Indian Institute of Management Calcutta

Working Paper Series

WPS No. 752
August 2014

The variable salvage value newsvendor model and its impact on supply contracts

Indranil Biswas
Doctoral Student, Indian Institute of Management Calcutta
D. H. Road, Joka, P.O. Kolkata 700 104
indranilb10@email.iimcal.ac.in

Balram Avittathur
Professor, Indian Institute of Management Calcutta
D. H. Road, Joka, P.O. Kolkata 700 104
balram@iimcal.ac.in

The Variable Salvage Value Newsvendor Model and its Impact on Supply Contracts

Indranil Biswas[a] and Balram Avittathur[b]

[a] Doctoral Student, Indian Institute of Management Calcutta, Kolkata 700104, India,
Email: indranilb10@email.iimcal.ac.in

[b] Professor, Indian Institute of Management Calcutta, Kolkata 700104, India,
Email: balram@iimcal.ac.in

Abstract

Clearance price depends on the leftover inventory in many real life contexts. The classical newsvendor model assumes a fixed salvage value for clearance sales. This assumption could lead to suboptimal order quantity and expected profit. We analytically prove that the newsvendor model with fixed salvage value results in over ordering and profit loss compared to a newsvendor model where salvage value is a function of leftover inventory. The conditions under which the variance in performance between the two newsvendor models is significant are determined. The paper also models, for variable salvage value, the wholesale price, buy-back, revenue sharing and sales rebate contracts for supply chain coordination. With regards to supply chain coordination, the variable salvage model displays similar behaviour as the fixed salvage model. Through numerical analysis we also illustrate the differences between the two salvage value models for the supply contracts considered.

Keywords: clearance pricing, coordination, newsvendor, inventory, supply chain contracts, variable salvage

1. Introduction

Managing uncertainty is a critical challenge faced by retailers of short product life cycle or single season products like apparel, consumer electronics, mobile phones, personal computers and event merchandise. Under ordering results in stock outs while over ordering results in leftover inventory that would have to be exhausted through a clearance sale. Fisher and Raman (2010) report that US retailers accumulate so much excess inventory that markdowns have increased from 8% in 1970 to nearly 30% at the turn of the century. Apart from this statistic, there is strong evidence to point that over ordering is today a common phenomenon in retailing. John Lewis posted record sale on the first clearance day after Christmas even during the economic downturn of 2008[1]. In 2012, Boxing Day posted record online clearance sale across UK retail websites[2]. The complexities of global supply chains force retailers to stock up products at the beginning of the season itself, with little information about what the demand would be, in spite of past ordeals of clearance sales. In order to avoid over-stocking retailers sometimes opt for advance discount (Prasad et al., 2011). However, post-season discounted-sale or clearance sale is the most prevalent form of clearing leftover inventory (Forest et al. 2003; Cachon and Kok, 2007; Wang and Webster, 2009). In recent times, it has been reported that retailers have started to carefully accommodate discount strategies in the pricing of the product itself, keeping their profit level unchanged[3]. Depending on user perception and estimation of location, companies have started offering varying discount rates[4]. In such a context the retailer should calculate her optimal stocking quantity keeping the discount strategy in mind. In situations like above, the famous newsvendor model that computes the optimal order quantity for a single period, would lead to sub-optimal solution as it assumes the salvage value to be fixed.

The newsvendor or the single period inventory model is one of the most extensively studied problems in operations management. In this model, a product procured at a fixed unit procurement cost, c, is sold during the period at a fixed unit price, p. The inventory leftover at the end of the period cannot be sold anymore at p and is cleared at a fixed unit salvage value, v, which is lesser than c. Before understanding the demand in the period, the newsvendor has to decide the order quantity that minimizes the sum of losses arising out of under- and over-stocking.

Many scholars (Hertz and Schaffir, 1960; Rozhon, 2005; Kratz, 2005) have indicated that the salvage price is variable and is dependent on the leftover inventory. For the purpose of modelling simplicity,

[1] Potter, M. (December 29, 2008) John Lewis posts record sales on 1st clearance day, Reuters, Retrieved from: http://uk.reuters.com/article/2008/12/29/uk-john-lewis-idUKTRE4BR2GN20081229.

[2] Peachey, K. (December 27, 2012) Sales shoppers set online 'Boxing Day record', BBC News, Retrieved from: http://www.bbc.com/news/business-20850299.

[3] Kapner, S. (November, 25 2013) The Dirty Secret of Black Friday 'Discounts': How Retailers Concoct 'Bargains' for the Holidays and Beyond, The Wall Street Journal, Retrieved from: http://online.wsj.com/news/articles/SB10001424052702304281004579217863262940166.

[4] Valentino-Devries, J., Singer-Vine, J., Soltani, A. (December 24, 2012) Websites Vary Prices, Deals Based on Users' Information, The Wall Street Journal, Retrieved from: http://online.wsj.com/news/articles/SB10001424127887323777204578189391813881534

literature has assumed a fixed unit salvage value. There are innumerable examples, ranging from clearance sales of perishable goods to end of season sales of fashion goods, where the price at which the inventory is cleared at the end of a period (season) is a function of the inventory leftover and the price elasticity of demand is not constant (Şen and Zhang, 2009; Caro and Gallien, 2012). In other words, apart from deciding the order quantity before the start of the period, the real life situation involves deciding the unit salvage value at the end of the period. The latter, which depends on the inventory at the end of the period, is a function of the initial order quantity decision as well as the actual demand that was observed in the period.

The change in optimal ordering decision due to incorporation of variable salvage value will influence the contract decision(s) between the retailer and the manufacturer as well. The newsvendor model results are extensively used in the design of supply contracts like wholesale price, buy-back, revenue sharing and sales rebate contracts. The fixed salvage value assumption has a bearing on the design of these contracts and could be a simplistic representation of what could be more complex in practice. In this paper, we model the newsvendor problem and a number of supply contracts for a variable salvage value that linearly decreases with the leftover inventory. In addition, we derive the conditions under which these supply contracts coordinate the supply chain.

Section 2 reviews the extant literature, §3 describes the newsvendor model for variable salvage value, §4 describes the modelling of certain supply contracts for variable salvage value and the conditions under which they coordinate the supply chain, §5 presents results of a numerical study, and the discussion of the results and conclusion are described in §6.

2. Literature Review

The extant literature can be divided into three broad categories: ones dealing with variations on the newsvendor model, ones that discuss various clearance pricing strategies and ones on supply chain coordination using newsvendor framework. Employing newsvendor framework, Pasternack (1985) shows that in a dyadic relationship an optimal return policy can achieve supply chain coordination where the supplier offers the retailer full credit for a partial return of goods. Petruzzi and Dada (1999) and Agrawal and Seshadri (2000) extend the newsvendor model where stocking quantity and selling price are set simultaneously. In both cases clearance prices are assumed to be exogenous and the decision maker chooses the regular selling price only. Carr and Lovejoy (2000) analyse the capacitated newsvendor problem where the firm chooses a demand distribution from available set of market opportunities. Dana and Petruzzi (2001) extend the classical newsvendor problem for endogenous demand where the uncertain demand is dependent upon both the price and the inventory level of a firm. With the help of single-period newsvendor model Lariviere and Porteus (2001) examine the change in wholesale price and order quantity as the market size changes. Eeckhoudt et al. (1995), Schweitzer and Cachon (2000), Raz and Porteus (2006), Besbes and Muharremoglu (2013) provide other extensions to the newsvendor model.

Hertz and Schaffir (1960) indicate that the salvage value of clearance inventory is dependent on the amount of leftover inventory. However, they do not estimate the salvage value and argue that constant salvage value is an adequate approximation. Recent works on multi-period newsvendor model have often taken similar assumption of fixed salvage value (Donohue, 2000; Fischer et al., 2001; Petruzzi and Dada, 2001). There are numerous articles that study markdown pricing under the assumption that the demand is independent across the time and do not allow for correlation in demand (Bitran and Mondschein, 1997; Smith and Achabal, 1998).

Cachon and Kok (2007) question the fixed salvage value assumption and describe the errors in decision making that are associated with this assumption. Instead of a constant salvage value, they propose a general clearance-pricing model that assumes iso-elastic clearance period demand functions. They also discuss four heuristic approaches to estimate a fixed salvage value that would result in better decision-making when using the newsvendor model. Among the heuristics proposed the weighted average salvage value heuristic results in newsvendor decisions that are closest to the optimal solution. Another heuristic, the marginal revenue heuristic, results in optimal order quantity but is difficult to estimate and is specific to the clearance pricing model proposed by the authors.

However, behavioural research and industry experts suggest that price elasticity changes with change in price level and also according to stages of product life cycle (Lilien et al., 1992). The iso-elastic or exponential demand functions exhibit constant price elasticity of demand with demand approaching infinity when the price approaches zero. Linear form of demand function often faces the criticism of being restrictive in terms of maximum permissible price (Varian, 1992; Huang et al., 2013). However, the same is not the case with modelling of clearance-sale demand. The clearance price or salvage value will always have an upper limit given by the normal period price (Cachon and Kok, 2007; Wang and Webster, 2009).

A large number of research papers address the issue of supply chain coordination through contracts in the presence of stochastic demand using newsvendor model. Lariviere and Porteus (2001) provide a detailed analysis of wholesale contract in the context of newsvendor problem. This contract though fails to coordinate a supply chain, generally serves as a benchmark case. In the similar context, Pasternack (1985) gives a complete analysis of buy-back contract and demonstrates how this contract helps in coordinating a supply chain. Cachon and Lariviere (2005) analyse revenue sharing contract in a generalized newsvendor setting and show how coordination is possible employing such a contract. Taylor (2002) and Krishnan et al. (2004) discuss the sales-rebate contract in the context of random retailer demand and prove the existence of coordinating contract mechanism. In the context of newsvendor framework, Cachon (2003) provides a comprehensive review of these types of contracts. All these contract analysis assume an exogenously decided constant salvage value. Wang and Webster (2009) assume the clearance pricing to be endogenous to the model and then compares between two different types of the contracts based on quantity and price markdown.

3. The newsvendor model for variable salvage value

We study the newsvendor model for a variable salvage value that decreases linearly with the leftover inventory. Like Cachon and Kok (2007), we too model the problem as one with two periods: a normal period (P_1) where the good is sold at a pre-determined price p and a clearance-sale period (P_2) where the good is cleared at a salvage value v, which is a function of inventory leftover at the end of the first period. As in the newsvendor model, the order quantity q that was procured at a marginal cost c is available for sale at the beginning of P_1. Unit selling price during the normal season is assumed to be fixed (Pasternack, 1985; Lariviere, 1999; Cachon and Kok, 2007; Wang and Webster, 2009) and is indicated by p such that $p > c$. The realised demand during P_1 is given by, x, and is distributed over $[0, q_{\max}]$. The realised demand is assumed to follow an increasing generalized failure rate (IGFR) distribution; the corresponding probability distribution and cumulative distribution are represented by $f(\cdot)$ and $F(\cdot)$, respectively. We further assume that $f(\cdot)$ and $F(\cdot)$ are differentiable over the entire range $[0, q_{\max}]$; $F(\cdot)$ is strictly increasing; the boundary conditions of the distribution are: $F(0) = 0$ and $F(q_{\max}) = 1$.

At the end of P_1, the leftover inventory, i, can be expressed as $i = (q - x)^+$. The leftover items are sold in the clearance-sale period P_2 at v, which can be expressed as $v = a_v - b_v i$, where $0 \le a_v \le p$ and $b_v > 0$. The P_2 revenue function, which will be referred to as salvage revenue, r_s, is given by the expression, $r_s = a_v i - b_v i^2$. The first order condition reveals that r_s is maximized at an inventory level given by $s = a_v / 2b_v$. We assume that when $i > s$, the inventory greater than s is disposed off at a salvage value of zero. This is observed often in real-life whenever there is a huge leftover inventory at the end of the season. In such situations a firm may adopt inventory disposal in the form of bundling apart from some portion being sold at a clearance price. Typical examples of bundling include packing the unsold good with another product of the firm or offering schemes like "buy two get one free" (Sarkis & Semple, 1999). We define $j = (q - s)^+$; then using these definitions the normal season demand, leftover inventory, clearance-sale revenue are as expressed in Table 1.

Table 1: Normal season and clearance-sale revenues

Demand in period $P_1(x)$	Leftover Inventory (I)	Clearance-sale volume	Stock to be disposed off at zero salvage value
$0 \le x \le j$	$i = q - x \ge s$	s	$i - s$
$j \le x \le q$	$0 \le i \le s$	i	–
$x > q$	–	–	–

The expected normal period revenue, $R_1(p,q)$, and the expected clearance-sale period revenue, $R_s(a_v,b_v,q)$, are defined by the following equations,

$$R_1(p,q) = \int_0^q pxf(x)dx + \int_q^{q_{\max}} pqf(x)dx = pq - p\int_0^q F(x)dx \quad \ldots \quad \ldots \quad (1)$$

$$R_s(a_v,b_v,q) = \int_0^q (a_v - b_v i)if(x)dx = (a_v - 2b_v q)\int_0^q F(x)dx + 2b_v\int_0^q xF(x)dx, \text{ for } q \le s \quad (2a)$$

$$= v_{\min} s\int_0^j f(x)dx + \int_j^q (a_v - b_v i)if(x)dx = (a_v - 2b_v q)\int_j^q F(x)dx + 2b_v\int_j^q xF(x)dx, \text{ for } q > s \quad (2b)$$

where, $v_{\min}$ is defined as, $v_{\min} = a_v - b_v s = a_v/2$. The expected profit is expressed by the equation $\pi = R_1 + R_s - cq$. Using the expression of R_1 and R_s, the expected profit is rewritten as follows:

$$\pi = (p-c)q - p\int_0^q F(x)dx + (a_v - 2b_v q)\int_0^q F(x)dx + 2b_v\int_0^q xF(x)dx, \text{ for } q \le s \quad \ldots \quad (3a)$$

$$= (p-c)q - p\int_0^q F(x)dx + (a_v - 2b_v q)\int_j^q F(x)dx + 2b_v\int_j^q xF(x)dx, \text{ for } q > s \quad \ldots \quad (3b)$$

The first and second derivatives of the profit function with respect to q are

$$\frac{d\pi}{dq} = (p-c) - (p-a_v)F(q) - 2b_v\int_0^q F(x)dx, \text{ for } q \le s \quad \ldots \quad \ldots \quad (4a)$$

$$= (p-c) - (p-a_v)F(q) - 2b_v\int_j^q F(x)dx, \text{ for } q > s \quad \ldots \quad \ldots \quad (4b)$$

$$\frac{d^2\pi}{dq^2} = -\{(p-a_v)f(q) + 2b_v F(q)\} < 0, \text{ for } q \le s \quad \ldots \quad \ldots \quad (5a)$$

$$= -[(p-a_v)f(q) + 2b_v\{F(q) - F(j)\}] < 0, \text{ for } q > s \quad \ldots \quad \ldots \quad (5b)$$

From the second order condition, it is evident that the expected profit, $\pi(p,q)$, is concave over $[0, q_{\max}]$. Denoting the optimal order quantity by $\hat{q}$, we have $d\pi/dq = 0$ at $q = \hat{q}$. Referring to (4a) and (4b), it can then be seen that $\hat{q}$ is increasing in a_v. As the highest value that can be taken by a_v is p, it can be observed from (4a) and (4b) that

$$\int_0^{\hat{q}} F(x)dx = \frac{(p-c)}{2b_v}, \text{ for } a_v = p \text{ and } \hat{q} \le s \quad \ldots \quad \ldots \quad \ldots \quad (6a)$$

$$\int_{\hat{j}}^{\hat{q}} F(x)dx = \frac{(p-c)}{2b_v}, \text{ for } a_v = p \text{ and } \hat{q} > s \quad \ldots \quad \ldots \quad \ldots \quad (6b)$$

where, $\hat{j} = \hat{q} - s$.

As $\hat{q}$ is increasing in a_v, the $\hat{q}$ for $a_v = p$ can be considered as $q_{\max}$. The search for $\hat{q}$, over the range $[0, q_{\max}]$, can be split into two sub-ranges: $[0, s]$ and $(s, q_{\max}]$ if $s < q_{\max}$. The exhaustive search starts with the first sub-range $[0, s]$; if the optimality condition does not hold in this range then the search extends to $(s, q_{\max}]$. The optimality condition for the order quantity in the presence of variable salvage value is presented in Theorem 1.

THEOREM 1: *The optimal order quantity, $\hat{q}$, for a newsvendor in presence of variable salvage value function, $v(i) = a_v - b_v i$, where $i = (q - x)^+$ denotes the leftover inventory, is determined according to the following:*

(i) *The necessary condition for $\hat{q}$ to lie in the range $[0, s]$ is given by,* $(p - a_v)\{1 - F(\frac{a_v}{2b_v})\} + (a_v - c) \le 0$*; otherwise it lies in the range* $(s, q_{\max}]$.

(ii) *In $[0, s]$ an exhaustive search over all the values of q will uniquely determine $\hat{q}$ such that*: $(p - a_v)F(q) + 2b_v \int_0^q F(x)dx = p - c$.

(iii) *In $(s, q_{\max}]$ an exhaustive search over all the values of q will uniquely determine $\hat{q}$ such that*: $(p - a_v)F(q) + 2b_v \int_{q-s}^q F(x)dx = p - c$.

(iv) *The sufficient condition for $\hat{q} \le q_{\max}$ is given by the inequality*: $a_v \le c + 2b_v \int_{q_{\max}-s}^{q_{\max}} F(x)dx$

PROOF: Please contact the authors for the working.

The optimal order quantity determined in Theorem 1 is unique in nature. In Theorem 2 the uniqueness of the optimal order quantity decision is established.

THEOREM 2: *The uniqueness of $\hat{q}$ over $[0, q_{\max}]$ is defined in the following way:*

(i) *If* $(p - a_v)\frac{df(q)}{dq} + 2b_v f(q) > 0$, $\frac{d\pi}{dq}$ *is monotone decreasing over* $[0, s]$

(ii) *If* $(p - a_v)\frac{df(q)}{dq} + 2b_v\{f(q) - f(j)\} > 0$, $\frac{d\pi}{dq}$ *is monotone decreasing over* $(s, q_{\max}]$

(iii) *The necessary condition for* $\hat{q} \in [0, s]$ *is given by*: $(p - a_v)\{1 - F(\frac{a_v}{2b_v})\} + (a_v - c) \leq 0$, *else* $\hat{q} \in (s, q_{\max}]$.

(iv) *Since* $\frac{d\pi}{dq}$ *is monotone decreasing over* $[0, q_{\max}]$ *and the necessary condition for* $\hat{q}$ *to lie in either of the sub-ranges is uniquely defined,* $\hat{q}$ *can be uniquely determined over* $[0, q_{\max}]$.

PROOF: Please contact the authors for the working.

As mentioned in §2, Cachon and Kok (2007) have proposed different heuristics for estimating the fixed salvage value. According to them, the weighted average salvage value (WASV) heuristic provides the solution that is closest to the optimal one. We evaluate the variable salvage value newsvendor model against the classical newsvendor model where salvage value is computed using the WASV heuristic. The WASV heuristic computes the fixed salvage value by the relationship $v(q^*) = R_s(q^*)/I(q^*)$, where q^* is the classical newsvendor optimal order quantity, $R_s(q^*)$ is the expected clearance-sale period revenue for order quantity q^* and $I(q^*)$[5] is the expected inventory at the end of normal sale for order quantity q^*. At the classical newsvendor optimal solution $F(q^*) = (p - c)/(p - v)$ or $v(q^*) = p - (p - c)/F(q^*)$, which also implies that *v* is increasing in q^*.

Using the expressions of R_s as shown in (2a) and (2b) and $I(q^*)$, $v(q^*)$ can be expressed as:

$$v(q^*) = a_v - \left\{ 2b_v \int_0^{q^*} (q^* - x) F(x) dx \Big/ \int_0^{q^*} F(x) dx \right\}, \text{for } q^* \leq s \qquad \ldots \qquad \ldots \qquad (7a)$$

$$= \left\{ a_v \int_{j^*}^{q^*} F(x) dx - 2b_v \int_{j^*}^{q^*} (q^* - x) F(x) dx \right\} \Big/ \int_0^{q^*} F(x) dx, \text{for } q^* > s \qquad \ldots \qquad (7b)$$

Observing (7a) and (7b), it can be noticed that q^* and, hence, $v(q^*)$ are increasing in a_v.

PROPOSITION 1: *The optimal profit in the variable salvage value newsvendor model will at least be equal to the optimal profit obtained for the classical newsvendor.*

---

[5] The expected inventory at the end of normal sale season, $I(q^*)$, is equal to $\int_0^q (q - x) f(x) dx$, which on simplification can be expressed as $\int_0^q F(x) dx$.

PROOF: As described above, the fixed salvage value as per WASV heuristic is $v = R_s(q^*)/I(q^*)$. At $q = q^*$, $\pi|_{q=q^*} = R_1(q^*) + R_s(q^*) - cq^* = p(q^* - I(q^*)) + vI(q^*) - cq^* = (p-c)q^* - (p-v)I(q^*)$, which is the classical newsvendor optimal profit. Thus, the variable salvage value newsvendor optimal profit is at least equal to the classical newsvendor optimal profit.

PROPOSITION 2: *The variable salvage value newsvendor optimal order quantity, $\hat{q}$, is always lesser than the classical newsvendor optimal order quantity, q^*, except when a_v = 0. For a_v = 0, $\hat{q} = q^*$.*

PROOF: Referring to the definitions of the normal period revenue, R_1, and the clearance-sale period revenue, R_s, indicated respectively by (1), (2a) and (2b), it can be seen that the difference between the variable salvage value newsvendor profit and the classical newsvendor profit is only in the R_s term for a given *p* and *q*. The R_s term in the classical newsvendor profit is equal to $vI(q)$, where $v = R_s(q^*)/I(q^*)$ as already explained earlier. Hence, the difference between the variable salvage value newsvendor profit and the classical newsvendor profit for a given *q*, indicated by $\Delta\pi(q)$, is:

$$\Delta\pi(q) = R_s(q) - vI(q) = R_s(q) - \frac{R_s(q^*)}{I(q^*)}I(q) = I(q)\left\{\frac{R_s(q)}{I(q)} - \frac{R_s(q^*)}{I(q^*)}\right\} \qquad \ldots \qquad (8)$$

By (8), $\Delta\pi(q)$ is non-negative for $0 \le q \le q^*$ if $R_s(q)/I(q)$ is decreasing in *q*. Referring to (2a) and (2b), it can be seen that $R_s(q)/I(q)$ is decreasing in *q* for a variety of distributions including uniform, normal and gamma distributions. It can further shown that $R_s(q)/I(q)$ decreases in *q* for any distribution that satisfies the condition $E(Z) + SD(Z) < 1$, where Z is a truncated distribution of the actual demand up to the order quantity q expressed over the range $[0,1]$. For the same reason, $\Delta\pi(q)$ is negative for $q > q^*$. As the variable salvage value newsvendor optimal profit is at least equal to the classical newsvendor optimal profit, it can be concluded that the variable salvage value newsvendor optimal order, $\hat{q}$, is such that $0 \le \hat{q} \le q^*$. When a_v = 0, *j* = *q* as *s* = 0. Hence, by (4b) $\hat{q} = F^{-1}((p-c)/p)$. Also, by (7b) $v(q^*) = 0$, which implies $F(q^*) = ((p-c)/p)$ or $q^* = F^{-1}((p-c)/p) = \hat{q}$.

It can be noted here that over the entire range $[0, q_{\max}]$ we have $\hat{q} < q^*$ and yet the profit yield is better with lesser optimal order quantity decision. Propositions 1 and 2 together establish how the variable salvage value model can offer better result than the WASV heuristic. Cachon and Kok (2007), using numerical experiments, had observed that a fixed salvage value estimate results in over ordering and lesser profits than the true optimal. Our propositions analytically reinforce these findings.

Figure 1: Comparison of the newsvendor models

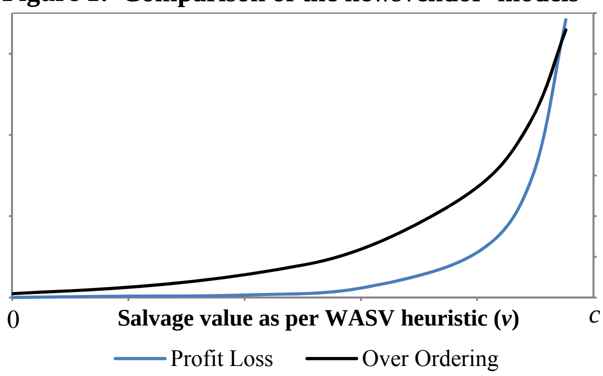


Figure 1 describes the over ordering and profit loss for a fixed salvage value newsvendor model against a variable salvage value newsvendor model. It can be seen that as the fixed salvage value considered increases from zero to c (the maximum value that fixed salvage value can take), the over ordering and profit loss increase exponentially. It is evident that the over ordering and profit loss owing to the classical newsvendor model are high for fixed salvage values that are closer to the marginal cost c. Further details are provided in the numerical study section.

4. Modelling supply contracts for variable salvage value

The optimal order $\hat{q}$ obtained for the newsvendor model with variable salvage value is also the optimal order quantity for a centralized supply chain comprising of a supplier (manufacturer) and a buyer (retailer). We denote this centralized supply chain optimal order quantity as $\hat{q}_c$ from here onwards. In this section we analyse different supply contracts for a variable salvage value in a decentralized supply chain setting. For the sake of simplicity we assume the buyer's marginal cost is zero and there is no penalty cost associated with every unit of under-stocking. Using the results obtained in the previous section, we attempt to understand how the contract structure changes with the introduction of variable salvage value and how coordination can be achieved between the supplier and buyer. The contracts studied are wholesale price contract, buy-back contract, revenue sharing contract and sales rebate contract.

4.1 Wholesale Price Contract

The wholesale price contract is the simplest of the contract forms. Here, the supplier charges the retailer a fixed per unit wholesale price, w_{WP}. The transfer payment is expressed by: $T_{WP}(q, w_{WP}) = w_{WP}q$.

The retailer profit function (π_R) takes up the following form:

$$\pi_R = (p - w_{WP})q - p\int_0^q F(x)dx + (a_v - 2b_v q)\int_0^q F(x)dx + 2b_v\int_0^q xF(x)dx, \text{ for } q \le s \quad \ldots \quad (9a)$$

$$= (p - w_{WP})q - p\int_0^q F(x)dx + (a_v - 2b_v q)\int_j^q F(x)dx + 2b_v\int_j^q xF(x)dx, \text{ for } q > s \quad \ldots \quad (9b)$$

Following newsvendor model presented in §3, it can be seen that the retailer profit function (π_R) is concave over the range $[0, q_{\max}]$ when the wholesale price contract is employed. The supplier profit function is of the form, $\pi_S = (w_{WP} - c)q$. Sequential optimization of the retailer profit function and the supplier profit function yields the optimal wholesale price expressed as a function of the optimal order quantity and is given by the following proposition.

PROPOSITION 3: *In wholesale price contract, the optimal wholesale price, $\hat{w}_{WP}$, can be expressed as a function of the retailer optimal order quantity, $\hat{q}_{WP}$, as follows:*

$$\hat{w}_{WP} = \hat{q}_{WP}\left\{(p - a_v)f(\hat{q}_{WP}) + 2b_v F(\hat{q}_{WP})\right\} + c, \text{ for } \hat{q}_{WP} \le s \quad \ldots \quad \ldots \quad (10a)$$

$$= \hat{q}_{WP}\left\{(p - a_v)f(\hat{q}_{WP}) + 2b_v\left(F(\hat{q}_{WP}) - F(\hat{j}_{WP})\right)\right\} + c, \text{ for } \hat{q}_{WP} > s \quad \ldots \quad (10b)$$

PROOF: Please contact the authors for the working.

In the presence of variable salvage value the optimal wholesale price takes distinct forms depending upon the value of $\hat{q}_{WP}$; this result differs from that of classical newsvendor model, as the optimal value takes two distinct forms over two mutually exclusive ranges. As shown by Cachon (2003), the wholesale price contract can coordinate only if the supplier earns a non-positive profit. For a positive supplier profit

($\hat{w}_{WP} > c$), the retailer would not order the centralized coordinating order quantity $\hat{q}_c$. Therefore this contract fails to coordinate the supply chain and this conclusion is consistent with a constant salvage value model (Lariviere and Porteus, 2001). The overall profit level of the supply chain reduces compared to the centralized case.

4.2 Buy-back Contract

In the buy-back contract the supplier charges the retailer a fixed per unit price w_{BB} and at the end of the normal period buys back the leftover inventory from the retailer at a per unit price of b. The supplier subsequently disposes off the leftover items during the clearance-sale period through a variable salvage value, $v(i) = a_v - b_v i$. The profit function of the retailer (π_R) is given by the following equation: $\pi_R = (p - w_{BB})q - (p - b)\int_0^q F(x)dx$. The optimal per unit price, $\hat{w}_{BB}$, for the buy-back contract is given by the following proposition.

PROPOSITION 4: *In buy-back contract, the optimal per unit price, $\hat{w}_{BB}$, can be expressed in terms of the optimal order quantity and exogenously decided buy-back price, b, in the following form*

$$\hat{w}_{BB} = c + (b - a_v)F(\hat{q}_c) + 2b_v \int_0^{\hat{q}_c} F(x)dx, \text{ for } \hat{q}_c \le s \qquad \ldots \qquad \ldots \qquad (11a)$$

$$= c + (b - a_v)F(\hat{q}_c) + (a_v - 2b_v s)F(\hat{j}) + 2b_v \int_{\hat{j}_c}^{\hat{q}_c} F(x)dx, \text{ for } \hat{q}_c > s \qquad \ldots \qquad (11b)$$

where, $\hat{q}_c$ is the optimal order quantity for the centralized case.

PROOF: Please contact the authors for the working.

The above form implies that there is a per unit price corresponding to a buy-back price that ensures that the retailer order quantity is $\hat{q}_c$. Hence, the overall supply chain profit is equal to that of the centralized case. Akin to the buy-back contract for the classical newsvendor model (Pasternack, 1985), the buy-back contract for a variable salvage value too coordinates the supply chain.

4.3 Revenue Sharing Contract

In the revenue sharing contract the supplier charges a fixed per unit price w_{RS} and the retailer gives the supplier a share, $\phi\,(0 \le \phi \le 1)$, of her total revenue, which is the sum of normal period revenue and the clearance-sale revenue. The profit function of the retailer is given by the following equation,

$$\pi_R = (\phi p - w)q - \phi\left\{(p - a_v + 2b_v q)\int_0^q F(x)dx - 2b_v\int_0^q xF(x)dx\right\}, \text{for } q \le s \quad \ldots \quad (12a)$$

$$= (\phi p - w)q - \phi\left\{p\int_0^q F(x)dx - (a_v - 2b_v q)\int_j^q F(x)dx - 2b_v\int_j^q xF(x)dx\right\}, \text{for } q > s \quad (12b)$$

Through sequential optimization, the optimal per unit price, $\hat{w}_{RS}$, for the revenue sharing contract is described in the following proposition.

PROPOSITION 5: *In revenue sharing contract, the optimal per unit price, $\hat{w}_{RS}$, can be expressed in terms of the optimal order quantity and exogenously decided revenue share, ϕ, in the following form*

$$\hat{w}_{RS} = c - (1-\phi)\left\{p - (p - a_v)F(\hat{q}_c) - 2b_v\int_0^{\hat{q}_c} F(x)dx\right\}, \text{for } \hat{q}_c \le s \quad \ldots \quad (13a)$$

$$= c - (1-\phi)\left\{p - (p - a_v)F(\hat{q}_c) - (a_v - 2b_v s)F(\hat{j}_c) - 2b_v\int_{\hat{j}_c}^{\hat{q}_c} F(x)dx\right\}, \text{for } \hat{q}_c > s \quad (13b)$$

PROOF: Please contact the authors for the working.

The revenue sharing contract will coordinate the supply chain when $\hat{w}_{RS} = \phi c$. This observation is similar to the result obtained for the classical newsvendor model (Cachon and Lariviere, 2005). Though the optimal per unit price takes two distinct forms as shown in 13a and 13b, the coordinating mechanism is same in both instances and is similar to that of the fixed salvage value classical newsvendor model framework.

4.4 Sales Rebate Contract

In sales-rebate contract the supplier charges a fixed per unit price, w_{SR}, and gives the retailer a per unit rebate, r, for every unit ordered above a threshold level of t. The profit function of the retailer is given by the following equation,

$$\pi_R = (p + r - w)q - rt - p\int_0^q F(x)dx - r\int_t^q F(x)dx + (a_v - 2b_v q)\int_0^q F(x)dx + 2b_v\int_0^q xF(x)dx, \text{for } q \le s \quad (14a)$$

$$= (p + r - w)q - rt - p\int_0^q F(x)dx - r\int_t^q F(x)dx + (a_v - 2b_v q)\int_j^q F(x)dx + 2b_v \int_j^q xF(x)dx, \text{ for } q > s \quad (14b)$$

The optimal per unit price, $\hat{w}_{SR}$, for sales-rebate contract that coordinates the supply chain is given by the following proposition.

PROPOSITION 6: *In sales-rebate contract, the optimal per unit price, $\hat{w}_{SR}$, can be expressed in terms of the optimal order quantity, exogenously decided rebate value, r , and threshold level, t , in the following form: $\hat{w}_{SR} = c + rF(t) + r\overline{F}(\hat{q}_c)$, where $\overline{F}(\hat{q}_c) = 1 - F(\hat{q}_c)$.*

PROOF: Please contact the authors for the working.

For a variable salvage value, the optimal per unit price, $\hat{w}_{SR}$, computed for sales-rebate contract is similar to the result obtained in the fixed salvage value case. The overall supply chain profit and the optimal order quantity are equal to those of the centralized case (Krishnan et al, 2004).

The above results indicate that modelling the salvage value as a function of leftover inventory results in optimal per unit prices that are distinct from the optimal per unit prices for a fixed salvage value model in the case of wholesale price and buy-back contracts. However, the optimal per unit price formulae are the same for variable and fixed salvage values in the case of revenue sharing and sales rebate contracts.

5. Numerical Study

A numerical study is conducted to understand the benefits of a variable salvage value modelling vis-à-vis the fixed salvage value model, and to compare supply contracts for variable salvage value against the same for fixed salvage value on wholesale prices, the retailer profits and the supplier profits. The parameter values considered are p = 20, c = 12, μ = 20 and σ = 5, where μ and σ are the mean and standard deviation of the normal period demand, respectively.

5.1 Effect of a_v on v and optimal order quantity

It has already been shown mathematically in §3 that v as well as variable salvage value newsvendor optimal order quantity are increasing in a_v. The numerical study in this sub-section, using above data, studies the effect of a_v on v and the optimal order quantity. The experiments are carried out for b_v values of 0.2, 0.6 and 1.0 and results are shown in Table 2.

Table 2: Effect of a_v on v and optimal order quantity

a_v	$b_v = 0.2$			$b_v = 0.6$			$b_v = 1.0$		
	v	$\hat{q}$	q^*	v	$\hat{q}$	q^*	v	$\hat{q}$	q^*
0	0.00	18.73	18.73	0.00	18.73	18.73	0.00	18.73	18.73
1	0.30	18.79	18.81	0.11	18.75	18.76	0.07	18.75	18.75
2	0.98	18.95	19.00	0.42	18.81	18.84	0.26	18.78	18.80
3	1.86	19.18	19.26	0.88	18.91	18.97	0.57	18.84	18.88
4	2.80	19.46	19.56	1.46	19.04	19.14	0.96	18.93	18.99
5	3.76	19.79	19.91	2.13	19.21	19.34	1.44	19.03	19.13
6	4.72	20.15	20.30	2.87	19.41	19.59	1.99	19.16	19.30
7	5.68	20.56	20.74	3.65	19.64	19.87	2.59	19.31	19.49
8	6.63	21.03	21.25	4.46	19.90	20.19	3.24	19.48	19.71
9	7.58	21.56	21.84	5.29	20.20	20.55	3.91	19.68	19.97
10	8.50	22.19	22.56	6.13	20.53	20.97	4.62	19.89	20.25
11	9.41	22.92	23.45	6.96	20.91	21.44	5.33	20.14	20.57
12	10.27	23.81	24.62	7.77	21.32	21.98	6.05	20.40	20.93
13	11.06	24.92	26.26	8.56	21.78	22.61	6.77	20.69	21.33
14	11.69	26.31	28.89	9.31	22.29	23.34	7.48	21.01	21.78
15	11.97	28.06	33.28	10.00	22.85	24.21	8.17	21.36	22.29
16	12.00	30.17	38.66	10.62	23.47	25.25	8.83	21.73	22.86
17	12.00	32.54	43.96	11.14	24.15	26.49	9.45	22.14	23.50
18	12.00	35.00	49.14	11.53	24.88	27.95	10.02	22.57	24.23
19	12.00	37.50	54.27	11.78	25.66	29.64	10.52	23.03	25.06
20	12.00	40.00	59.37	11.91	26.47	31.48	10.95	23.52	25.98

Based on the above results, the behaviour of v and over ordering ($q^* - \hat{q}$) with respect to a_v is plotted in Figures 2 and 3, respectively. From Figure 2 it can be seen that as a_v increases from 0 to p, v increases from 0 and can take a maximum value of c.

Figure 2: v versus a_v

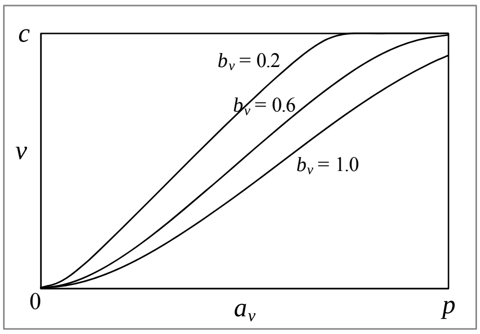


Figure 3: Over Ordering versus a_v

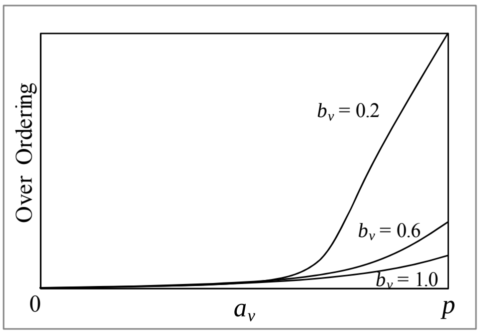


Figure 3 shows that the over ordering increases rapidly once a_v is greater than $p/2$. The over ordering is greater as b_v decreases. The above observations clearly indicate that the classical newsvendor optimal order quantity is inaccurate for high a_v and low b_v values. Similar observations were noticed when the demand was assumed to follow a uniform distribution. As already seen in Figure 1, over ordering is associated with profit loss. In many retail contexts, including online retailing, it is observed that retailers start off with a low discount rate, which is indicative of a high a_v value. Such a discount strategy is also known as *contingent discount policy* (Aviv & Pazgal, 2008). In such situations, the classical newsvendor approach reduces the retailer profit while making the retailer over order and makes the case of using the variable salvage value newsvendor model very strong.

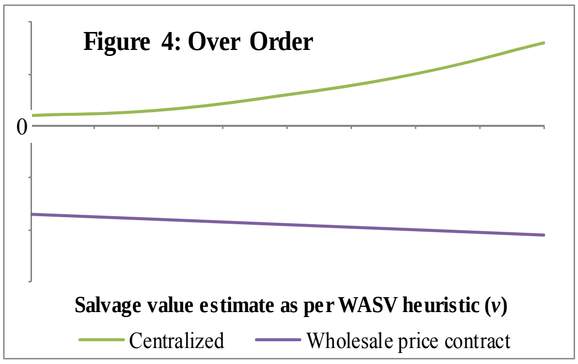
5.2 Comparison of wholesale price contracts against the centralized supply chain

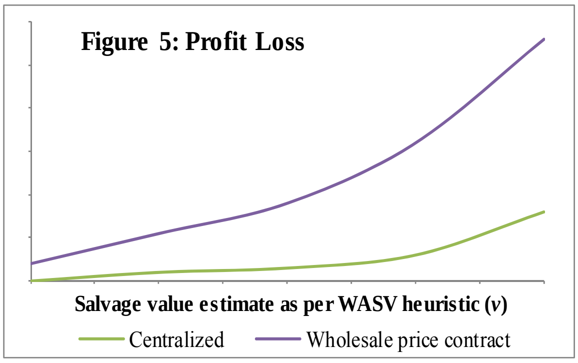
We consider five combinations of a_v and b_v values that would, for the demand data above, yield salvage value estimates (v) of 2, 4, 6, 8 and 10 when the WASV heuristic is used.

Table 3: Comparison of wholesale price contract against centralized supply chain

No.	a_v	b_v	s	v	Centralized supply chain				Wholesale price contract					
					Constant v		Variable v		Constant v			Variable v		
					q^o	π^o_{SC}	q^o	π^o_{SC}	q^o	π^o_{SC}	π^o_R	q^o	π^o_{SC}	π^o_R
1	13	5.31	1.22	2	19.3	124.4	19.1	124.4	12.8	99.7	14.5	12.9	100.1	14.3
2	14	2.60	2.69	4	20.0	128.1	19.7	128.3	13.2	102.1	15.0	13.3	103.3	14.5
3	15	1.73	4.33	6	20.9	132.5	20.3	132.8	13.6	105.2	15.5	13.7	107.0	14.9
4	16	1.23	6.52	8	22.2	138.2	21.2	138.8	14.0	108.7	16.0	14.4	111.9	15.4
5	17	0.86	9.86	10	24.2	146.0	22.6	147.6	14.6	113.0	16.7	15.2	118.6	16.4

Looking at the centralized supply chain results in Table 3 (the superscript '*o*' indicates optimal value), it is seen that over ordering (order quantity for fixed salvage value in excess of order quantity for variable salvage value) and profit loss (profit for variable salvage value in excess of profit for fixed salvage value) as a result of assuming a fixed salvage value increases with v as was already shown in §3. However, while employing the wholesale price contract a contrasting observation is noticed regarding order quantity. The optimal order quantity is greater when the salvage value is variable and, hence, wholesale price contract results in under ordering (see Figure 4) when the salvage value is assumed constant. However, supply chain profit continues to be higher when the salvage value is variable. Interestingly, the profit loss while employing the wholesale price contract as a result of assuming a fixed salvage value is higher than that observed in centralized supply chain (see Figure 5). In other words, the impact on profit as a result of assuming a fixed salvage value is greater for wholesale price contracts vis-à-vis the centralized supply chains.





5.3 Effect of variable salvage value on the coordinating contracts

It has been shown in §4 that the buy-back, revenue sharing and sales rebate contracts coordinate the supply chains when the salvage value is variable. Here we attempt to understand the effect on retailer and supplier profits as a result of a variable salvage value. We continue with the p, c, μ and σ values used in the previous analyses. We assume a_v = 15 and b_v = 0.6, which results in a s value of 12.5 units and a salvage value estimate of 10 when the WASV heuristic is used. The data considered results in optimal order quantities of 24.2 units and 22.9 units in the constant and variable salvage value centralized supply chains, respectively. The centralized supply chain profits are 146.0 and 147.1 for constant and variable salvage values, respectively. The revenue sharing contract is not considered in the numerical study as the share of revenue that the retailer receives is also the retailer's share of the supply chain profit in both the salvage value cases.

In the fixed salvage value case, the retailer profit decreases from 100% to 0% of the supply chain profit as the buy-back price increases from v to p. Referring to Table 4, the per unit price is greater for the variable salvage case except when the buy-back price is equal to p. Hence, the retailer profit as a share of the supply chain profit is lesser for the variable salvage case. Thus for a given buy-back price, the variable salvage value benefits the supplier greater. In other words, a supplier targeting a particular profit level can settle for a lower buy-back price in the variable salvage value case in comparison to a fixed salvage value case.

Table 4: Effect of buy-back price

b	Constant v		Variable v	
	w^o	π_R^o	w^o	
10	12.0	146.0	12.8	126.2
11	12.8	131.4	13.6	113.6
12	13.6	116.8	14.3	101.0
13	14.4	102.2	15.0	88.3
14	15.2	87.6	15.7	75.7
15	16.0	73.0	16.4	63.1
16	16.8	58.4	17.1	50.5
17	17.6	43.8	17.9	37.9
18	18.4	29.2	18.6	25.2
19	19.2	14.6	19.3	12.6
20	20.0	0.0	20.0	0.0

Table 5: Effect of sales rebate

t	r	Constant v		Variable v	
		w^o	π_R^o	w^o	π_R^o
15	1	12.4	142.2	12.4	141.5
15	2	12.7	138.4	12.9	135.9
15	3	13.1	134.5	13.3	130.3
15	4	13.4	130.7	13.8	124.7
15	5	13.8	126.9	14.2	119.2
20	1	12.7	130.5	12.8	130.3
20	2	13.4	115.0	13.6	113.5
20	3	14.1	99.5	14.4	96.7
20	4	14.8	84.0	15.1	79.9
20	5	15.5	68.5	15.9	63.0

As observed for the buy-back price contract, the per unit price is greater for the variable salvage case in the sales rebate contract too (see Table 5). Hence, the retailer profit as a share of the supply chain profit is lesser for the variable salvage case for a given combination of r and t. Thus for a given r and t combination, the variable salvage value benefits the supplier greater. In other words, a supplier targeting a particular profit level can settle for a lower buy-back price in the variable salvage value case in comparison to a fixed salvage value case.

6. Discussion and Conclusions

Clearance price is a function of inventory leftover in many real life contexts. However, the classical newsvendor model assumes a fixed or constant salvage value for clearance sales. We analytically prove that the newsvendor model with constant salvage value results in over ordering and profit loss compared to a newsvendor model where salvage value is assumed to be a function of inventory leftover at the end of the period. Though Cachon and Kok (2007) have numerically reported these findings and have argued about the importance of estimating salvage value accurately, we have not come across any work that has established these results analytically.

If goods are sold at a fixed discount rate or at a fixed salvage value, the seller cannot maximize her expected revenue (Aviv & Pazgal, 2008). With a fixed salvage value, manager over estimates the expected demand and it leads to over ordering. Subsequently, it leads not only to the wastage of capacity for the producer, but also to dilution of the brand value of the product as higher clearance inventory ensues. In order to increase the perceived value of a product, scarcity of goods is often employed as a marketing strategy. Zara, a Spanish apparel producer, restricts the number of products in their stores to create urgency among her customers (Ghemawat & Nueno, 2003) and similar strategy is adopted by World Co. Ltd., a Japanese retailer for women clothing (Raman et al., 2001).

However, the seller may consider the salvage value to be fixed when the conditions are such that the WASV heuristic yields a v value that is lesser than $c/2$. Such situations arise when the product gets obsolete rapidly at the end of season (products with low a_v) or is quickly perishable (certain categories of food products with short shelf lives). Sellers should certainly consider the variable salvage value when their estimate of a_v is at least $p/2$ and particularly when b_v, the clearance price sensitivity to leftover inventory, takes a lower value. As mentioned in the previous section, there are many such retail situations and using the classical newsvendor model would reduce the retailer profit while making her over order.

The variable salvage model displays similar behaviour as the fixed salvage model with regards to coordination through supply contracts. As in the fixed salvage case, the wholesale price contract does not coordinate the supply chain while the buy-back, revenue sharing and sales rebate contracts coordinate the supply chain when the salvage value is variable. For a variable salvage value, the wholesale price contract results in higher order quantity and profit. The buy-back contract results in a higher per unit price for a given buy-back price or higher profit for the supplier when the salvage value is variable. Similar observations are noticed in the sales rebate contract too.

7. References

1. Agrawal, V., & Seshadri, S. (2000). Impact of uncertainty and risk aversion on price and order quantity in the newsvendor problem. *Manufacturing & Service Operations Management*, 2(4), 410-423.

2. Besbes, O., & Muharremoglu, A. (2013). On implications of demand censoring in the newsvendor problem. *Management Science*, 59(6), 1407-1424.
3. Bitran, G. B., & S. V. Mondschein. (1997). Periodic pricing of seasonal products in retailing. *Management Science*, 43 (1), 64–79.
4. Cachon, G. P. (2003). Supply chain coordination with contracts. *Handbooks in operations research and management science*, 11, 227-339.
5. Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management science*, 51(1), 30-44.
6. Cachon, G. P., & Kok, A. G. (2007). How to (and how not to) estimate the salvage value in the newsvendor model. *Manufacturing & Service Operations Management*, 9(3), 276-290.
7. Caro, F., & Gallien, J. (2012). Clearance pricing optimization for a fast-fashion retailer. *Operations Research*, 60(6), 1404-1422.
8. Carr, S., & Lovejoy, W. (2000). The inverse newsvendor problem: Choosing an optimal demand portfolio for capacitated resources. *Management Science*, 46(7), 912-927.
9. Dana Jr, J. D., & Petruzzi, N. C. (2001). Note: The newsvendor model with endogenous demand. *Management Science*, 47(11), 1488-1497.
10. Donohue, K. (2000). Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management Science*, 46(11), 1397–1411.
11. Eeckhoudt, L., Gollier, C., & Schlesinger, H. (1995). The risk-averse (and prudent) newsboy. *Management Science*, 41(5), 786-794.
12. Fisher, M.L., Rajaram, K., & Raman, A. (2001). Optimizing inventory replenishment of retail fashion products. *Manufacturing & Service Operation Management*, 3(3), 230–241.
13. Fisher, M.L., & Raman, A. (2010). The new science of retailing: how analytics are transforming the supply chain and improving performance. Harvard Business School Publishing, Boston.
14. Forest, S. A., Zellner, W., & Palmer, A. T. (2003). Giving away the store. *Business Week,* June, 2, 51.

15. Hertz, D. B., K. H. Schaffir. (1960). A forecasting method for management of seasonal style goods inventories. *Operations Research*, 8(1), 45–52.

16. Huang, J., Leng, M., & Parlar, M. (2013). Demand Functions in Decision Modeling: A Comprehensive Survey and Research Directions. *Decision Sciences*, 44(3), 557-609.

17. Kratz, E. F. (2005). Marked down. *Fortune*, 152(4), 103 - 107.

18. Krishnan, H., Kapuscinski, R., & Butz, D. A. (2004). Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management science*, 50(1), 48-63.

19. Lariviere, M. A., & Porteus, E. L. (2001). Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management*, 3(4), 293-305.

20. Lilien, G.L., Kotler, P., & Moorthy, K.S. (1992). Marketing Models. Prentice Hall, Englewood Cliffs, NJ.

21. Pasternack, B. A. (1985). Optimal pricing and return policies for perishable commodities. *Marketing Science*, 4(2), 166-176.

22. Petruzzi, N. C., & Dada, M. (1999). Pricing and the newsvendor problem: A review with extensions. *Operations Research*, 47(2), 183-194.

23. Petruzzi, N., & M. Dada. (2001). Information and inventory recourse for a two-market, price setting retailer. *Manufacturing & Service Operation Management*, 3(3), 242–263.

24. Prasad, A., Stecke, K. E., & Zhao, X. (2011). Advance selling by a newsvendor retailer. *Production and Operations Management*, 20(1), 129-142.

25. Raz, G., & Porteus, E. L. (2006). A fractiles perspective to the joint price/quantity newsvendor model. *Management Science*, 52(11), 1764-1777.

26. Rozhon, T. (2005). First the markdown, then the showdown. *New York Times*, (February 25), C1.

27. Sarkis, J., & Semple, J. H. (1999). Vendor Selection with Bundling: A Comment. *Decision Sciences*, 30(1), 265-271.

28. Schweitzer, M. E., & Cachon, G. P. (2000). Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence. *Management Science*, 46(3), 404-420.

29. Şen, A., & Zhang, A. X. (2009). Style goods pricing with demand learning. *European Journal of Operational Research*, 196(3), 1058-1075.
30. Smith, S., & D. Achabal. (1998). Clearance pricing and inventory policies for retail chains. *Management Science*, 44(3), 285–300.
31. Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. *Management science*, 48(8), 992-1007.
32. Varian, H. R. (1992). *Microeconomic analysis* (Vol. 2). New York: Norton.
33. Wang, C. X., & Webster, S. (2009). Markdown money contracts for perishable goods with clearance pricing. *European Journal of Operational Research*, 196(3), 1113-1122.
34. Aviv, Y., & Pazgal, A. (2008). Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing & Service Operations Management*, 10(3), 339-359.
35. Ghemawat, P., Nueno, J. L., & Dailey, M. (2003). *Zara: fast fashion* (pp. 1-35). Harvard Business School.
36. Raman, A., Fisher, M., & McClelland, A. (2001). *Supply Chain Management at World Co., Ltd* (p. 21). Boston, MA: Harvard Business School.