chain. Though wholesale price contract cannot coordinate a supply chain (Cachon, 2003), it enjoys huge popularity across industries for its easy implement-ability. This contract mechanism also attributes to conflicts of interest among supply chain agents, resulting in sub-optimal decisions for supplier or buyers/ retailers or both.

In the context of dyadic relationship between a supplier and a retailer, option contract provides us with a simple mechanism such that the retailer can commit to a quantity at early stage of production so that the supplier can plan for her production and reduce her production related risks (Zhao et al., 2010). In the contexts of different industries like fashion apparel, toys, electronics etc option contract has been extensively applied (Eppen and Iyer, 1997; Carbone, 2001; Billington, 2002; Barnes-Schuster et al., 2002). Most of these industries face high degree of market uncertainty industry and large inventory carrying cost. Option contract is characterized by two parameters: (i) option price, $o$, and (ii) exercise price, $e$. The option price designates an allowance paid by a buyer to the supplier for reserving one unit of the production capacity. While exercising the option, buyer has to pay the exercise price to the supplier for purchase of

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[1] "Apparel Retailers Full of Hot Air", Published on Febuary 6, 2012. WSJ Report. Retrieved from: http://blogs.wsj.com/overheard/2012/02/06/apparel-retailers-full-of-hot-air/

every unit of product. However, the extant literature on option contract mostly focuses on dyadic supply chain relationship.

Motivated by these observations, we look into option contract mechanism for answering two key questions – (i) whether a channel coordinating contract can be designed for a supply chain network such that the supplier can hedge for her production and (ii) whether optimal option contract provides the supplier with any flexibility in terms of profit allocation, compared to other coordinating contracts. In this article we focus on the coordination of a supply chain network comprising of one supplier and multiple heterogeneous buyers through option contract. We analyze under what condition(s) pure-strategy Nash equilibrium can be achieved in such scenario. We subsequently look into what such equilibrium means in the context of price competition and inventory competition. This exercise helps us to analytically demonstrate the flexibility of option contract in terms of profit allocation and also to identify the limitation of this contract form.

The rest of the article is organized as follows: We review the extant literature of contracts in Section 2. We describe the option contract model, retailer's game, supplier's game, and existence of pure-strategy Nash equilibrium in Section 3. In Section 4 we discuss the application of option contract in the context of price competition, inventory competition, linear deterministic demand function, and the special case of identical retailers with proportional allocation of demand. We conclude in Section 5 by presenting managerial implications and directions of future research.

## 2. Literature Review

We review the extant literature that explores the use of option contract in the context of supply chain coordination. Usage of options in supply chain primarily focuses on operational flexibility and economic flexibility (Zhao et al., 2010). To demonstrate how option contract holds the advantage of adding flexibility to coordination mechanism, we also briefly review other coordinating contract mechanisms.

In one of the early works on option, Eppen and Iyer (1997) propose backup agreement contract and show that such agreement influences the expected profit level of supply chain members. Cachon and Lariviere (2001) investigate option contract under voluntary compliance and prove that under such condition supplier has substantial flexibility. Wu et al. (2002) design

optimal bidding strategies for a dyadic relationship between one supplier and one buyer. Barnes-Schuster et al. (2002) have demonstrated that options can provide flexibility while facing demand uncertainty and can achieve channel coordination. Cheng et al. (2003) consider an option model where the buyer would commit to a certain minimum quantity and would have additional option of acquiring an additional quantity from the supplier, by procuring options. Brown and Lee (2003) analytically examine the impact of demand information updating and signal quality on ordering decisions with an options-futures contract. Spinler et al. (2003) develop an analytical framework for the valuation of options contracts for physical delivery that enable risk-sharing between the trading partners by considering spot market price risk and the seller's marginal cost risk. Shen and Pang (2004) develop a capacity options model in the two-echelon supply chain where procurement happens from the spot market. They incorporate both demand uncertainty and supply uncertainty in their model and through Stackelberg game approach they investigate the effect of the uncertainties on the optimal decisions. Wu and Kleindorfer (2005) develop an analytical framework for analyzing business-to-business (B2B) supply chains by integrating contract procurement markets with spot markets using options and forwards. They consider the structure of the optimal portfolios of contracting and spot market transactions for the buyer and seller, and the market equilibrium pricing. Wang and Tsao (2006) analyze the buyer's perspective in a bidirectional option such that the buyer is able to adjust the initial order quantity in either upwards or downward direction. Wang and Liu (2007) model channel coordination and risk sharing in a retailer-led supply chain and prove that option contract brings benefit to each party.

We now review a few representative studies in the area of supply chain coordination with contracts. Pasternack (1985) analytically proves that supply chain coordination between a buyer and a supplier can be achieved with buy-back contracts, while facing stochastic demand. Cachon and Lariviere (2005) demonstrate that revenue sharing can coordinate a two-echelon supply chain. Taylor (2002) models sales-rebate contracts with sales effort effects and shows that when demand is influenced by sales effort, sales-rebate contract can achieve channel coordination. Other channel coordinating contracts include quantity discounts (Weng, 1995) and quantity–flexibility contracts (Tsay, 1999; Tsay and Lovejoy, 1999). All these contract forms can coordinate the supply chain that faces stochastic demand though their profit allocation levels differ. Cachon (2003) provides a review of supply chain coordination with contracts. The

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difference in profit allocation leads to conflict among supply chain members about agreeing to a single contract form that will be accepted by all the members for implementation.

In this paper we focus on the economic efficiency of options and how it can successfully resolve the supplier's problem while facing demand from a group of heterogeneous buyers. In this paper we develop a relatively simple option contract model and demonstrate that pure-strategy Nash equilibrium exists in a single supplier multiple buyer game. Subsequently we apply the results in the context of price competition game and inventory competition game and show how a coordinating option contract will behave under such scenarios. We also highlight what is the potential limitation of applying option contract form for coordinating a supply network. We present our analysis through analytically derived results and closed-form solutions. While most of the extant literature focuses on the flexibility aspect of option contract (Zhao et al., 2010; Wang and Liu, 2007), there is no work that highlights on the aspect that option contract can face limitation while coordinating a supply network.

## 3. Model Description

We consider a two-echelon supply chain consisting of a single supplier and *n* retailers. The retailers could be geographically distributed and format-wise diverse (for example, dedicated franchise shops, online retail outlet, and supermarket etc.). The supplier's product is sold through these various types of retailers in the market. Prior to the selling season, the supplier calculates her optimal contract and subsequently announces the option price, $o_i$, and exercise price, $e_i$, to each retailer *i* for *i = 1,…,n*. Contract parameters can be different for each retailer. Subsequently, the retailers simultaneously make decisions about order quantities and prices before the realization of the demand. Retailers book their respective order quantities to the supplier by paying option price. Option Price represents an allowance paid by the retailer to the supplier for reserving one unit of the production capacity (Zhao et al., 2010). During the selling season, the retailers obtain their realized demand quantity from the supplier by paying the exercise price. We assume that there is no lead time associated with product delivery. All the price and cost parameters are adjusted to inventory-clearing salvage value. Thereby the assumptions to avoid triviality of results: (i) $p_i > c_i > v$ ,(ii) $0 \leq o_i < c_i - v$, and (iii) $e_i > v$ get simplified to take the following forms: (I) $p_i > c_i > 0$,(II) $0 \leq o_i < c_i$ and (III) $e_i > 0$, where supplier's adjusted

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marginal cost of production for $i^{th}$ retailer is $c_i$ and $c_i$'s are considered to be different for different retailers for the purpose of generalizability. As retailers are geographically dispersed as well as format-wise different, therefore the marginal cost incurred by the supplier towards different retail can be different. The first assumption indicates that supplier is not risk-free for her production and the second assumption represents that the retailers do not always exercise all the options purchased by them.

## 3.1. Retailer's Game

The demand function of the $i^{th}$ retailer has the following form, $L_i(\vec{p}) + \varepsilon_i$, where $\vec{p} = (p_1, p_2, ..., p_n)$. $L_i(\vec{p})$ represents the deterministic part of the demand, $\varepsilon_i$ represents the price-independent stochastic part of the demand, and $(\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)$ follows independent and known continuous demand distributions with positive support. $\varepsilon_i$ has a probability distribution function (pdf), $f_i(\cdot)$, and cumulative distribution function (cdf), $F_i(\cdot)$. Since the retailers are assumed to be geographically diverse, therefore the deterministic demand and the stochastic component of it are different for different retailers. $L_i(\vec{p})$ captures the economics of price competition among the retailers $L_i^{(i)}(\vec{p}) \equiv \partial[L_i(\vec{p})]/\partial p_i < 0$ and $L_i^{(j)}(\vec{p}) \equiv \partial[L_i(\vec{p})]/\partial p_j > 0$. This functional form and assumptions are consistent with the price-setting newsvendor model literature (Petruzzi and Dada, 1999; Agrawal and Seshadri, 2000; Zhao and Atkins, 2008; Zhao, 2008). Retailer $i$ makes the following decisions: (i) retail price ( $p_i$ ) and (ii) safety stock ( $y_i$ ), before realization of demand. The safety stock $y_i$ protects the retailer $i$ against demand uncertainty and establishes a certain service level. Therefore the total inventory level to be maintained by the $i^{th}$ retailer is, $Y_i = L_i(\vec{p}) + y_i$.

We assume that a fixed exogenous proportion, $\gamma_{ji}$, of the lost sales from retailer $j$ switches to retailer $i$ and this proportion is called *spill rate*. $\gamma_{ji}$ is independent of the price levels $p_i$ and $p_j$ (Zhao and Atkins, 2008; Zhao, 2008). Therefore, the actual stochastic demand faced by retailer $i$ is given by, $D_i^s(y_{-i}) = \varepsilon_i + \sum_{j \neq i} \gamma_{ji}(\varepsilon_j - y_j)^+$, where $y_{-i} = (y_1, ..., y_{i-1}, y_{i+1}, ..., y_n)$. This representation is consistent with the *effective stochastic component of demand* definition of Zhao

and Atkins (2008). The pdf and cdf of $D_i^s(y_{-i})$ are given by $f_{D_i^s}(\cdot)$ and $F_{D_i^s}(\cdot)$ respectively. Thus the total effective demand faced by the $i^{th}$ retailer

$$D_i(y_{-i}) = L_i(\vec{p}) + D_i^s(y_{-i}) = L_i(\vec{p}) + \varepsilon_i + \sum_{j \neq i} \gamma_{ji} (\varepsilon_j - y_j)^+ \tag{1}$$

Retailer $i$'s problem is to choose, $(p_i, y_i)$, such that she maximizes her expected profit function,

$$E[\pi_{oi}(\vec{p}, \vec{y})] = (p_i - e_i) E\{\min[D_i(y_{-i}), Y_i]\} - o_i Y_i$$

$$= \pi_{oi}^d(\vec{p}) - o_i y_i + (p_i - e_i) E\{\min[D_i^s(y_{-i}), y_i]\} \tag{2}$$

where, $\pi_{oi}^d(\vec{p}) = (p_i - o_i - e_i) L_i(\vec{p})$, represents the profit of the deterministic part of the demand. The expectation function of equation (2) is defined as follows,

$$E[\min\{D_i^s(y_{-i}), y_i\}] = D_i^s(y_{-i}) \Pr\{D_i^s(y_{-i}) < y_i\} + y_i \Pr\{D_i^s(y_{-i}) > y_i\}.$$

The retailers' strategy sets are compact: $\{(p_i, y_i) : c_i \leq p_i \leq p_i^{max}, 0 \leq y_i \leq y_i^{max}\}$. The upper limits $p_i^{max}$ and $y_i^{max}$ are large enough to not to restrict either the retailers or the mathematical optima (Cachon and Netessine, 2004). We subsequently discuss different special cases of this general model in Section 4.

## 3.2. Supplier's Game

We are primarily interested in finding whether the supply chain can be coordinated by an option contract mechanism in the presence of heterogeneity among retailers. In the presence of simultaneous price and inventory competition, we aim to find a supply contract that maximizes the total system-wide profit (Zhao, 2008). In an integrated system, the expected profit is given by, $E(\Pi_C) = \sum_{i=1}^{n} E(\pi_i^C)$ where, $E(\pi_i^C)$ is the expected profit by serving the customers of the $i^{th}$ retailer.

$$E(\pi_i^C) = \pi_i^{C(d)} - c_i y_i + p_i E\{\min[D_i^s(y_{-i}), y_i]\} \tag{3}$$

where $c_i$ is the marginal cost associated with retailer $i$ and $\pi_i^{C(d)} = (p_i - c_i) L_i(\vec{p})$. Therefore the total expected profit of the supply chain is given by,

$$E(\Pi_C) = \sum_{i=1}^{n} E(\pi_i^C) = \sum_{i=1}^{n} \left( \pi_i^{C(d)} - c_i y_i + p_i E\{\min[D_i^s(y_{-i}), y_i]\} \right)$$

$$= \sum_{i=1}^{n}(p_i - c_i)L_i(\vec{p}) - \sum_{i=1}^{n} c_i y_i + \sum_{i=1}^{n} p_i E\{\min[D_i^s(y_{-i}), y_i]\} \qquad (4)$$

The supplier's problem is to optimally set $(o_i, e_i)$, so that it coordinates the entire supply chain. In a decentralized supply chain, by entering into an option contract agreement with $i^{th}$ retailer, the profit made the supplier is presented by the following equation

$$E(\pi_i) = (o_i - c_i)Y_i + e_i E\{\min[D_i(y_{-i}), Y_i]\}$$

$$= (o_i + e_i - c_i)L_i(\vec{p}) + (o_i - c_i)y_i + e_i E\{\min[D_i^s(y_{-i}), y_i]\} \qquad (5)$$

The total expected profit made the supplier is given by, $E(\Pi) = \sum_{i=1}^{n} E(\pi_i)$. In the following section, we investigate the existence of a pure strategy Nash equilibrium of the aforementioned game.

### 3.3. Pure Strategy Nash Equilibrium

Two additional conditions are required to be satisfied for the existence of a Nash equilibrium and they are as follows:

(a)     $\partial^2 \pi_i^d(\vec{p})/\partial p_i^2 < 0$ and $\partial^3 \pi_i^d(\vec{p})/\partial p_i^3 < 0$     (6)

(b)     The distribution of $D_i^s(\cdot)$ is IFR, so that the failure rate, defined by

$$r_{D_i^s}(\cdot) = \frac{f_{D_i^s}(\cdot)}{1 - F_{D_i^s}(\cdot)}, \text{ is increasing in nature} \qquad (7)$$

These assumptions are consistent with those presented in Zhao (2008) and Zhao and Atkins (2008). Assuming that conditions (a) and (b) hold, then using a mathematical lemma proposed by Zhao and Atkins (2008, pp. 541), we can establish that pure-strategy unique Nash equilibrium exists in the presence of option contract and combined price-inventory competition. The Nash equilibrium is described in Theorem 1.

**THEOREM 1:** *If (6) and (7) hold, then*

*(i)* *The retailer i's expected profit function,* $E[\pi_{oi}(\vec{p}, \vec{y})]$, *is jointly quasi-concave in*

$(p_i, y_i)$, *and therefore a pure strategy Nash equilibrium, where* $\dfrac{\partial E[\pi_{oi}(\vec{p}, \vec{y})]}{\partial p_i} = 0$

*and* $\dfrac{\partial E[\pi_{oi}(\vec{p}, \vec{y})]}{\partial y_i} = 0$ *exists.*

*(ii)* *The best response of retailer i is given by the solution of (8) and (9).*

$$\partial E[\pi_{oi}(\vec{p}, \vec{y})]/\partial p_i = [\partial \pi_{oi}^d(\vec{p})/\partial p_i] + E[\min(D_i^s, y_i)] = 0 \tag{8}$$

$$\partial E[\pi_{oi}(\vec{p}, \vec{y})]/\partial y_i = -o_i + (p_i - e_i)\Pr(D_i^s > y_i) = 0 \tag{9}$$

*(iii)* *The sufficiency condition for existence of unique Nash equilibrium is as follows:*

$$\{\partial^2 \pi_{oi}^d(\vec{p})/\partial p_i^2\} + \Pr[D_i^s > y_i(p_i)]/\{(p_i - e_i)r_{D_i^s(y_{-i})}[y_i(p_i)]\} < 0 \tag{10}$$

**PROOF OF THEOREM 1:** The proof is provided in the Appendix. □

Theorem 1 gives the pricing and inventory equilibrium with exogenous contract parameters. From Theorem 1, we can easily derive the fractile solution of optimal ordering quantity for retailer *i*. It is given by, $F(y_i) = (p_i - e_i - o_i)/(p_i - e_i)$. The obtained result also conforms to the solution form presented by Zhao et al. (2010) in the context of dyadic relationship of supplier and retailer. We subsequently study channel coordination in a decentralized supply chain with simultaneous price and inventory competition among independent retailers. For an integrated supply chain, total expected profit function is given by (4). The global optimal solution or the solution to the integrated supply chain is given by the first order conditions, as expressed by (11) and (12).

$$\partial E(\Pi_C)/\partial p_i = L_i(\vec{p}) + (p_i - c_i)L_i^{(i)}(\vec{p}) + \sum_{j=1, j \neq i}^{n}(p_j - c_j)L_j^{(i)}(\vec{p}) + E\{\min[D_i^s, y_i]\} = 0 \quad (11)$$

$$\partial E(\Pi_C)/\partial y_i = -c_i + p_i \Pr(D_i^s > y_i) - \sum_{j=1, j \neq i}^{n}p_j \gamma_{ji} \Pr(D_j^s < y_j, \varepsilon_i > y_i) = 0 \tag{12}$$

By comparing them with equations (8) and (9), we identify the difference between the global optima and a competitive equilibrium in the presence of option contract. It is indicative of the source of inefficiency when option contract tries to enforce channel coordination. This comparison is represented by the following set of equations.

$$\partial E(\Pi_C)/\partial p_i = \{\partial E[\pi_{oi}(\vec{p},\vec{y})]/\partial p_i\} - (c_i - o_i - e_i)L_i^{(i)}(\vec{p}) + \sum_{j=1, j\neq i}^{n}(p_j - c_j)L_j^{(i)}(\vec{p}) \quad (13)$$

$$\partial E(\Pi_C)/\partial y_i = \{\partial E[\pi_{oi}(\vec{p},\vec{y})]/\partial y_i\} - (c_i - o_i) + e_i \Pr(D_i^s > y_i) - \sum_{j=1, j\neq i}^{n} p_j \gamma_{ji} \Pr(D_j^s < y_j, \varepsilon_i > y_i) \quad (14)$$

From (13) and (14) we can see that the effect of double marginalization and competition are identical to those observed in other channel coordinating contracts like buy back. However, we will subsequently show that option contract provides the supplier with more flexibility compared to a channel coordinating wholesale price or buy back contract. $(\vec{p}^C, \vec{y}^C) = \{(p_i^C), (y_i^C)\}$ ($\forall i = 1,2,...,n$) denotes the global optima that maximizes the total supply chain profit; this global optima satisfies (11) and (12). Using this definition we obtain the channel coordinating option contract and it is represented by Theorem 2.

**THEOREM 2:** *The unique option contract mechanism,* $(o_i^*, e_i^*)$, *that coordinates the entire supply chain is characterized as follows:*

(i) $\quad 0 \leq o_i^* < c_i$ *and* $e_i^* > 0$

(ii) $\quad o_i^* = \dfrac{\Pr(D_i^s > y_i^C)}{1 - \Pr(D_i^s > y_i^C)}\left\{(p_i^C - c_i) + \left[\sum_{j=1, j\neq i}^{n}(p_j^C - c_j)L_j^{(i)}(\vec{p}^C)\Big/L_i^{(i)}(\vec{p}^C)\right]\right\}$

(iii) $\quad e_i^* = p_i^C - [o_i^*/\Pr(D_i^s > y_i^C)]$

*where* $(\vec{p}^C, \vec{y}^C) = \{(p_i^C), (y_i^C)\}$ ($\forall i = 1,2,...,n$) *represents the global optimum solution, that maximizes the system-wide profit of an integrated supply chain.*

**PROOF OF THEOREM 2:** The proof is provided in the Appendix. □

It is possible to have multiple equilibria under the existence of coordinating contract, $(o_i^*, e_i^*)$. However in the case of symmetric retailers there exists a unique equilibrium for the retailer's game. We will explore the properties of equilibrium and coordination contract through the symmetric retail case in the following section.

## 4. Discussion

We consider a few special cases, like linear deterministic demand, price competition, inventory competition, and identical n retailer problem to draw managerial insights. From the general pure-

strategy solution, presented in Section 3, it is not possible to comment on whether supplier behavior alters in the presence of channel coordinating option contract.

## 4.1. Price Competition

Price competition is one extreme case of the general model presented in Section 3. A *price competition game* has the following property: $\gamma_{ji} = 0$ for all $i$, $j$. Therefore the effective stochastic component of demand is presented by, $D_i^s = \varepsilon_i$. In a price competition game, the optimal contract is characterized by Proposition 1.

**PROPOSITION 1:** *In a price competition game, the channel coordinating option contract is presented by,*

(i) $\quad o_i^* = p_i^C \Pr\!\left(D_i^s > y_i^C\right) + \frac{\Pr\!\left(D_i^s > y_i^C\right)}{1 - \Pr\!\left(D_i^s > y_i^C\right)} \left[ \sum_{j=1, j \neq i}^{n} \left(p_j^C - c_j\right) L_j^{(i)}\!\left(\vec{p}^C\right) \Big/ L_i^{(i)}\!\left(\vec{p}^C\right) \right]$

(ii) $\quad e_i^* = \dfrac{\left(c_i - o_i^*\right)}{\Pr\!\left(D_i^s > y_i^C\right)} > 0$

**PROOF OF PROPOSITION 1:** The proof is provided in the Appendix. □

Proposition 1 clearly indicates that, price competition increases the option price level. As a result, the supplier increasingly becomes risk-free for her production. Since supplier is able to charge the highest possible option price, the retailers are forced to charge higher prices to the end customers.

## 4.2. Inventory Competition

Inventory competition is the other extreme case of the general model presented in Section 3. An *inventory competition game* has the following property: $L_i^{(j)}\!\left(\vec{p}\right) = 0$ and $L_i\!\left(\vec{p}\right) = L_i\!\left(p_i\right)$. The optimal contract for an inventory competition game is characterized by Proposition 2.

**PROPOSITION 2:** *In an inventory competition game, the channel coordinating option contract is presented by,*

(i) $\quad o_i^* = \dfrac{\Pr\!\left(D_i^s > y_i^C\right)}{1 - \Pr\!\left(D_i^s > y_i^C\right)} \left(p_i^C - c_i\right)$

(ii) $\quad e_i^* = -\left[p_i^C \Pr\!\left(D_i^s > y_i^C\right) + c_i\right] \Big/ \Pr\!\left(D_i^s < y_i^C\right) < 0$

**PROOF OF PROPOSITION 2:** The proof is provided in the Appendix. □

In the case of inventory competition, the supplier additional collects revenue from the leftover inventory of the retailers. As the retailers are required to pay the supplier for overstocking, such a scheme requires extensive inventory monitoring by the supplier.

### 4.3. *Linear Deterministic Demand*

We consider a particular case where deterministic portion of the demand assumes a linear form as follows: $L_i(\vec{p}) = a - bp_i + \sum_{j \neq i} \theta p_j$. Similar demand functions are used in the analysis of operations management (Tsay and Agrawal, 2000; Anderson and Bao, 2010). In this case we have: $L_i^{(i)}(\vec{p}) = -b$ and $L_j^{(i)}(\vec{p}) = \theta$. Using these relations, the optimal option price takes a simpler form as follows: $o_i^* = \dfrac{\Pr(D_i^s > y_i^C)}{1 - \Pr(D_i^s > y_i^C)}\left\{(p_i^C - c_i) - \dfrac{(n-1)\theta}{b}\right\}$. From this expression, the impact of $\theta$ on optimal contract form can be summarized in the following way.

**PROPOSITION 3:** *Option price, $o_i^*$, is decreasing in $\theta$ and exercise price, $e_i^*$, is increasing in $\theta$.*

**PROOF OF PROPOSITION 3:** The proof is provided in the Appendix. □

This relation signifies that with increase in competition the option price for booking the supplier's capacity would decrease and at the time of demand realization the retailers have to purchase products at a higher rate. In the case of symmetric price competition the following conditions are satisfied: $\gamma_{ji} = 0$ and $D_i^s = \varepsilon_i$. Therefore under optimality the order quantity is given by, $\Pr(\varepsilon_i < y_i) = (p_i - c_i)/p_i$. Under the condition of identical retail price and symmetry, we further have: $p_j - c_j = p_i - c_i = p - c$. The optimal contract form of such a game is: $o^* = c\{1 - [(n-1)\theta/b]\}$ and $e^* = p(n-1)\theta/b$. Though exercise price is always positive for $n > 1$ but option price is positive till $n < 1 + (b/\theta)$. Therefore only with a limited number of retailers, supply chain can be coordinated by an option contract.

### 4.4. *The Case of n Identical Retailers*

We consider a completely decentralized supply chain network consisting of a single supplier and $n$ retailers. We assume that the supplier's marginal production costs for different retailers are identical and it is presented by $c$. The retail demand is assumed to be divided among $n$ retailers according to proportions of their stocking quantities. This assumption of proportional allocation is consistent with those of Wang and Gerchak (2001). Due to proportional allocation rule, the integrated supply chain faces a single newsvendor problem and the optimal order quantity is defined by, $F(q_C) = \dfrac{p-c}{p}$, where $p$ is the retail price and $q_C$ is the order quantity for the central planner. The overall profit of the integrated supply chain is given by,

$$\Pi(q_C) = (p-c)q_C - p\int_0^{q_C} F(x)dx.$$ If the supplier offers same contract terms to all the retailers and if all the retailers place identical orders, then the profit allocation among the supplier and the retailers for two channel coordinating contracts, namely option and buyback, takes a simple form and they are presented by the following proposition.

**PROPOSITION 4:** *Under a channel coordinating buyback contract, the $i^{th}$ retailer's and supplier's profits are given by:* $\pi_{Ri}^{BB} = \dfrac{1}{n^2}\left(1 - \dfrac{b}{p}\right)\Pi(q_C)$ *and* $\pi_S^{BB} = \left\{1 - \dfrac{1}{n}\left(1 - \dfrac{b}{p}\right)\right\}\Pi(q_C)$,

*respectively, where $b$ is the unit buyback price. Under a channel coordinating option contract the $i^{th}$ retailer's and supplier's profits are given by:* $\pi_{Ri}^{OC} = \dfrac{\lambda}{n}\Pi(q_C)$ *and* $\pi_S^{OC} = (1-\lambda)\Pi(q_C)$,

*where $M = \{(o,e) : o = \lambda c, e = (1-\lambda)p, \lambda \in [0,1)\}$ is the coordinating option contract set and $o$ and $e$ designate the option price and the exercise price, respectively.*

**PROOF OF PROPOSITION 4:** The proof is provided in the Appendix. □

From Proposition 4, we can see an option contract yields a supplier profit at least equal to that of a buyback contract at $\lambda = \dfrac{1}{n}\left(1 - \dfrac{b}{p}\right)$, for a given buyback rate. Every retailer maximizes her individual profit under coordinating wholesale contract and this profit level is given by,

$\pi_{Ri}^{BB}\big|_{b=0} = \dfrac{\Pi(q_C)}{n^2}$. At $\lambda = \dfrac{1}{n}$, an option contract also yields the same profit. Therefore, option contract facilitates the supplier to yield more profit than a buyback contract as well as to

coordinate the overall supply chain for $\lambda \in \left(0, \dfrac{1}{n}\left[1 - \dfrac{b}{p}\right]\right)$. When $\lambda \in \left(\dfrac{1}{n}, 1\right)$, retailers can fetch more profit from option contract compared to a coordinating wholesale price contract. Thus, coordinating option contract captures the entire gamut of conventional coordination mechanisms. It further provides additional contract forms where, without losing the advantage of coordination, either supplier or retailers stand a chance to extract higher profit. Thus option contract can act as a more flexible instrument for designing supply chain coordination. This result further establishes that the option contract coordinating mechanism, as proposed by Zhao et al. (2010), can be extended to single manufacturer multiple retailer supply chain network as well.

### 4.5. *Numerical Analysis*

In this sub-section, we numerically analyze the individual retailer's profit and integrated supply chain profit. We show that the retailer's expected profit function $\left[E\{\pi_{oi}(\vec{p}, \vec{y})\}\right]$ is jointly quasi-concave in her decision variables: (i) retail price ( $p_i$ ) and (ii) safety stock ( $y_i$ ). We further demonstrate that joint concavity of the system-wide profit function $\left[E(\Pi_C)\right]$ of an integrated supply chain holds in the central planner's decision set, $\left(\vec{p}^C, \vec{y}^C\right)$.

We set the following parameter values for the purpose of the numerical study. We consider a supply chain comprising a single supplier and two retailers. The retailers face identical linear deterministic demand function, given as below

$$L_1(p_1, p_2) = 100 - p_1 - 0.5p_2 \text{ and } L_2(p_1, p_2) = 100 - p_2 - 0.5p_1$$

The supplier incurs a marginal costs, $c_1 = 40$ and $c_2 = 50$, for producing the order that she receives from retailer 1 and retailer 2 respectively. The stochastic component of the market demand can be a maximum of 10% of the deterministic demand component for retailer 1 and that for retailer 2 is 20%. The safety stock policy for a retailer varies between 0% - 15% of the deterministic demand component. For expositional simplicity, marginal cost of the retailer and the salvage price are taken to be zero.

For the vertically integrated system, the behavior of the system-wide profit function is presented through Figures 1 – 3.
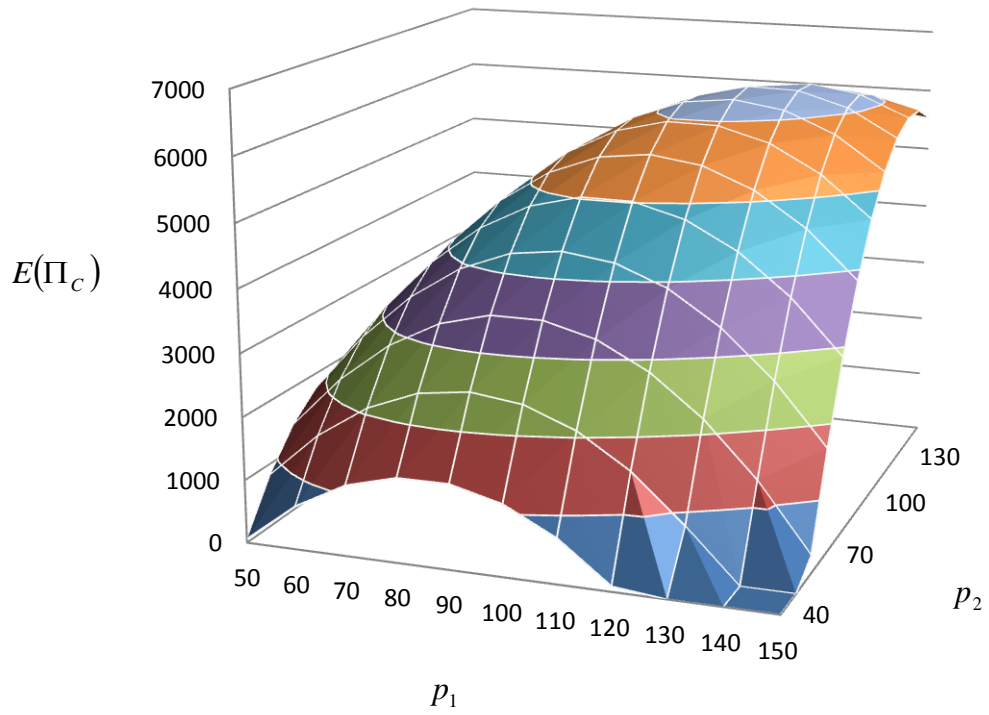
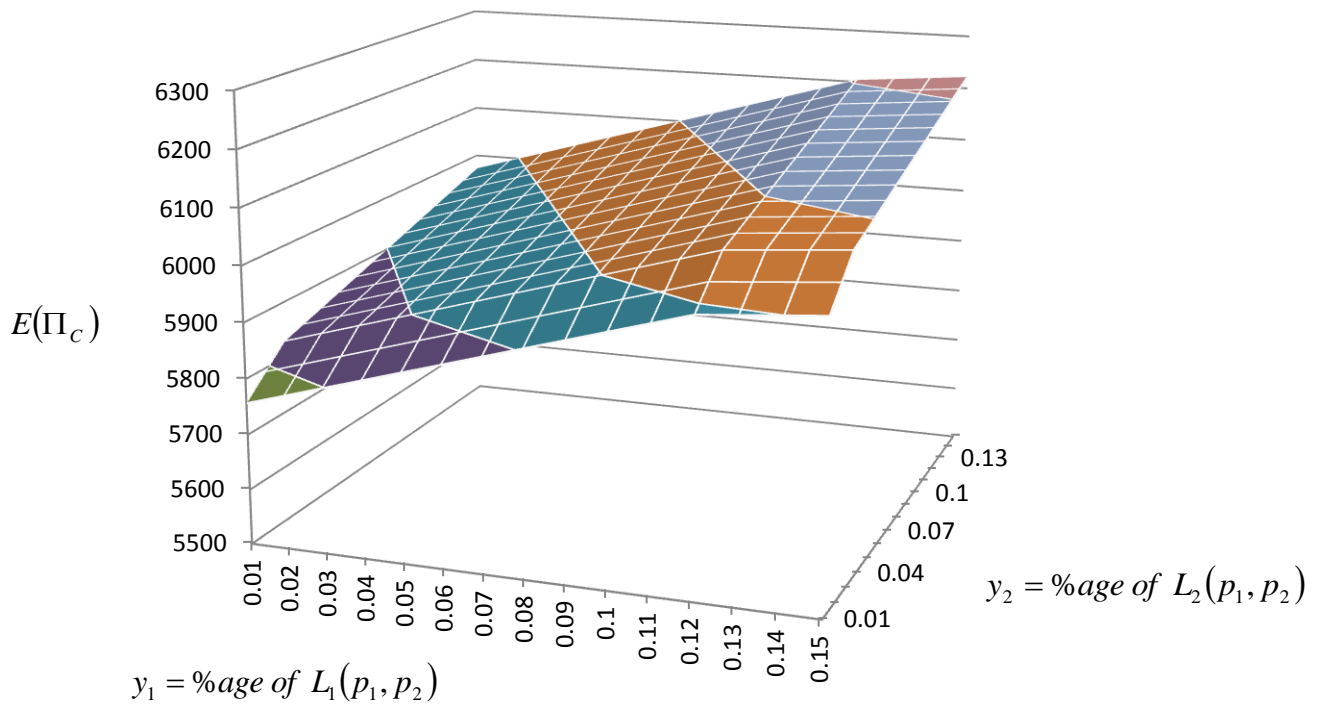**Figure 1:** Integrated Supply Chain Profit $\left[E(\Pi_C)\right]$ vs Price Vector $(p_1, p_2)$



**Figure 2:** Integrated Supply Chain Profit $\left[E(\Pi_C)\right]$ vs Safety Stock Vector $(y_1, y_2)$
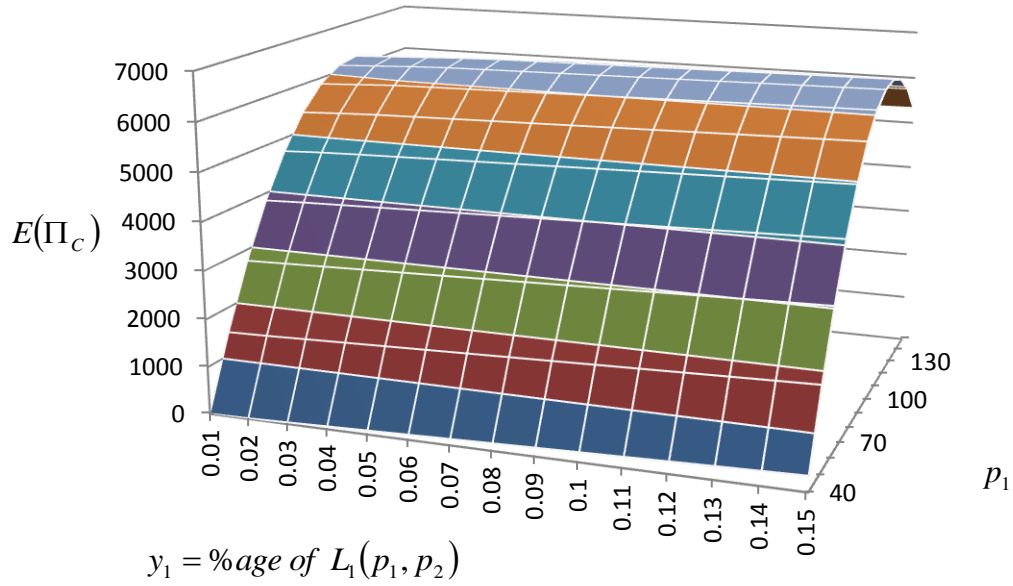
**Figure 3:** Integrated Supply Chain Profit $\left[E(\Pi_C)\right]$ vs Decision Variables for Product 1 $(p_1, y_1)$

From the Figures (1) – (3) we can observe that system-wide profit function $\left[E(\Pi_C)\right]$ is jointly concave in the central planner's decision set, $\left(\vec{p}^C, \vec{y}^C\right)$.

We next look at the behavior of the retailer's expected profit function $\left[E\{\pi_{oi}(\vec{p}, \vec{y})\}\right]$ in her decision variables: (i) retail price ($p_i$) and (ii) safety stock ($y_i$); they are presented through Figures (4) and (5).
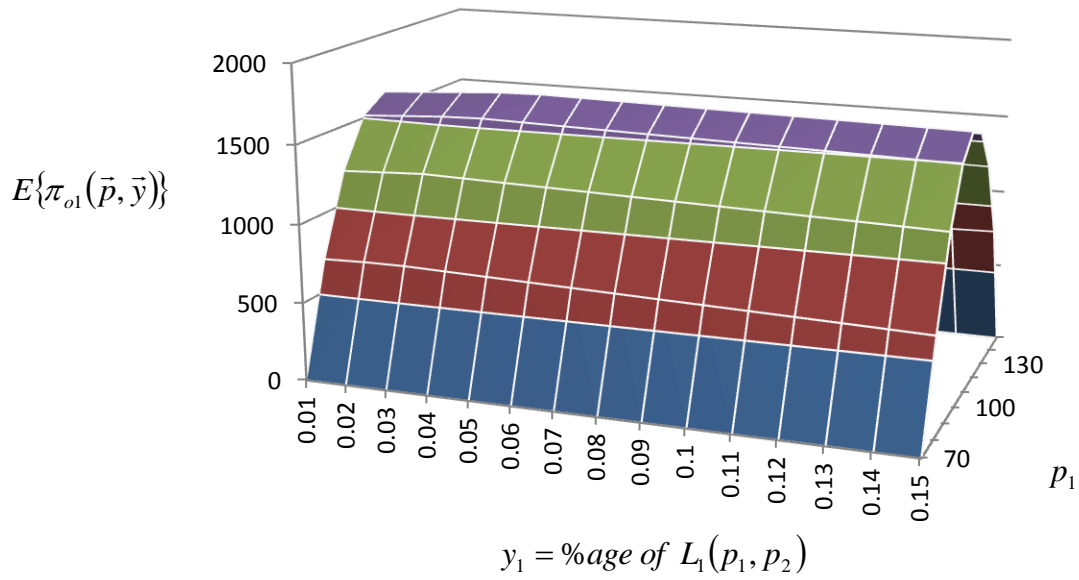
**Figure 4:** Expected Profit $\left[E\{\pi_{o1}(\vec{p}, \vec{y})\}\right]$ vs Decision Variables $(p_1, y_1)$ (Retailer 1)
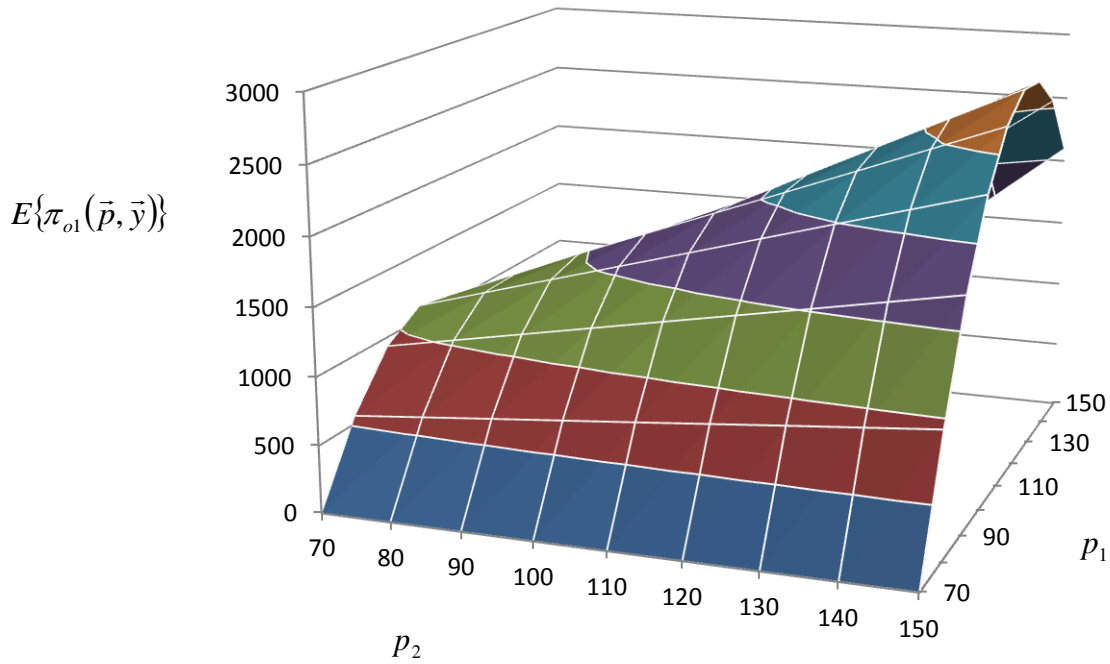


**Figure 5:** Expected Profit $\left[E\{\pi_{o1}(\vec{p}, \vec{y})\}\right]$ vs Price Vector $(p_1, p_2)$ (Retailer 1)

From Figures (4) and (5) we observe that the retailer's individual profit is jointly concave in the retail price and the safety stock. For retailer 2 we also have similar observation.

Through the numerical analysis we see that both the system-wide profit of an integrated supply chain and the individual profits of the retailers of a decentralized supply chain hold the joint quasi-concavity property in the retail price and safety stock. Therefore there exists a global optimum solution $\left(\vec{p}^{C}, \vec{y}^{C}\right)$, that maximizes the system-wide profit of the integrated supply chain and this solution is given by: $\left(\vec{p}^{C}, \vec{y}^{C}\right) = \left\{\left(p_1^{C}, p_2^{C}\right), \left(y_1^{C}, y_2^{C}\right)\right\} = \left\{(120,120), (0.15,0.15)\right\}$. Using this global optimum solution we can calculate the optimal option contract mechanism and. Optimal option contracts that the supplier would offer to the buyer 1 is given by: $\left(o_1^{*}, e_1^{*}\right) = (20,60)$. For the same setting of the supply chain the optimum wholesale price that maximizes the supplier's profit is $w^{*} = 66.67$. Therefore through option contract, the supplier can not only coordinate the supply chain but also extract an extra profit margin of $(20+60) - 66.67 = 13.33$ units for every unit of realized demand from buyer 1. Similarly supplier can have excess profit margin form buyer 2 as well. It presents us with the insight that option contract can help the supplier to extract larger profit level from the retailers, while coordinating the entire supply chain.

## 6. Conclusion

Our analysis reveals that, an option contract mechanism can coordinate a supply chain network of single supplier multiple buyers with Pareto-improvement. Under simplifying condition, we analytically establish that option contract provides a supplier with more flexibility in terms of profit allocation. We have also explored the issues concerning implementation of the coordinating option contract form in the context of price competition and inventory competition.

Our study demonstrates that, in the presence of heterogeneous buyers it is possible to have multiple equilibria under the existence of coordinating contract. However in the case of symmetric retailers there exists a unique equilibrium for the retailer's game. This result is similar to those reported by Zhao and Atkins (2008) for simultaneous price and inventory game. In the case of price competition game, competition increases the option price level. As a result, the supplier increasingly becomes risk-free for her production. As the supplier would be able to charge higher option price, the buyers are forced to charge higher prices to the end customers. In the case of inventory competition, the retailers are required to pay the supplier for overstocking.

Such a scheme requires extensive inventory monitoring by the supplier and might not be feasible to implement. Therefore in the case of price competition option contract might prove to be an implementable coordinating mechanism.

In the case of linear deterministic demand, we observe that, under the condition of symmetry, the exercise price is always positive when the supply chain consists of multiple buyers. However, the option price is positive only to a limit and this limit is a function of the own-price and cross-price elasticity of demand. In other words, depending on the level of competition, only up to a certain limit the supply chain can be coordinated by an option contract mechanism. This characteristic is unlike any other coordinating contract form like buy-back, revenue-sharing, quantity-flexibility etc, where the coordination mechanism is not limited to a number of buyers. This characteristic points to the limitation of option contract as a mechanism to coordinate a supply chain network.

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