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Product Greening, Competition and Cooperation under Environmental Regulations

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Abstract

In this paper we explore the effect of environmental regulations and costs of greening on firms. Our problem deals with the case of a single firm, a duopoly and the case of cooperation between them. We use these market settings to study the pricing and greening decision of players under environmental regulations and increasing costs. We also analyze their impact on consumers. Further, we explore several contracts between the players under competition. Through this problem we address the burgeoning challenges that firms face in the presence of competition and environmental regulations. This research lays the platform for future work in the area of 'green' product pricing, environmental contract design mechanisms and study of impact of environmental regulations on firms and supply chains.

Keywords: Green Product, Environmental Legislation, Contracts, Game theory

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1 Introduction

During the past two decades 'sustainability' has emerged as the new paradigm of conducting business. From increased Government pressure to societies demanding more responsible behaviour from corporate houses towards environmental issues, the voices are pouring in. Under such growing demand for change in products and processes of organisations, businesses realise that the dynamics of world economy are changing. Those companies which cannot fully leverage this may have serious socio-economic manifestations in the form of over-dependence on resources which may be very scarce and costly to procure and utilize. Further, sustainable development has the potential to change the economics of supply chains and may compromise the competitiveness of companies by affecting the cost structure of industries and restricting market access. Policies of governments across the world are rapidly changing course and the implications of such change for companies can be far fetched. Those companies which already envision a change in the policy, will undertake investments much earlier than their competitors. These investments will have significant impact in terms of product prices, strategic decisions on product improvement levels and so on. On the other hand, Government legislations, once they come into force, will increase the costs of firms several folds. Given these prospects, we explored various policy changes that Governments across the world have initiated fairly recently and we zeroed down to two observations.

Our problem is primarily motivated by the recent developments in two markets of the world namely, India and the United States. The Finance bill 2010-11 in India created a corpus called "*National Clean Energy Fund which will invest in entrepreneurial ventures and research in the field of clean energy technologies. The money for this will be garnered through a so-called 'clean energy cess' of Rs* 50 *on every tonne of coal, both domestic and imported.* " (*Economic Times*, Feb 2010). In the United States, 'Corporate Average Fuel Economy(CAFE)' regulations underwent a sea change when they included light trucks under the stringent CAFE standards. The rules state that " *if the average fuel economy of a manufacturer's annual fleet of car and/or truck production falls below the defined* *standard, the manufacturer must pay a penalty, currently \$5.50 USD per* 0.1 *mpg under the standard, multiplied by the manufacturer's total production for the U.S. domestic market*" (*www.nhtsa.gov*). In the U.S, the National Highway Traffic Safety Administration (NHTSA) regulates CAFE standards and in return helps automobile manufacturers in several ways. The administration advertises on its website those companies which follow these norms, displays to the consumer the total fines collected from various auto makers and assigns a green score to each vehicle type from each auto maker. The aim is to increase awareness of the consumer towards greener vehicles and also help the complying auto makers generate more sales. The increasing consumer preference towards green vehicles is an important consideration in this study.

It was further observed that for each model year heavy fines were collected from leading auto makers like Porsche, Fiat, Mercedes-Benz, Daimler Chrysler,Volkswagen, Aston Martin, Jaguar and many more. Surprisingly from model year 1983 till 2003, auto maker Toyota had not been fined. Studies on the highest quality standards of Toyota definitely speak volumes in support of this observation. Also, the cost of greening for Toyota, subject to these regulations may be far lesser as compared to that of its competitors. The differentiated cost of greening is another important consideration in our study. [3].

From the above discussion, the following key inferences are:

1. Government norms for pollution/fuel efficiency .
2. Increasing consumer preference towards less polluting (greener) vehicles.
3. Differentiated cost of greening between competitors.

The inferences although primarily derived from the auto sector are prevalent in several other sectors like steel manufacturing , consumer goods production , chemical and dye manufacturing etc. These industries are typically characterized by price competition and now, increased competition in greening their products. In this study, we consider the impact of competition on product greening levels and prices of the green product.

Inspite of the intense competition, interestingly several companies within these industries have come together to counteract the Government legislations. Several of them have

[3] *Note however, that recently Toyota has been involved in several product recalls raising questions on the quality standards maintained by the company (www.economist.com/blogs/schumpeter/2014/04/toyota)*

formed joint ventures in order to develop cleaner technologies, few have invested in third party research organisations to develop environmental friendly technologies while some have shared best practices and knowledge of their processes with their competitors and suppliers in order to build a better knowledge base for green technologies and products. To cite few examples, project ULCOS (*Ultra Low CO_2 emissions*) is a research venture by major EU steel companies and TATA Steel aimed at developing technologies for reduction of carbon dioxide emissions by at least 50 percent. The companies would be financing the research conducted by scientists and research schools in Europe (*www.ulcos.org*). In another example, in a different set up of co-operative model, General Motors Corp. and Ford Motor Co., teamed up in a unique partnership to develop a new six-speed automatic transmission. The two companies cooperated on designing, engineering and testing the new transmission as well as working with suppliers to develop and buy components. The high-volume, front-wheel-drive transmission offers an estimated 4 - 8 percent improved fuel economy over traditional four-speed transmissions in front-wheel-drive cars (*www.bnet.com*).

Thus, several questions arise. What is the impact of Government regulation and cost of greening on a firm's decision on the level of product greening to be achieved. Further, how do they impact the price of the green product? What happens when there is price and greening competition between two manufacturers as noticed in the auto sector? What is the impact on the level of greening and price of the green products in such a case. Are the results any different when the two players cooperate in the market to develop a cleaner technology/product but compete on prices? How do contracts between two competitor firms impact their decisions? Which strategy can best suit a firm under prevailing regulations and costs?

In order to answer the above questions we adopt an analytical approach. We first analyse the case of a single firm incurring greening costs and facing Government penalty. We extend this model to a duopoly under price and greening competition. We study several contracts in this context which can be entered into by competing firms. We finally study the case of cooperation between the two players. The analytical modeling approach helps understand the greening issue that organisations face from a basic stand point of costs and government penalty. The contract analysis throws interesting insights into how contractual terms impact greening decisions. Finally, we explore collaboration between firms under greening and study a cost sharing contract between collaborating firms. Our work should help organisations analyse the impact of government penalty and greening costs on their strategic decisions and profitability. Further, study of various contracts should help managers design and implement contractual terms with competing firms.

The rest of the paper is organised as follows. In the next section we discuss the background literature for this paper. Subsequently we discuss the case of a single firm and a duopoly in sections 3 and 4. In section 5 we discuss the case of cooperation in a duopoly. The last section discusses limitations of our work, managerial conclusions and future research ideas.

2 Literature Review

The literature related to the problem considered in this paper comes from three streams: (i) operations and supply chain management (ii) government legislations studies in economics and industrial organization and (iii) marketing . In what follows, we present an overview of the literature related to the three streams closely addressing our work in this paper.

Spence (1975) deals with market problems when a monopoly sets some aspect of product quality and price. The market problem discussed is the inability of the prices to convey information about the value of attached to quality by inframarginal consumers. The problem is also faced by the regulator for whom the challenge is to estimate the average value of quality over all consumers in the market. Spence (1976) discusses welfare aspects of product differentiation and monopolistic competition in a market system. The author discusses welfare problems in product differentiation in absence of price discrimination and derives functional forms for demands for product as a function of their attributes.In another work, Singh and Vives (1984) consider a differentiated duopoly where the demand function considered is linear and allows the the goods to be substitutes or complements. The authors show that in this setting, Bertrand competition is more efficient than Cournot competition in that the consumer surplus and total surplus are higher regardless of whether the goods are substitutes or complements. The findings of the authors sparked a large stream of literature comparing Bertrand and Cournot competition (Vives 1985, Bonanno 1986, Champsaur and Rochet 1989, Choi and Shin 1992, Motta M 1992a,1992b). To illustrate one, Motta (1993) compares Bertrand and Cournot competition with fixed and variable costs of quality. Their estimate of welfare indicates that not only consumer surplus but even producer surplus is higher under Bertrand competition than Cournot competition. In our paper we consider price and greening competition and use welfare definitions as cited in some of these previous works.

In the supply chain management literature, among early works Thomas (1970) discusses price and production decisions for a single product with a known deterministic demand function. Under the profit maximisation objective, the author finds optimal pricing decisions and planning horizons. The paper suggests an efficient forward algorithm combining these decisions. More recently, Karmarkar and Pitbladdo (1997) represent quality through two dimensions namely "class" attribute and "conformance" attribute and discuss various implications for a monopolist, perfect competition and oligopoly. They also discuss the case of a monopolist where consumer preferences for the product are uncertain. Our work is close to two significant papers on duopoly and competition. Banker, Khosla and Sinha (1998) study how quality is influenced by competitive intensity. Competitive intensity is discussed in terms of market share in a duopoly and oligopoly setting. The authors assume a linear demand curve as a function of prices and quality. They consider fixed and variable costs of achieving quality. The equilibrium results are analysed in the light of increased competition between firms. The authors also model a symmetric duopoly where firms cooperate in setting quality levels. Our modeling approach though quite similar to theirs addresses a different problem of greening under government legislation. We incorporate price and greening competition under the presence of government penalisation. The complexity of the problem increases manifold as even under linear demand and deterministic settings, the impact of government legislation is now considered along with price and product competition. Further, we do not model competitive intensity in terms of market share as greening initiatives are still an evolving process where competitive intensity has set in more in terms of pricing and greening levels of the product. Tsay and Agrawal (2000) study a distribution system in which a manufacturer supplies a common product to two independent retailers who use service and retail price to directly compete for the end customers. The authors study the impact of competitive intensity on total sales, market share and profitability. The authors also introduce wholesale price contract as a means to coordinate the channel between the manufacturer and two retailers. In contrast we study price and product competition under government legislation between two manufacturers under varying costs of greening, cooperation and contracts. We first study a single firm setting and extend this to a duopoly. Corbett and Karmarkar (2001) examine the impact of fixed and variable costs on the structure and competitiveness of supply chains with a serial structure and price sensitive linear demand. The authors derive price and production quantity decisions based on the number of entrants at each tier in the supply chain. The model competition in supply chain through number of players in each tier. Chen, Federgruen and Zheng (2001) model a two echelon distribution system in which the sales volumes of the retailers are endogenously determined on the basis of known demand functions. The demand of the retail market is assumed to be a decreasing function of the retail price in the market. The authors characterise the centralized channel and the decentralized channel optimal strategies. The authors propose a fixed fee contract and discount schemes through which the channel can be coordinated.

In the marketing stream, channel literature dealing with competition between two manufacturers or retailers have been dealt with extensively. Jeuland and Shugan (1983) did early work dedicated to channel coordination with prices as endogenous variables. They analyse a single manufacturer - retailer set up where they discuss optimal strategies with respect to three variables namely, prices, quality and sales effort. The authors demonstrate that without joint ownership and coordination, the manufacturer has the power and profit maximizing incentive to raise his margin above the level at which total channel profits are maximized. He also has the incentive to lower product quality below the joint maximum level and lower all other promotional decision variables at this disposal. The authors also discuss channel coordination mechanisms like joint ownership, simple contracts, implicit understanding, revenue sharing and quantity discounts. McGuire and Staelin (1983) discuss an exclusive dealer channel where two firms manufacturing differentiated but competing products sell their products through exclusive dealers. The authors discuss three cases: exclusive Manufacturer- exclusive retailer; two separate manufacturers owning the stores ; one manufacturer selling through a private retailer and the other selling through a company store. Coughlan and Wernerfelt (1989) discuss various class of distribution models between manufacturers and competing retailers that have been used in literature. They use price as the strategic decision variable. Choi (1991) studies a duopoly model of manufacturers who sell their products through a common independent retailer. He models three cases of Manufacturer Stackelberg(MS), Retailer Stackelberg(RS) and Vertical Nash(VN) equilibrium. The decision variables are whole sale prices for the manufacturers and the retail prices for the retailer. Ingene and Parry (1995) discuss channel coordination by a manufacturer that sells through competing retailers. Each retailer faces a downward sloping demand curve in prices. The authors find that a two part tariff is unable to coordinate the channel whereas a quantity discount schedule coordinates the channel. Choi (1996) discusses price competition in a duopoly common retailer channel. Author includes price competition between duopoly common retailer. He captures product differentiation and store differentiation. Lee and Staelin (1997) discuss vertical strategic interaction and implications for channel pricing strategy. Vertical Strategic Interaction means the direction of a channel member's reaction to the actions of its channel partner within a given demand structure. The authors analyse optimal strategies of two manufacturers selling competing products both carried by two competing retailers. Padmanabhan and Png (1997) discuss manufacturer's return policies with uncertain demands, limited shelf life and retail competition. The retailers compete in prices. The authors discuss various cases under which the returns policy is profitable for the manufacturer. Trivedi (1998)discusses various models of distribution channels , one of them being two competing manufacturers and two competing retailers. Using linear demand function, the competition is modeled on prices. The author analyses the impact of competitive intensity on both profits and prices. Iyer (1998) studies price and service competition between a single manufacturer and two retailer channel. The author represents individual consumer behaviour in terms of value of service and disutility of travel and from this derives each retailer's demand function. The author also discusses various channel coordination mechanisms.

Our work largely focusses on greening as a product attribute and models pricing and greening strategies of firms under rising costs and government penalty. Further, we address concerns of firms in designing contractual terms with their competitors to undergo greening. We also evaluate the surplus generated for consumers as a result of penalisation of firms. Lastly, we evaluate the impact of collaboration between competing firms on greening investment decisions and also explore a contract for greening cost sharing between the partner firms.

3 The Case of a Single Firm

We begin our analysis with the case of a single firm. We assume that all the activities in the market happen within a single period (Tsay and Agrawal, 2000). In our model, θ denotes the 'level of greening' decision by the firm and is continuous. The demand faced by the firm under product greening is assumed to be linearly decreasing in prices and increasing in the 'level of greening'. Linear demand models although a simplification are often considered for analytical tractability as such models throw interesting insights into problem parameters. The demand faced by the firm is given by

$$q = a - bp + \alpha\theta \qquad \text{where} \qquad a > bp, \alpha, b > 0 \tag{1}$$

Here a denotes the total market demand faced by the firm, p denotes the price of the product and θ denotes the 'level of greening' of the product. Further, 'b' and 'α' denote the demand sensitivity to price and 'greening level' respectively. The above equation captures the phenomenon of increased consumer demand achieved as a result of greening. We further model Government penalisation similar to the one levied under CAFE Standards. Let 'K' denote the penalty levied per unit difference in greening standards per unit produced. We assume that the Government set environmental standard is given by 'θ_0'. Under such a taxation scheme, the profit function of the firm can be written as :

$$\Pi_{SF} = (p - c)q - I\theta^2 - K(\theta_0 - \theta)q \tag{2}$$

$$s.t.$$

$$\theta \leq \theta_0$$

$$\theta, p \geq 0$$

The index SF denotes a single firm in our case. The above model captures two phenomena. Firstly, the firm incurs a cost of greening given by $I\theta^2$ which is increasing in the level of greening θ and convex. I is an investment parameter here. Convex costs reflect diminishing returns from R&D expenditures. Convexity of costs are often attributed to diseconomies of scale where investment efforts are involved. To explain further, we estimate that the 'low hanging fruit' during greening would be plucked much easily by the firms while subsequent improvements may become progressively more difficult. Secondly, the Government levies a linear penalty for every unit of greening that the firm falls short of multiplied by the total production quantity of the firm. The firm's variable cost of production, denoted by c here is assumed to be constant. It is further assumed that the greening improvement modelled here does not affect the firm's marginal costs. The greening improvement that we model here refers to a product attribute such that once the improvement comes into being, it makes the older product obsolete. Bhaskaran and Krishnan (2009) and Abbott (1953) refer to such improvements as "innovation quality dimensions" which when introduced cost no more to produce thus turning the older quality obsolete. It is to be noted that our model specifically addresses the problem where the firm falls short of the Government mandated greening standards, a significantly widespread problem as illustrated through the case of CAFE fines.

The firm has two decisions to make. How much 'price' to charge and the 'level of greening' improvements to achieve. The firm's objective is to maximise (2) with respect to these key decision variables under Government penalty and investments in greening. The decision making by the firm follows the following sequence:

(i) The firm selects the 'level of greening' and decides on the price of its green product

(ii) Demand is realised based on the price and greening level set by the firm.

The above optimisation problem is solved with respect to the decision variables. However we first propose here a few results with respect to the nature of the optimisation problem and then proceed to derive the equilibrium values.

Lemma 1. *The deterministic model given by equation 2 is a convex program.*

Proof. The objective function function is concave for $\frac{\partial^2 \Pi_{SF}}{\partial p^2} = -2b < 0$; $\frac{\partial^2 \Pi_{SF}}{\partial \theta^2} = -2(I - K\alpha) < 0$ and $|H| = 4Ib - (\alpha + Kb)^2 > 0$. The constraint is linear in the decision variable of the model. Hence, the deterministic constrained profit maximisation problem for the single firm is a convex program.

Since, the deterministic model is a convex program, Karush-Kuhn-Tucker(KKT) optimality conditions are necessary and sufficient to obtain optimal solution for the problem. Using the KKT optimality conditions for the constrained optimization problem, the optimal solution for the firm's problem is given as follows

$$a - 2bp^* + bc + \alpha\theta^* + Kb(\theta_0 - \theta^*) = 0 \tag{3}$$

$$(p^* - c + K)(a - bp^* + \alpha\theta^*) + \alpha(p^* - c - K(\theta_0 - \theta^*)) - 2I\theta^* - \lambda^* = 0 \tag{4}$$

$$\theta_0 - \theta^* \geq 0 \tag{5}$$

$$\lambda^*(\theta_0 - \theta^*) = 0 \tag{6}$$

where p^* , θ^* and λ^* denote the optimal levels of p, θ and λ respectively. Solving the above expressions we get

$$\lambda^* = \begin{cases} 0 \quad if \quad (\theta_0 - \theta^*) > 0 \\ \dfrac{(a - bc + \alpha\theta_0)}{2}(\dfrac{\alpha}{b} + K) - 2I\theta_0 \quad otherwise \end{cases} \tag{7}$$

$$p^* = \begin{cases} \dfrac{a + b(c + K\theta_0) + \theta^*(\alpha - Kb)}{2b} \quad if \quad (\theta_0 - \theta^*) > 0 \\ \dfrac{a + bc + \alpha\theta_0}{2b} \quad otherwise \end{cases} \tag{8}$$

$$\theta^* = \begin{cases} \dfrac{K(a - bp^*) + \alpha(p^* - (c + K\theta_0))}{2(I - \alpha K)} \quad if \quad (\theta_0 - \theta^*) > 0 \\ \theta_0 \quad otherwise \end{cases} \tag{9}$$

Solving for p and θ values simultaneously, we get the following equilibrium values :

Proposition 1. *The optimal level of greening achieved by the firm is*

$$\theta_{SF} = \begin{cases} \dfrac{(\alpha + Kb)[a - b(c + K\theta_0)]}{4Ib - (\alpha + Kb)^2} \quad if \quad I > A_1 \\ \theta_0 \quad if \quad I \leq A_1 \end{cases} \tag{10}$$

where

$$A_1 = \frac{(\alpha + Kb)(a - bc + \alpha\theta_0)}{4b\theta_0} \tag{11}$$

For non-negativity of θ_{SF}, we assume $a > b(c + K\theta_0)$. Thus the two assumptions in this model are:

Assumption: $a > b(c + K\theta_0)$

Assumption : $4Ib - (\alpha + bK)^2 > 0$

It can be inferred from the above proposition that when the cost of greening is quite high, the firm falls short of the Government mandated standards. However, when the cost of 'greening' is less than the bound given by A_1, the firm would attain the Government decided 'level of greening'. Note that the bound given by A_1 is increasing in the penalty levied (K) and decreasing in Government decided environmental standard θ_0. (The partial derivative of A_1 w.r.t K is positive and the partial derivative of A_1 w.r.t θ_0 is given by $\frac{-(a-bc)(\alpha+Kb)}{4b\theta_0^2}$ which is negative).

Lemma 2. *θ_{SF} is decreasing in the cost of greening(I) and increasing in consumer sensitivity towards greening(α).*

Proof: The derivative of θ_{SF} w.r.t I gives $\frac{\partial\theta_{SF}}{\partial I} = \frac{-4b(\alpha+Kb)(a-b(c+K\theta_0))}{(4Ib-(\alpha-Kb)^2)^2} < 0$. Also, the derivative of θ_{SF} w.r.t α gives $\frac{\partial\theta_{SF}}{\partial\alpha} = \frac{(a-b(c+K\theta_0))((\alpha+Kb)^2+4Ib)}{(4Ib-(\alpha-Kb)^2)^2} > 0$.

Thus, θ_{SF} decreases with cost of greening(I). This is a consequence of the fact that when the cost rises, the firm cannot afford higher levels of greening. Refer figure 1. Additionally, θ_{SF} increases with consumer sensitivity towards greening(α). Higher consumer sensitivity to greening provides the required impetus to achieve higher levels of greening as through marginal increase in greening levels, the demand increases manifolds. The plot of level of greening to the ratio α/β shows that as the ratio increases(by increasing α) the level of greening achieved by the firm rises. Refer figure 5. The argument reveals why Governments should make consumers environmentally conscious while simultaneously taxing product manufacturers.

Lemma 3. *Under the given assumptions, the corresponding values of price, quantity and profit of the firm are*

$$p_{SF} = \begin{cases} \dfrac{2I(a + b(c + K\theta_0)) - (\alpha + Kb)(aK + \alpha(c + K\theta_0))}{4Ib - (\alpha + Kb)^2} & if \quad I > A_1 \\ \dfrac{a + bc + \alpha\theta_0}{2b} \quad if \quad I \leq A_1 \end{cases} \tag{12}$$

$$q_{SF} = \begin{cases} \dfrac{2Ib(a - b(c + K\theta_0))}{4Ib - (\alpha + Kb)^2} & if \quad I > A_1 \\ \dfrac{a - bc + \alpha\theta_0}{2} \quad if \quad I \leq A_1 \end{cases} \tag{13}$$

$$\Pi_{SF} = \begin{cases} \dfrac{[(a - b(c + K\theta_0)]^2 I}{4Ib - (\alpha + Kb)^2} & if \quad I > A_1 \\ \dfrac{(a - bc + \alpha\theta_0)^2 - 4Ib\theta_0^2}{4b} \quad if \quad I \leq A_1 \end{cases} \tag{14}$$

The above results are derived by substituting the optimal value of θ_{SF} into the expressions for prices, quantity and profits.

Lemma 4. *The price of the green product is increasing in the cost of greening(I) while the total quantity and profit of the firm are decreasing in the cost of greening(I).*

Proof: The partial derivatives of the variables with respect to I gives $\dfrac{\partial p_{SF}}{\partial I} = \dfrac{2(a - b(c + K\theta_0))(Kb + \alpha)(Kb - \alpha)}{(4Ib - (\alpha + Kb)^2)^2} > 0$, $\dfrac{\partial q_{SF}}{\partial I} = \dfrac{-2(a - b(c + K\theta_0))(Kb + \alpha)^2 b}{(4Ib - (\alpha + Kb)^2)^2} < 0$, $\dfrac{\partial \Pi_{SF}}{\partial I} = \dfrac{-(a - b(c + K\theta_0))^2(Kb + \alpha)^2}{(4Ib - (\alpha + Kb)^2)^2} < 0$.

The impact of increased cost of greening on the various firm level outcomes are expressed in the above result. Our results corroborate the concerns of managers over greening costs. Our results analytically support managerial decision making based on the total costs incurred and other parametric values. Refer figures 2, 3 and 4.

The structural results are followed by numerical analysis in the following section.

3.1 Numerical Analysis

To study the impact of Government levied penalty(K) and consumer sensitivity towards greening(α), we conduct various sensitivity analyses in this section.

Impact of consumer sensitivity towards greening(α) : We conduct numerical analysis where the parametric values are the following based on the model assumptions, $a = 4000, b = 50, c = 6, I = 950, K = 5, \theta_0 = 8, \alpha$ is varied from 40-94. It is observed that price is decreasing in the consumer sensitivity towards greening (α). Refer Fig 6. With increased sensitivity of consumers towards greening, the quantity demanded rises and the firm subsequently quotes a lower price for its product. Additionally, the quantity demanded for the green product increases with the increase in consumer sensitivity towards green products. Refer Fig 7. The profit of the firm also increases with increase in (α), significantly influenced by the increase in demand for the green product. Refer Fig 8.

Impact of penalty(K) : The Government's linear penalization of firms for falling short of the mandated environmental greening standards has interesting implications. To study the impact of Government penalty(K) we assume the following parametric values: $a = 4000, \alpha = 40, c = 6, b = 50, I = 960, \theta_0 = 8, K = 3 - 6.8$. It can be inferred that the producer's profit is decreasing in penalty as with increasing penalization the producer earns less profits. Refer Fig 12. Interestingly, *high* government penalty(K) leads to lower price and higher quantity. Refer Figures 10 and 11. This is attributed to the impact of government penalty(K), which leads to higher greening levels. Refer Fig 9. Increase in greening levels lead to higher demand and subsequently lower prices.

3.2 Surplus

Government taxation has received huge attention in economics literature in the past. In our study we evaluate the consumer surplus under government taxation and product greening. In what follows we derive the consumer surplus for a monopoly market and evaluate results on the social surplus generated out of government penalization and product greening. The consumer surplus denoted by CS is given as:

$$CS = \int_0^{q_{SF}} P(x, \theta_{SF})\, dx \ - p_{SF} q_{SF} \tag{15}$$

where $P(x, \theta_{SF}) = \dfrac{(a - x + \alpha\theta_{SF})}{b}$ and x denotes quantity. Substituting the values of θ_{SF}, q_{SF} and p_{SF} from the single firm's decisions, we obtain consumer surplus as

$$\begin{aligned} CS &= \frac{2Ib(a - b(c + K\theta_0))[a(3I - K(\alpha + Kb)) + (c + K\theta_0)(b(I - \alpha K) - \alpha^2)]}{(4Ib - (\alpha + Kb)^2)^2} \\ &\quad - [2Ib(a - b(c + K\theta_0))][\frac{2I(a + b(c + K\theta_0)) - (aK + \alpha(c + K\theta_0))(\alpha + Kb)]}{(4Ib - (\alpha + Kb)^2)^2} \\ &= \frac{2I^2 b[a - b(c + K\theta_0)]^2}{(4Ib - (\alpha + Kb)^2)^2} \end{aligned} \tag{16}$$

Lemma 5. *The consumer surplus is decreasing in the cost of greening(I) and increasing in the consumer sensitivity to greening(α).*

Proof. The first order derivatives of CS w.r.t I and α gives the following $\dfrac{\partial CS}{\partial I} = \dfrac{-4Ib(a - b(c + K\theta_0))^2(\alpha + Kb)^2}{(4Ib - (\alpha + Kb)^2)^3} < 0$ and $\dfrac{\partial CS}{\partial \alpha} = \dfrac{8I^2 b(a - b(c + K\theta_0))^2(\alpha + Kb)}{(4Ib - (\alpha + Kb)^2)^3} > 0$. Thus, consumer surplus is decreasing in the cost of greening(I) and increasing in the consumer sensitivity to green product(α). In summary, higher costs of greening decrease the producer and consumer surplus and stand as a major challenge for companies and societies today. However, a more green conscious consumer base can mitigate the cost impact to a large extent. This also explains why NHTSA makes the consumers aware of the vehicle manufacturers adhering to the CAFE legislations.

The above analysis of the manufacturer's profit and consumer surplus necessitates the importance of evaluating the overall social surplus. The social surplus (SS) is derived by summing the consumer and producer surplus, government earnings and environmental pollution, given by

$$SS = CS + \Pi_{SF} + GovernmentTaxation - EnvironmentalDamage$$

Government earnings is given by $K(\theta_0 - \theta_{SF})q_{SF}$ and Environmental Damage is calculated as $E(\theta_0 - \theta_{SF})q_{SF}$ where E is the damage to environment per unit of greening level difference per unit produced. We assume here that the environmental damage is linear in nature and a function of the difference in greening levels and total production. E is exogenously determined. Thus, the social surplus is calculated as

$$\begin{aligned} SS &= CS + \Pi_{SF} + K(\theta_0 - \theta)q_{SF} - E(\theta_0 - \theta)q_{SF} \\ &= \frac{I(a - b(c + K\theta_0))}{(4Ib - (\alpha + Kb)^2)^2}[2Eb\{(\alpha + Kb)(a - bc + \alpha\theta_0) - 4Ib\theta_0\} \\ &\quad + (a - bc)\{6Ib - Kb(4\alpha + 3Kb)\} + Kb^2\theta_0(2I + K^2b) - \alpha^2\{a - b(c + K\theta_0)\}] \end{aligned} \tag{17}$$

In order to answer whether the greening level improvements and quantity produced by the monopolist is equal to the socially desirable values, we derive the socially optimal values of greening and quantity.

$$\begin{aligned} \max_{q,\theta} SS &= \int_0^q P(x,\theta)\,dx \ - C(q,\theta) \\ &= \int_0^q P(x,\theta)\,dx \ - cq - I\theta^2 - E(\theta_0 - \theta)q \\ &= \frac{aq + \alpha\theta q - q^2/2}{b} - cq - I\theta^2 - E(\theta_0 - \theta)q \end{aligned} \tag{18}$$

The first order conditions are

$$\frac{\partial SS}{\partial q} = (a + \alpha\theta - q)/b - c - E(\theta_0 - \theta)$$

$$\frac{\partial SS}{\partial \theta} = \frac{\alpha q}{b} - 2I\theta + Eq$$

The second order conditions are

$$\frac{\partial^2 SS}{\partial q^2} = -\frac{1}{b} \tag{19}$$

$$\frac{\partial^2 SS}{\partial \theta^2} = -2I \tag{20}$$

$$\frac{\partial^2 SS}{\partial q \partial \theta} = \frac{\alpha}{b} + E \tag{21}$$

The Hessian is positive for $I > \frac{b}{2}(\frac{\alpha}{b} + E)^2$. Thus, equating the first order conditions to zero and solving for the socially optimal θ, quantity and price gives

$$\theta_{SS} = \frac{(\alpha + bE)(a - b(c + E\theta_0))}{2Ib - (\alpha + bE)^2} \tag{22}$$

$$q_{SS} = \frac{(a - b(c + E\theta_0))2Ib}{2Ib - (\alpha + bE)^2} \tag{23}$$

$$p_{SS} = \frac{aE(\alpha + bE) + (c + E\theta_0)(2Ib - \alpha(\alpha + bE))}{2Ib - (\alpha + bE)^2} \tag{24}$$

Proposition 2. *For K=E and* $I > \frac{(\alpha + Kb)^2}{2b}$, $\theta_{SS} > \theta_{SF}$ *and* $q_{SS} > q_{SF}$

Proof : When government penalization (K) is equal to the environmental damage (E), it can be inferred that $2Ib - (\alpha + Kb)^2 < 4Ib - (\alpha + Kb)^2$. Hence, θ_{SS} and q_{SS} are greater than the optimal values derived for the monopolist. The results imply that the monopolist falls short of the socially desirable outcomes.

4 The case of a Duopoly

In this section, we extend our to a price and greening competition in a duopoly setting. There are two firms labelled $1, 2$. Each firm has one product for which it decides the 'greening levels' and prices. The two firms compete on prices and greening levels in the market. The demand faced by each firm is given by

$$q_i = a - bp_i + \gamma p_j + \alpha\theta_i - \beta\theta_j \tag{25}$$
$$\text{where} \qquad b > \gamma > 0, \alpha > \beta > 0, i \neq j, i, j = 1, 2$$

The above assumptions are necessary so that the effect of Firm i's own price on its quantity demanded is greater than that of its competitor and similarly the effect of greening level on quantity demanded for Firm i is greater than that of its competitor (Banker et.al,1998). Further a denotes the total market demand faced by each firm which is assumed to be equal for both firms considering fairly large players in the market. Also, we intend to study competition between firms in pricing and greening levels under government penalty and not competition in market share which Banker et.al (1998) study. Tsay and Agarwal (2000) on the other hand study price and service competition but in a different set up of one manufacturer serving a market through two competing retailers. The decision making by the firms follow the following sequence:
(i) The firms simultaneously select the 'levels of greening'
(ii) They observe each others greening level, then decide the price of the green product
(iii) Demand is realised based on the prices and greening levels set by the firms.
In our model, p_i and p_j denote the price of each firm's product and θ_i and θ_j denote the 'level of greening' of the product. Further, 'b', 'γ' and 'α', 'β' denote the demand sensitivity to price and 'greening level' respectively. We model Government penalization similar to the one modelled in the case of a single firm. Let 'K' denote the penalty levied per unit gap in greening standards per unit produced. It is to be noted here, that the amount of penalty levied by the Government is incurred by each firm to the tune of not meeting the Government standard given by 'θ_0' . Under such a taxation scheme, the objective of each firm is the following

$$\max_{p_i,\theta_i} \Pi_i = (p_i - c)q_i - I_i\theta_i^2 - K(\theta_0 - \theta_i)q_i \tag{26}$$
$$s.t.$$

$$\theta_i \leq \theta_0$$

$$\theta_i, p_i \geq 0$$

$$i \neq j, i, j = 1, 2$$

In the above equation, the variable cost of production 'c' for each firm is assumed to be same. Since we are interested in analysing the impact of cost of greening on each firm's strategic decision, we vary the cost of greening 'I' for each firm. Also, similar to the case of a single firm, each firm in the duopoly incurs an increasing and convex cost of greening which is represented by a quadratic form here. Further, each firm incurs a penalty when it falls short of the Government determined standards. Using the KKT conditions as outlined in the case of a single firm, we find

The optimal level of greening(θ^{NC}) is derived as:

$$\theta_i^{NC} = \frac{BX[(I_jW^2 - Z) - BY]}{(I_iW^2 - Z)(I_jW^2 - Z) - B^2Y^2} \tag{27}$$

$$= b[(S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW))(b(S_1 + KW)(2S_2 + T) + bS_2T - 2I_jW^2)]$$

$$/[b^2T^2(S_1 + S_2 + KW)^2 + 4bS_2KW^3(I_i + I_j) - 4(I_jW^2 - bS_1S_2)(I_iW^2 - bS_1S_2)$$

$$- 4b^2S_2^2KW(2S_1 - KW)]$$

when,

$$\textbf{Condition}: I_i > [bG_2(b(S_1 + KW)(2S_2 + T) + bS_2T) - \theta_0(b^2T^2(S_1 + S_2 + KW)^2$$

$$-4b^2S_2^2KW(2S_1 + KW)) - 2bG_2I_jW^2 - \theta_0 4bS_2KW^3I_j - 4\theta_0 bS_1S_2(I_jW^2 - bS_1S_2)]$$

$$/[\theta_0(4bS_2KW^3 - 4W^2(I_jW^2 - bS_1S_2))]$$

This can be simplified to :

$$\theta_i^{NC} = \frac{bG_2G_3}{G_1} \tag{28}$$

From the optimal greening level, the price, quantity and profit function of Firm i, i $\neq$ j, i,j= $1,2$ under competition is derived as:

$$p_i^{NC} = \frac{A_1 + \dfrac{G_2G_3bS_1}{G_1} - \dfrac{bTG_2G_4}{G_1}}{W} \tag{29}$$

$$q_i^{NC} = b(\frac{A_2 + \dfrac{G_2G_3bS_2}{G_1} - \dfrac{bTG_2G_4}{G_1})}{W} \tag{30}$$

$$\Pi_i^{NC} = \frac{b[A_1 - Wc + G_2G_3bS_1/G_1 - bG_2(G_4T/G_1)][A_2 + G_2G_3bS_2/G_1 - bG_2(G_4T/G_1)]}{W^2} - \frac{I_iG_2^2G_3^2b^2}{G_1^2} - Kb[(\theta_0 - \frac{G_2G_3b}{G_1})]\frac{[A_2 + \dfrac{G_2G_3bS_2}{G_1} - bG_2(G_4T/G_1)]}{W} \tag{31}$$

where

$G_1 = b^2T^2(S_1 + S_2 + KW)^2 + 4bS_2KW^3(I_i + I_j) - 4(I_jW^2 - bS_1S_2)(I_iW^2 - bS_1S_2) - 4b^2S_2^2KW(2S_1 + KW)$

$G_2 = A_2(KW + S_1) + (S_2(A_1 - W(c + K\theta_0)))$

$G_3 = b(S_1 + KW)(2S_2 + T) + bS_2T - 2I_jW^2$

$G_4 = b(S_1 + KW)(2S_2 + T) + bS_2T - 2I_iW^2$

Lemma 6. *Given the cost of greening(I_j) of its competitor, the greening level of Firm i is decreasing in its own cost I_i.*

Proof: The first order derivative w.r.t I_i is
$\dfrac{\partial\theta_i^{NC}}{\partial I_i} = -\dfrac{BXW^2(I_jW^2 - Z)((I_jW^2 - Z) - BY)}{((I_iW^2 - Z)(I_jW^2 - Z) - B^2Y^2)^2} < 0.$
The result corroborates our finding as in the single firm's case, that the equilibrium greening levels of firms are decreasing in their own costs of greening.

When

$$\textbf{Condition}: I_i \leq [bG_2(b(S_1+KW)(2S_2+T)+bS_2T)-\theta_0(b^2T^2(S_1+S_2+KW)^2$$
$$-4b^2S_2^2KW(2S_1+KW))-2bG_2I_jW^2-\theta_0 4bS_2KW^3I_j-4\theta_0 bS_1S_2(I_jW^2-bS_1S_2)]$$
$$/[\theta_0(4bS_2KW^3-4W^2(I_jW^2-bS_1S_2))]$$

The equilibrium values of greening levels, price, quantity and profit for Firm i, $i \neq j, i, j = 1, 2$ are given as

$$\theta_i^{NC} = \theta_0$$
$$p_i^{NC} = \frac{A_1+\theta_0(S_1-T)}{W}$$
$$q_i^{NC} = b(\frac{A_2+\theta_0(S_2-T))}{W}$$
$$\Pi_i^{NC} = b[\frac{A_1+\theta_0(S_1-T)}{W}-c][\frac{A_2+\theta_0(S_2-T)}{W}]-I_i\theta_0^2$$

In the above result, the equilibrium price, quantity and profit of the firm is derived by substituting $\theta = \theta_0$ in the expressions for price, quantity and profit.

Proposition 3. *The relative greening level difference between the two firms is increasing in the cost of greening difference.*

Proof: We derive

$$\frac{|\theta_i-\theta_j|}{\theta_i+\theta_j} = \frac{|\Delta\theta|}{\theta_T}$$
$$= \frac{W^2\,|(I_j-I_i)|}{W^2(I_i+I_j)-2(Z+BY)}$$
$$= \frac{W^2\,|(I_j-I_i)|}{2[W^2\frac{(I_i+I_j)}{2}-(Z+BY)]}$$

Keeping the cost averages constant, it is observed that the relative greening difference is increasing in the cost of greening difference. Thus, the Firm with a lower cost of greening would be able to provide a higher greening level in comparison to its competitor. This result confirms our understanding of the CAFE legislations where Toyota had not paid any fine over a period of twenty years while its competitors who had significantly higher costs of greening had been fined and provided lower levels of greening(fuel economy) in the vehicles they produced.

4.1 When Firms have equal costs of Greening

In this section, we deal with the case when cost of greening for both the firms are equal. The results derived in this section are important that they help analyse the impact of cooperation on cost of greening and greening level decisions in a later section.
When $I_i = I_j$, we get :

$$\theta^N = \frac{BX}{IW^2 - Z + BY}$$

Substituting the values of B,X,Y and Z in the expression for θ^N, we get

$$\theta^N = \frac{b[S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW)]}{2IW^2 - 2bS_2(S_1 + KW) + bT(S_1 + S_2 + KW)}$$

When,

$$\theta^N < \theta_0$$
$$\Rightarrow \frac{BX}{IW^2 - Z + BY} < \theta_0$$
$$\Rightarrow I > (Z + \frac{B(X - \theta_0 Y)}{\theta_0})/W^2$$
$$\Rightarrow I > \frac{b}{2\theta_0 W^2}[S_2(A_1 - W(c - K\theta_0)) + (S_1 + KW)(A_2 - T\theta_0) + S_2\theta_0(2S_1 - T)]$$

Thus, for **Condition**: I > $\frac{b}{2\theta_0 W^2}[S_2(A_1 - W(c - K\theta_0)) + (S_1 + KW)(A_2 - T\theta_0) + S_2\theta_0(2S_1 - T)]$

The equilibrium values of prices, quantities and profits are derived as:

$$p^N = [\frac{A_1 + (S_1 - T)\frac{b[S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW)]}{2IW^2 - 2bS_2(S_1 + KW) + bT(S_1 + S_2 + KW)}}{W}]$$

$$q^N = b[\frac{A_2 + (S_2 - T)\frac{b[S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW)]}{2IW^2 - 2bS_2(S_1 + KW) + bT(S_1 + S_2 + KW)}}{W}]$$

$$\begin{aligned}\Pi^N &= [(2IW^2(A_1 - c) + 2bS_2WK(Wc - A_1K) - A_1bS_1S_2 \\ &+ (S_1 + KW)(A_1bT + bA_2(S_1 - T)) + WbS_1c(S_2 - T) - W^2bcK(2S_2 - T) \\ &- bS_2KW\theta_0(S_1 - T))(A_2 + (1/2)(\frac{bM(S_2 - T)}{N}))b]/[W^2(2IW^2 - 2bS_2(S_1 + KW) \\ &+ bT(S_1 + S_2 + KW))] \\ &- (1/4)(\frac{Ib^2N^2}{M^2}) - \frac{K(\theta_0 - (1/2)(\frac{bN}{M}))b(A_2 + (1/2)(\frac{bN(S_2 - T)}{M}))}{W} \qquad where,\end{aligned}$$

$$\begin{aligned}&M = IW^2 - bS_2(S_1 + KW) + \frac{1}{2}bT(S_1 + S_2 + KW) \\ &and \\ &N = S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW)\end{aligned}$$

However, for **Condition**: I $\leq \frac{b}{2\theta_0 W^2}[S_2(A_1 - W(c - K\theta_0)) + (S_1 + KW)(A_2 - T\theta_0) + S_2\theta_0(2S_1 - T)]$

$$\begin{aligned}\theta^N &= \theta_0 \\ p^N &= \frac{A_1 + \theta_0(S_1 - T)}{W} \\ q^N &= b(\frac{A_2 + \theta_0(S_2 - T))}{W} \\ \Pi^N &= b[\frac{A_1 + \theta_0(S_1 - T)}{W} - c][\frac{A_2 + \theta_0(S_2 - T)}{W}] - I\theta_0^2\end{aligned}$$

4.2 Contract Analysis and Greening

In the following sections we consider few contracts which impact the decision making of firms under greening and government legislations. Our scope of study limits itself to two competing firms facing government legislations. In that perspective we deal with contracts which help share the burden of development of the greening innovation between both the firms. We study a fixed fee contract and revenue sharing contract in this section. In another section we study a cost sharing contract under cooperation. As outlined previously, there are several examples of firms participating in the joint development of the green product or sharing the cost of development of the technology or sharing revenues generated through the development of the green technology with the partner firm. Tsay,Nahmias and Aggarwal(1999) and Cachon(2003) provide a detailed review of various supply chain contracts. We refer to these contracts in the case of a duopoly. In the next section we study a fixed fee contract.

4.2.1 Greening through Fixed Fee Contract

Decision making under the fixed fee contract follows the following sequence :

1: Firm j offers a fixed fee F for utilizing the green technology that Firm i solely develops.

2: Firm i decides to accept or reject the contract. If Firm i accepts the offer, then based on the fixed fee, Firm i decides on the level of greening to achieve. It also incurs the cost of greening.

3: Finally, both the firms compete on prices and demand is realised based on the prices and greening level.

Given the three stage game, the objective of Firm i is:

$$\max_{p_i,\theta} \Pi_i = (p_i - c - K(\theta_0 - \theta))q_i - I\theta^2 + F$$

$$s.t.$$

$$\theta \leq \theta_0$$

and the objective of Firm j is :

$$\max_{p_j} \Pi_j = (p_j - c - K(\theta_0 - \theta))q_j - F$$

The demand realised is :

$q_i = a - bp_i + \gamma p_j + \theta(\alpha - \beta)$ where $i \neq j$ and $i, j = 1, 2$

Solving for the optimum level of greening (θ^F) gives:

$$\theta^F = \frac{N_3 b N_2}{N_1}$$

for

$$\textbf{Condition} : I > \frac{bN_3[N_2 + \theta_0 N_3]}{\theta_0(4b(b-\gamma) + \gamma^2)}$$

Substituting the optimum greening level(θ^F) into the profit function of Firm i gives:

$$\Pi_i^F = [N_5 - c - N_4][a - (b-\gamma)N_5 + \frac{N_3 b N_2(\alpha - \beta)}{N_1}] - \frac{I_i N_3^2 b^2 N_2^2}{N_1^2} + F$$

Substituting the optimum greening level(θ^F) into the profit function of Firm j gives:

$$\Pi_j^F = [N_5 - c - N_4][a - (b-\gamma)N_5 + \frac{N_3 b N_2(\alpha - \beta)}{N_1}] - F$$

where

$N_1 = I(4b(b-\gamma) + \gamma^2) - b((\alpha - \beta) + K(b-\gamma))^2$

$N_2 = a - (b-\gamma)(c + K\theta_0)$

$N_3 = \alpha - \beta + K(b-\gamma)$

$N_4 = K(\theta_0 - \frac{N_3 b N_2}{N_1})$

$N_5 = \frac{a + \frac{N_3 b N_2(\alpha - \beta)}{N_1} + b(c + N_4)}{2b - \gamma}$

Both Firm i and j would participate in the fixed fee contract when their profits through the contract are greater than the profits in the non-contractual case . Thus, Firms would participate when

$$\Pi_i^F \geq \Pi_i^{NC} \qquad and$$
$$\Pi_j^F \geq \Pi_j^{NC}$$

Solving for Firm i's case, we derive

$$F \geq \frac{b[A_1 - Wc + G_2G_3bS_1/G_1 - bG_2(G_4T/G_1)][A_2 + G_2G_3bS_2/G_1 - bG_2(G_4T/G_1)]}{W^2}$$

$$- \frac{I_iG_2^2G_3^2b^2}{G_1^2} - Kb[(\theta_0 - \frac{G_2G_3b}{G_1})]\frac{[A_2 + \frac{G_2G_3bS_2}{G_1} - bG_2(G_4T/G_1)]}{W}$$
$$- \left[[N_5 - c - N_4][a - (b-\gamma)N_5 + \frac{N_3bN_2(\alpha-\beta)}{N_1}] - \frac{I_iN_3^2b^2N_2^2}{N_1^2}\right]$$

Proposition 4. *The greening level achieved(θ^N), when firms have equal costs of greening is higher than the greening level achieved(θ^F) under fixed fee contract.*

Proof:

$$\theta^N - \theta^F = \frac{(2b-\gamma)^2(2b\beta - \gamma(\alpha - Kb))Ib(a-(b-\gamma)(c+K\theta_0))}{N_1(2IW^2 - 2bS_2(S_1+KW) + bT(S_1+S_2+KW))} > 0$$

Under equal costs of greening, price and greening competition under government legislations lead to a higher equilibrium level of greening than a single firm developing the green technology/product under a fixed fee contract with its competitor firm in the market. The result interestingly points out that contractual terms may result in increased surplus for the firms but the equilibrium levels of greening remain higher under competition between the firms.

4.2.2 Greening through Revenue Sharing

We discuss another mechanism of greening where one of the firms offers a revenue sharing contract in return for leasing/usage of green technology/product that the other firm develops. Revenue sharing contracts have been dealt with in detail by Cachon and Lariviere(2005). However the authors discuss the contract in the context of a supply chain whereas we apply the revenue sharing contract in the case of a duopoly with price and greening competition. Decision making under the revenue sharing contract follows the following sequence :

1: Firm j offers a portion ω of its revenues to Firm i for utilizing the green technology/product that Firm i solely develops.

2: Firm i decides to accept or reject the revenue sharing contract. If Firm i accepts the offer, then based on the portion of revenues shared by Firm j, Firm i decides on the level of greening to achieve. It also incurs the cost of greening.

3: Both the firms compete on prices and demand is realised based on the prices and greening level.

Given the three stage game, the objective of Firm i is:

$$\max_{p_i,\theta} \Pi_i = (p_i - c - K(\theta_0 - \theta))q_i - I\theta^2 + (1-\omega)p_j q_j$$

$$s.t.$$

$$\theta \leq \theta_0$$

and the objective of Firm j is :

$$\max_{p_j} \Pi_j = \omega(p_j q_j) - (c + K(\theta_0 - \theta))q_j$$

The demand realised is :

$q_i = a - bp_i + \gamma p_j + \theta(\alpha - \beta)$ where $i \neq j$ and $i, j = 1, 2$.

The optimal greening levels and profit functions of each firm is derived as

$$\theta^{RS} = (1/2)(\frac{S_{12}}{S_{11}})$$

Substituting the above value of (θ^{RS}) into the profit function of each firm gives

$$\Pi_i^{RS} = (\frac{S_{13}}{\omega S_1} - c)S_{10} - 1/4(\frac{I_i S_{12}^2}{S_{11}^2}) - K(\theta_0 - 1/2(\frac{S_{12}}{S_{11}}))S_{10}$$
$$+\frac{(1-\omega)S_{14}(a - \frac{bS_{14}}{\omega S_1} + \frac{\gamma S_{13}}{\omega S_1} + 1/2\frac{S_{12}(\alpha-\beta)}{S_{11}})}{\omega S_1}$$

$$\Pi_j^{RS} = \frac{(S_{15}S_{16})}{S_1} - cS_{16} - K(\theta_0 - 1/2(\frac{S_{12}}{S_{11}}))S_{16}$$

where $S_1 = 4b^2 - \gamma^2(2-\omega)$
$S_{10} = (a - \frac{bS_{13}}{\omega S_1} + \frac{\gamma S_{14}}{\omega S_1} + 1/2\frac{S_{12}(\alpha-\beta)}{S_{11}})$
$S_{11} = \%1$
$S_{12} = \%2$
$S_{13} = \%4 = (\omega a + bc)\gamma(2-\omega) + 2\omega b(a+bc) + \gamma K b\theta_0(2-\omega) - \frac{\gamma K b S_{12}}{S_{11}} + 2\omega K b^2\theta_0 + \frac{\omega S_{12}(\alpha-\beta)(\beta+\gamma)}{S_{11}} - \frac{\omega K b^2 S_{12}}{S_{11}} - (1/2)\frac{\omega^2\gamma S_{12}(\alpha-\beta)}{S_{11}} + (1/2)\frac{\omega\gamma K b S_{12}}{S_{11}}$
$S_{14} = \%5 = \omega 2ab + 2Kb^2\theta_0 + \frac{\omega b S_{12}(\alpha-\beta)}{S_{11}} + 2b^2c + \omega\gamma(a + b(c + K\theta_0)) + (1/2)\frac{\omega\gamma S_{12}(\alpha-\beta)}{S_{11}} - (1/2)\frac{\omega\gamma K b S_{12}}{S_{11}} - \frac{Kb^2 S_{12}}{S_{11}}$
$S_{15} = \%4_j = \omega 2ab + 2b^2(c + K\theta_0) + \frac{\omega b S_{12}(\alpha-\beta)}{S_{11}} - (1/2)\frac{\omega\gamma K b S_{12}}{S_{11}} - \frac{Kb^2 S_{12}}{S_{11}} + (1/2)\frac{\omega\gamma S_{12}(\alpha-\beta)}{S_{11}} + \omega\gamma(a + b(c + K\theta_0))$
$S_{16} = \%5_j = a - \frac{bS_{15}}{\omega S_1} + \frac{\gamma}{\omega S_1}[2\gamma bc + 2b\omega(a+bc) - (1/2)\frac{\omega^2\gamma S_{12}\alpha}{S_{11}} + \gamma K b\theta_0(2-\omega) + 2\omega(\gamma a + Kb^2\theta_0) - \frac{\gamma K b S_{12}}{S_{11}} - \omega\gamma(\omega a + bc) + \frac{b\omega S_{12}(\alpha-\beta)}{S_{11}} - \frac{\omega K b^2 S_{12}}{S_{11}} + \frac{\omega\gamma S_{12}(\alpha-\beta)}{S_{11}} + (1/2)(\frac{\omega\gamma S_{12}}{S_{11}})(\omega\beta + Kb)] + (1/2)\frac{S_{12}(\alpha-\beta)}{S_{11}}$

4.2.3 Greening through cost sharing contract

We address the question of what happens to the choice of greening level when firms decide to co-operate. Subsequently we find the impact of greening levels on the price of the product. One of the reasons cited in literature for co-operation is the reduced cost of development (Banker,Khosla and Sinha, 1998). We model the reduced cost of development in the following way. The reduced cost of development is given by I_c where the index c stands for co-operation.

$$I_c = 2I\delta \qquad where \qquad 0 \leq \delta \leq 1$$

The above model of cost under co-operation indicates that the cost of greening under co-operation is certain fraction of the total cost of greening when firms work individually. The decision making between the two firms follows the following sequence in our model:

1. The two firms jointly select their greening levels.
2. The firms then compete on their prices.
3. Demand is realised based on the choice of prices and greening levels.

We assume that the total cost of greening under co-operation given by I_c is shared between the two firms such that firm i incurs ϕ portion of the cost while firm j incurs $(1-\phi)$. The parameter ϕ is assumed to be decided exogenously. In another model we discuss the implications of ϕ being decided endogenously by one of the firms. For our model, given greening levels, we find that the equilibrium prices of each firm are . We assume the two firms cooperate in choosing the greening levels and hence $\theta_i = \theta_j = \theta^C$. On substituting the same, the two firms jointly maximise their profits given by:

$$\begin{aligned}\Pi^C(\theta) &= \Pi_1^C(\theta) + \Pi_2^C(\theta) \\ &= \frac{b(A_1 - Wc + \theta(S_1 - T))(A_2 + \theta(S_2 - T))}{W^2} - \phi I_c \theta^2 - \frac{bK(\theta_0 - \theta)(A_2 + \theta(S_2 - T))}{W} \\ &+ \frac{b(A_1 - Wc + \theta(S_1 - T))(A_2 + \theta(S_2 - T))}{W^2} - (1-\phi) I_c \theta^2 - \frac{bK(\theta_0 - \theta)(A_2 + \theta(S_2 - T))}{W}\end{aligned}$$

$$= \frac{2b(A_1 - Wc + \theta(S_1 - T))(A_2 + \theta(S_2 - T))}{W^2} - I_c\theta^2 - \frac{2bK(\theta_0 - \theta)(A_2 + \theta(S_2 - T))}{W}$$

Finding the first order condition and equating it to zero we get,

$$\begin{aligned}\theta^C &= \frac{b[(S_1 - T)A_2 - KW(S_2 - T)\theta_0 + KWA_2 + (S_2 - T)(A_1 - Wc)]}{[I_cW^2 - 2b(S_1 - T)(S_2 - T) - 2bKW(S_2 - T)]} \\ &= \frac{b[(S_2 - T)(A_1 - W(c + K\theta_0)) + A_2(KW + (S_1 - T))]}{[I_cW^2 - 2b(S_2 - T)((S_1 - T) + KW)]}\end{aligned}$$

for

$$\begin{aligned}\textbf{Condition}: I_c > \frac{b}{W^2\theta_0}&[(S_2 - T)(A_1 - W(c + K\theta_0)) \\ &+ KWA_2 + (S_1 - T)A_2 + 2\theta_0(S_1 - T + KW)(S_2 - T)]\end{aligned}$$

Substituting the above value of θ^C into the prices and quantities of each firm we get,

$$\begin{aligned}p^C(\theta) &= \frac{(A_1 + (S_1 - T)\theta^C)}{W} \\ &= [I_cA_1W^2 - b(S_1 - T)(S_2 - T)W(K\theta_0 + c) \\ &- bA_1(S_2 - T)(S_1 - T + 2KW) + bA_2(S_1 - T)(S_1 - T + KW)] \\ &/[W(I_CW^2 - 2b(S_1 - T)(S_2 - T) - 2bKW(S_2 - T))]\end{aligned}$$

$$\begin{aligned}q^C(\theta) &= b\frac{(A_2 + (S_2 - T)\theta^C)}{W} \\ &= [I_cA_2W^2 - bA_2(S_1 - T)(S_2 - T) - bKWA_2(S_2 - T) \\ &+ b(S_2 - T)^2(A_1 - W(c + K\theta_0))] \\ &/[W(I_CW^2 - 2b(S_1 - T)(S_2 - T) - 2bKW(S_2 - T))]\end{aligned}$$

The profit of each firm is given as:

$$\Pi_i^C(\theta) = \frac{b(A_1 - Wc + \theta(S_1 - T))(A_2 + \theta(S_2 - T))}{W^2} - \phi I_c \theta^2 - \frac{bK(\theta_0 - \theta)(A_2 + \theta(S_2 - T))}{W}$$

and

$$\Pi_j^C(\theta) = \frac{b(A_1 - Wc + \theta(S_1 - T))(A_2 + \theta(S_2 - T))}{W^2} - (1 - \phi) I_c \theta^2 - \frac{bK(\theta_0 - \theta)(A_2 + \theta(S_2 - T))}{W}$$

where, θ is given by equilibrium value of θ^c.

When

$$\textbf{Condition}: I_c \leq \frac{b}{W^2\theta_0}[(S_2 - T)(A_1 - W(c + K\theta_0)) + KWA_2 + (S_1 - T)A_2 + 2\theta_0(S_1 - T + KW)(S_2 - T)]$$

$$\theta^C = \theta_0$$

$$p^C = \frac{A_1 + \theta_0(S_1 - T)}{W}$$

$$q^C = b\frac{A_2 + \theta_0(S_2 - T)}{W}$$

$$\Pi_i^C = b[\frac{A_1 + \theta_0(S_1 - T)}{W} - c][\frac{A_2 + \theta_0(S_2 - T)}{W}] - \phi I_c \theta_0^2$$

Proposition 5. *When* $I_c = 2I$ *where* $I_i = I_j = I$; $\theta^C < \theta^N$

Proof: From the expressions of θ^C and θ^N it is seen that $[(S_2 - T)(A_1 - W(c + K\theta_0)) + A_2(KW + (S_1 - T))] < [S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW)]$ since $T > 0$. Comparing $[I_cW^2 - 2b(S_1 - T)(S_2 - T) - 2bKW(S_2 - T)]$ and $[2IW^2 - 2bS_2(S_1 + KW) + bT(S_1 + S_2 + KW)]$ for the case when $I_c = 2I$, we get, $[2IW^2 - 2b(S_1 - T)(S_2 - T) - 2bKW(S_2 - T)]$ - $[2IW^2 - 2bS_2(S_1 + KW) + bT(S_1 + S_2 + KW)] = S_1 + S_2 + KW - 2T$. Substituting the values of S_1, S_2, W and T gives $2(2b + \gamma)\{(\alpha - \beta) + K(b - \gamma)\} > 0$. Thus, combining the two results, we derive, $\theta^C < \theta^N$.

The result indicates that contrary to expectations, when cost of greening under cooperation equals the sum of costs of each firm, the greening level achieved is less than the case when each firm decides its own greening level for its product, given equal costs of greening. Thus cooperation results in a higher greening level, only when the costs of greening due to cooperation are significantly reduced.

5 Conclusion

In this paper, we address the concerns of firms on optimal decision making in the face of rising costs and environmental regulation. We study three set ups namely, a single firm, a duopoly and cooperation between them. We derive the strategic decisions of firms under each set up. It is found that environmental regulation does serve the required purpose of forcing firms to provide higher greening levels. However it is only restricted to certain range of greening values. Further, greening costs do restrict firms from going green. However, firms can make optimal decisions on greening and pricing based on our analytical results. We also analyze various contracts which competitor firms can enter into under greening. However it is found that the fixed fee contract where one firm pays a fixed amount for using the green product/technology, is not beneficial from a greening perspective since greening investments under competition is found to be higher. In the case of a revenue sharing contract it is found that the greening levels under certain ranges of the contract parameter results in higher greening level than that under competition. It is also found that unless cooperation leads to significant reduction in costs for firms, competition between firms is more beneficial from a greening perspective.

The contribution of our research lies in the various optimal decision making models incorporating greening costs, environmental regulation and consumer demand. Although the models are borne out of specific industry examples, they can be applied to several other industries undergoing greening. Further, contracts have been an interesting area of research in operations management. Additionally, we analyze these contracts in the light of greening issues. Lastly, the issues arising out of greening initiatives need an analytical approach to understanding and simplify them. We believe that our research lays down such a platform for researchers and practitioners alike.

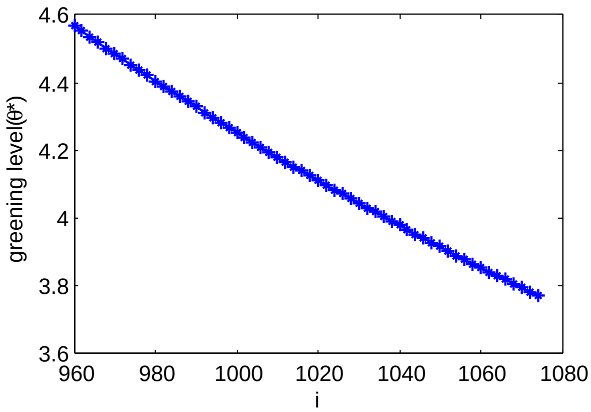

Figure 1: θ_{SF} vs I

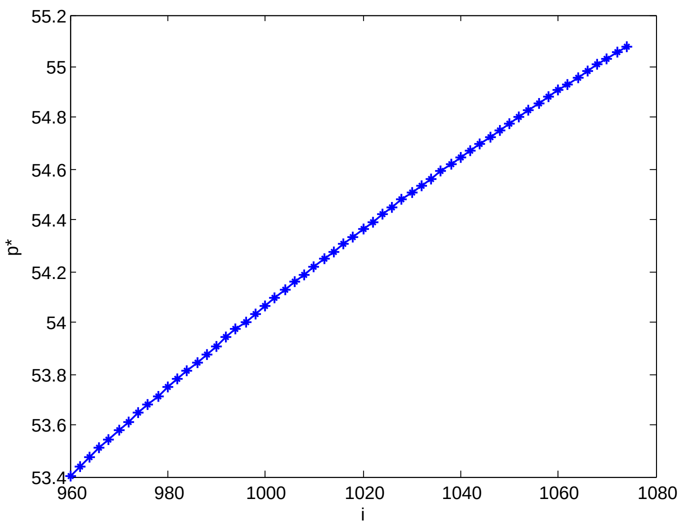

Figure 2: p_{SF} vs I

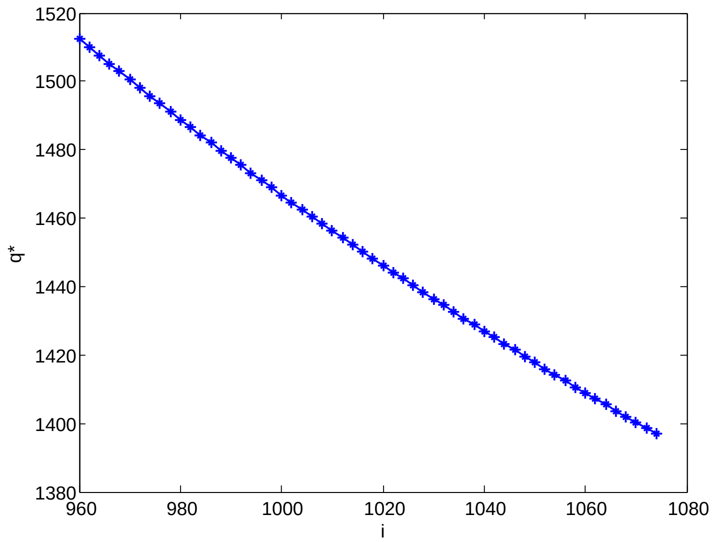

Figure 3: q_{SF} vs I

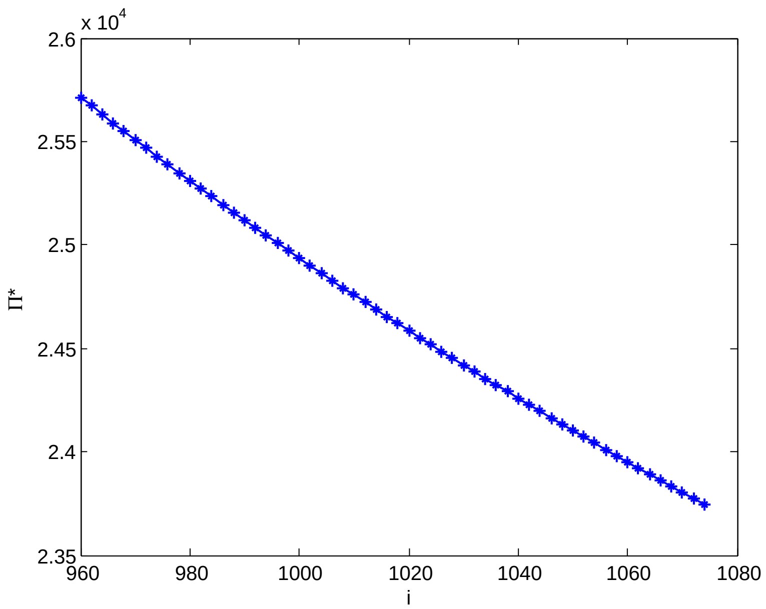

Figure 4: Π_{SF} vs I

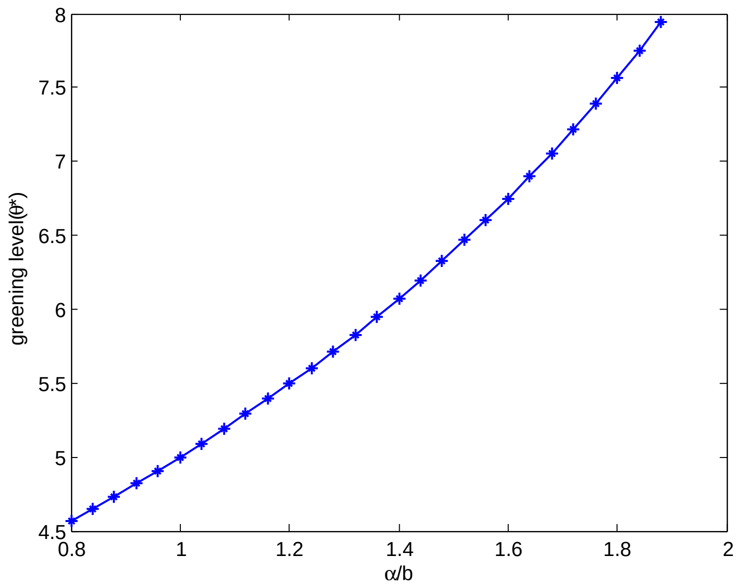

Figure 5: θ_{SF} vs α

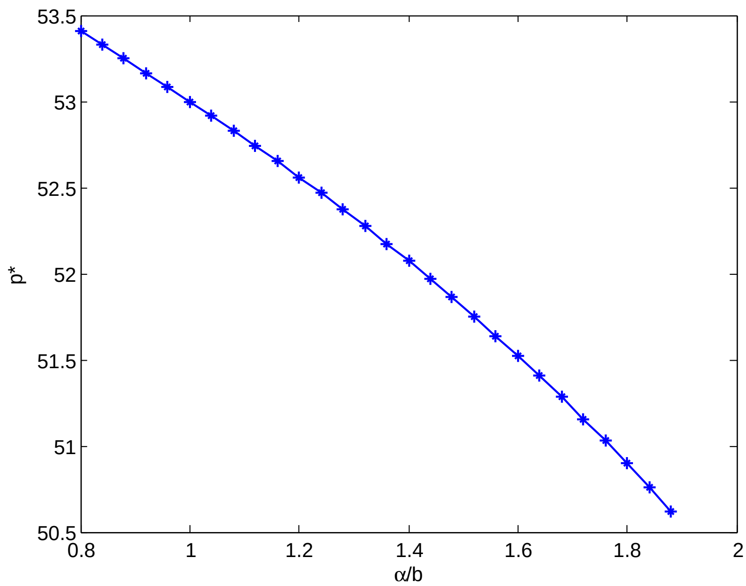

Figure 6: p_{SF} vs α

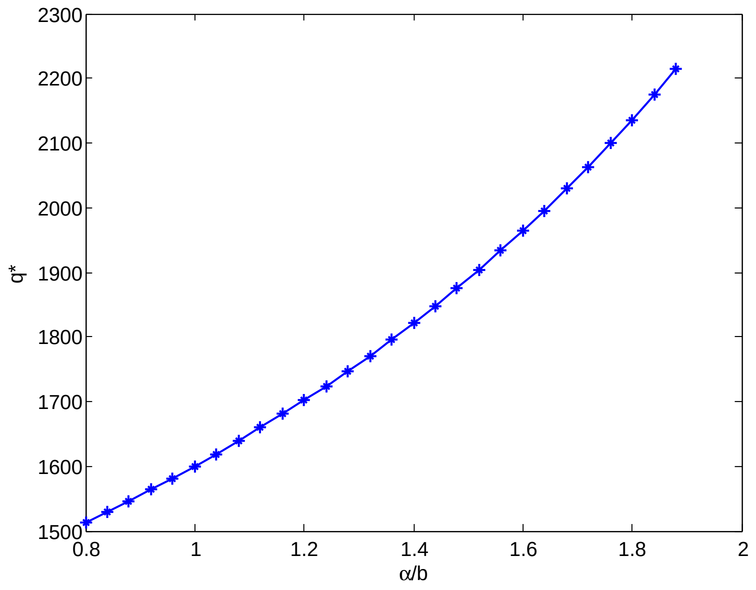

Figure 7: q_{SF} vs α

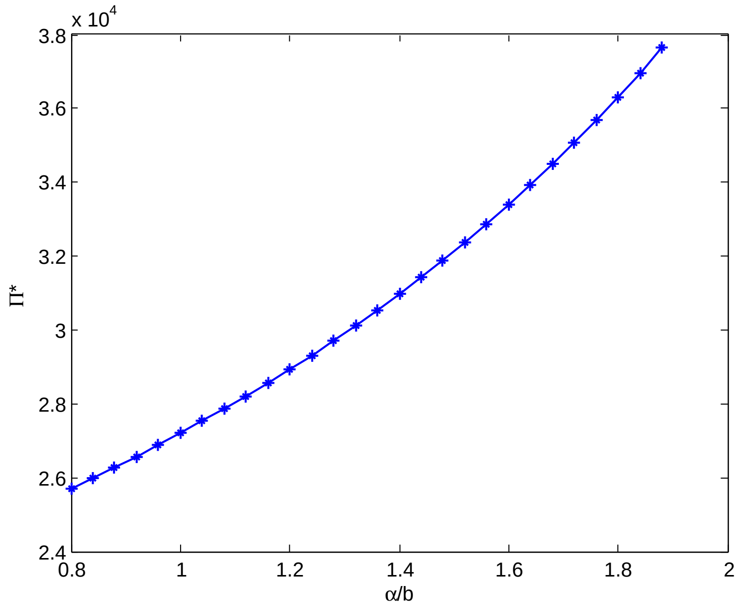

Figure 8: Π_k vs α

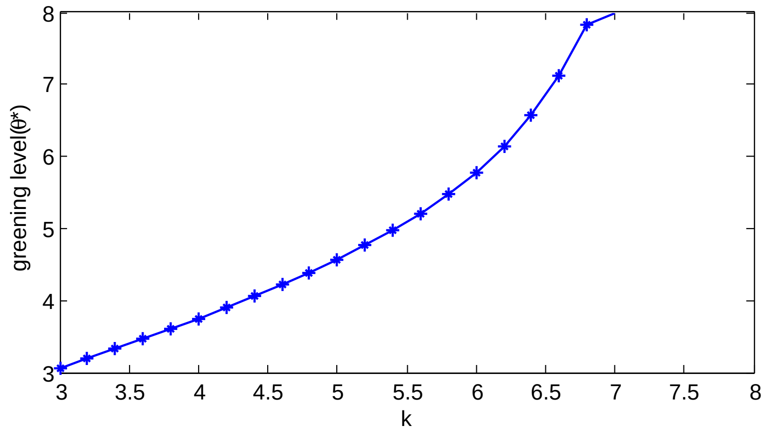

Figure 9: θ_{SF} vs k

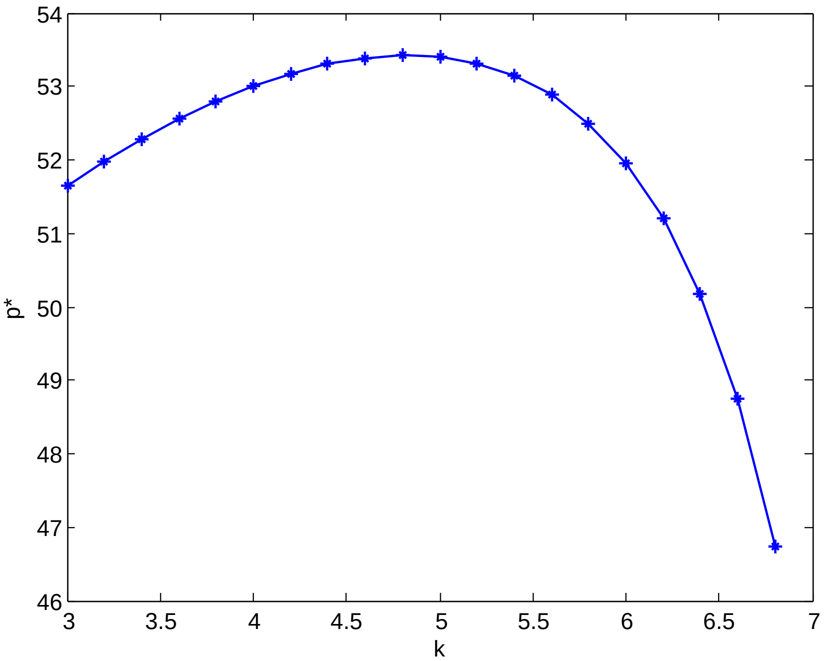

Figure 10: p_{SF} vs k

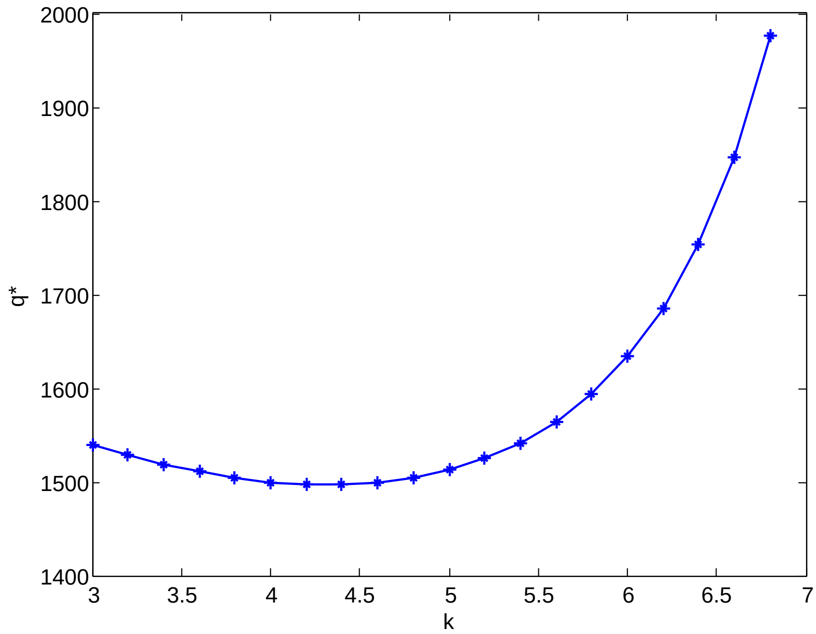

Figure 11: q_{SF} vs k

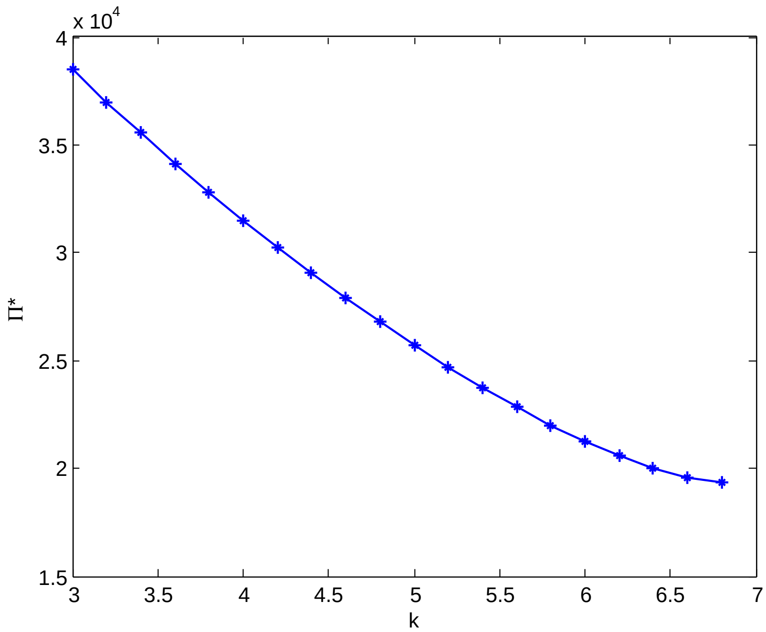

Figure 12: Π_{SF} vs k

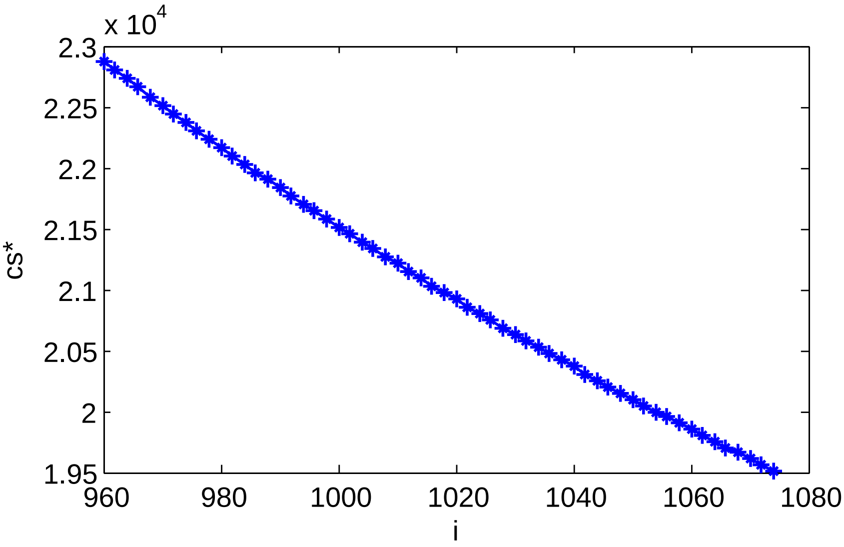

Figure 13: CS_k vs I

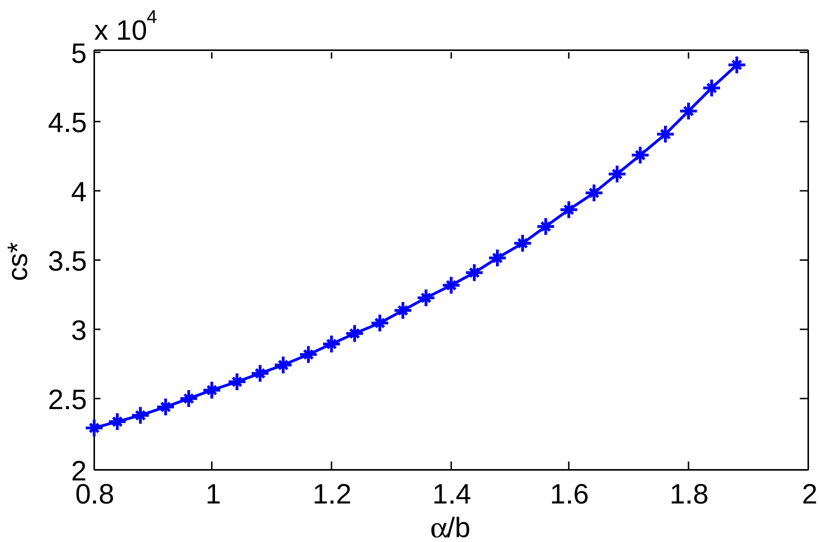

Figure 14: CS_k vs α

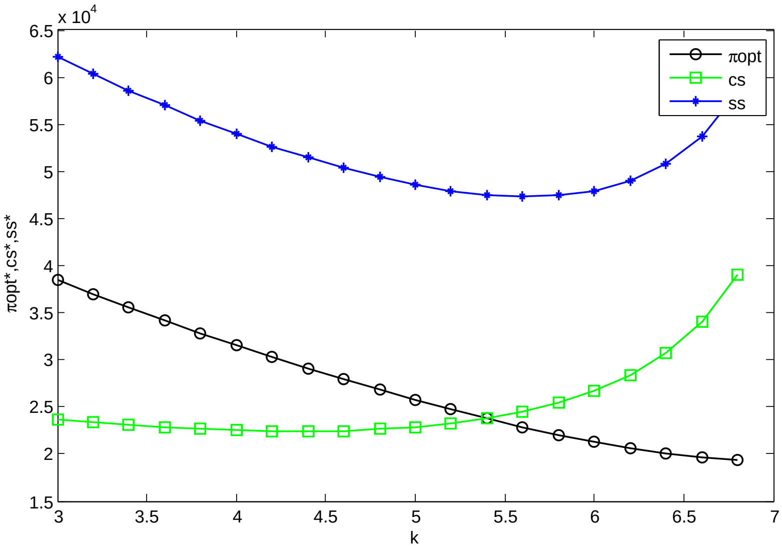

Figure 15: Π_k, CS_k, SS_k vs k

Appendix

The case of a Duopoly

We employ backward induction method to solve the second problem. We first find out the equilibrium prices given greening levels θ_i, θ_j. We derive,

$$\Pi_i(\theta_i,\theta_j) = (p_i - c - K(\theta_0 - \theta_i))(a - bp_i + \gamma p_j + \alpha\theta_i - \beta\theta_j) - I_i\theta_i^2$$

The first order condition is

$$\frac{\partial}{\partial p_i}\Pi_i(\theta_i,\theta_j) = -2bp_i + a + \gamma p_j + \alpha\theta_i - \beta\theta_j + bc + Kb(\theta_0 - \theta)$$
$$= a - 2bp_i + \gamma p_j + \theta_i(\alpha - Kb) - \beta\theta_j + b(c + K\theta_0)$$

The second order condition is

$$\frac{\partial^2}{\partial p_i^2}\Pi_i(\theta_i,\theta_j) = -2b < 0$$

Thus, Firm i's profit function is strictly concave in 'p_i'. Equating the first order condition to zero, we get,

$$p_i(\theta_i,\theta_j) = \frac{(a + \theta_i(\alpha - Kb) + b(c + K\theta_0) + \gamma p_j - \beta\theta_j)}{2b}$$

Solving for p_i and p_j simultaneously, we obtain the equilibrium price for each firm:

$$p_i^*(\theta_i,\theta_j) = \frac{(2b+\gamma)(a + b(c + k\theta_0)) + \theta_i(2b(\alpha - Kb) - \gamma\beta) - \theta_j(2b\beta - \gamma(\alpha - Kb))}{4b^2 - \gamma^2}$$

which is further simplified as:

$$p_i^*(\theta_i,\theta_j) = \frac{(A_1 + S_1\theta_i - T\theta_j)}{W} \qquad \text{where}$$
$$W = (4b^2 - \gamma^2)$$
$$A_1 = (2b+\gamma)(a + b(c + K\theta_0))$$
$$S_1 = (2b(\alpha - Kb) - \gamma\beta)$$
$$T = 2b\beta - \gamma(\alpha - Kb)$$
$$i \neq j, i, j = 1, 2$$

The corresponding values of quantities and profits at the equilibrium prices are:

$$q_i^*(\theta_i,\theta_j) = \frac{b(A_2 + S_2\theta_i - T\theta_j)}{W} \qquad \text{where}$$
$$W = (4b^2 - \gamma^2)$$
$$A_2 = (2b+\gamma)(a - (b-\gamma)(c + K\theta_0))$$
$$S_2 = 2b(\alpha + Kb) - \gamma(\beta + K\gamma)$$
$$T = 2b\beta - \gamma(\alpha - Kb)$$
$$i \neq j, i, j = 1, 2$$

$$\Pi_i^*(\theta_i,\theta_j) = \frac{b(A_1 - Wc + \theta_i S_1 - \theta_j T)(A_2 + \theta_i S_2 - \theta_j T)}{W^2} - I_i\theta_i^2 - \frac{bK(\theta_0 - \theta_i)(A_2 + \theta_i S_2 - \theta_j T)}{W}$$

We need the following assumptions:
Assumption : When $\theta_i = \theta_j = 0$, we should have positive quantity and prices. Hence, $A_1 > 0$ and $A_2 > 0$.
Assumption : We observe that if $T < 0$, the Firm i's prices and quantities increase in the greening level of its competitor Firm j, which is not the market scenario. Hence, $T > 0$.
Assumption : The impact of Firm i's own greening level on its prices and quantities should be higher than that of its competitor. Hence, $S_1 > T$ and $S_2 > T$.
To solve for the optimum 'level of greening' , we differentiate the profit function of the firm with respect to θ_i and equating it to zero, obtain the best action for Firm i given that Firm j chooses θ_j. The equilibrium 'level of greening' for Firm i is :

$$\theta_i = \frac{b[S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW) - \theta_j T(S_1 + S_2 + KW)]}{2(I_i W^2 - bS_1S_2 - KbS_2W)}; \qquad i \neq j, i, j = 1, 2$$

The second order differentiation of the profit function reveals

$$\frac{\partial^2}{\partial^2\theta_i}\Pi_i = 2\left(\frac{bS_1S_2}{W^2} - I_i + \frac{KbS_2}{W}\right)$$

The profit of the Firm is strictly concave in the level of greening θ_i when

Condition : $I_i > \dfrac{bS_2(S_1 + KW)}{W^2}, \qquad i \neq j, i, j = 1, 2$

To simplify the expression for the equilibrium value of θ_i further, let

$$X = S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW)$$
$$Y = T(S_1 + S_2 + KW)$$
$$B = b/2$$
$$Z = bS_2(S_1 + KW)$$

Thus,

$$\theta_i = \frac{B[X - \theta_j Y]}{I_i W^2 - Z} \qquad i \neq j, i, j = 1, 2$$

Now, solving the two simultaneous equations in θ_i and θ_j, we get the equilibrium 'levels of greening' as:

$$\theta_i^{NC} = \frac{BX[(I_j W^2 - Z) - BY]}{(I_i W^2 - Z)(I_j W^2 - Z) - B^2 Y^2}$$
$$= b[(S_2(A_1 - W(c + K\theta_0)) + A_2(S_1 + KW))(b(S_1 + KW)(2S_2 + T) + bS_2 T - 2I_j W^2)]/[b^2 T^2 (S_1 + S_2 + KW)^2$$
$$+ 4bS_2 KW^3 (I_i + I_j) - 4(I_j W^2 - bS_1 S_2)(I_i W^2 - bS_1 S_2) - 4b^2 S_2^2 KW(2S_1 - KW)]$$

where NC denotes the Nash Equilibrium under competition. To ensure $\theta_i^{NC} > 0$ we need,

$$\textbf{Condition} : I_j > \frac{BY + Z}{W^2}$$
$$\Rightarrow I_j > \frac{b[TS_2 + (2S_2 + T)(S_1 + KW)]}{2W^2}$$

Now, $\theta_i^{NC} < \theta_0$ which gives the condition

$$\textbf{Condition} : I_i > [bG_2(b(S_1 + KW)(2S_2 + T) + bS_2 T) - \theta_0(b^2 T^2 (S_1 + S_2 + KW)^2 - 4b^2 S_2^2 KW(2S_1 + KW))$$
$$-2bG_2 I_j W^2 - \theta_0 4bS_2 KW^3 I_j - 4\theta_0 bS_1 S_2 (I_j W^2 - bS_1 S_2)]/[\theta_0(4bS_2 KW^3 - 4W^2(I_j W^2 - bS_1 S_2))]$$

Greening through Fixed Fee Contract

We solve the problem in two stages. In the first stage, we solve an unconstrained optimization problem and in the second stage, subject the equilibrium value of θ_F to the constraint to derive bounds on greening costs. The profit function of Firm i is given as:

$$\Pi_i = (p_i - c - K(\theta_0 - \theta))q_i - I\theta^2 + F$$

and the profit function of Firm j is given as:

$$\Pi_j = (p_j - c - K(\theta_0 - \theta))q_j - F$$

The first order conditions of the profit functions w.r.t to prices are:

$$\frac{\partial \Pi_i}{\partial p_i} = a - 2bp_i + \gamma p_j + \theta(\alpha - \beta) + b(c + K(\theta_0 - \theta))$$
$$\frac{\partial \Pi_j}{\partial p_j} = a - 2bp_j + \gamma p_i + \theta(\alpha - \beta) + b(c + K(\theta_0 - \theta))$$

The second order conditions of the profit functions w.r.t to prices are :

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -2b < 0$$
$$\frac{\partial^2 \Pi_j}{\partial p_j^2} = -2b < 0$$

Thus, the profit functions are concave in prices. Equating the first order conditions to zero and solving the two simultaneous equations reveals

$$p_i = \frac{a + b(c + K(\theta_0 - \theta)) + \theta(\alpha - \beta)}{2b - \gamma} \qquad where \qquad i \neq j, i, j = 1, 2$$

Substituting the values of p_i and p_j into the demand function gives

$$q_i = \frac{ab}{2b - \gamma} - \frac{(b - \gamma)(b(c + K(\theta_0 - \theta)) + \theta(\alpha - \beta))}{2b - \gamma} + \theta(\alpha - \beta)$$

Substituting the price and quantity values into the profit equation of Firm i gives:

$$\Pi_i = (p_i - c - K(\theta_0 - \theta))q_i - I\theta^2 + F$$
$$= [\frac{a + b(c + K(\theta_0 - \theta)) + \theta(\alpha - \beta)}{2b - \gamma} - c - K(\theta_0 - \theta)][\frac{ab}{2b - \gamma} - \frac{(b - \gamma)(b(c + K(\theta_0 - \theta)) + \theta(\alpha - \beta))}{2b - \gamma} + \theta(\alpha - \beta)] - I\theta^2 + F$$

The first order condition gives :

$$\frac{\partial \Pi}{\partial \theta} = (\frac{\alpha-\beta-bK}{2b-\gamma} + K)(a - \frac{(b-\gamma)\%1}{2b-\gamma} + \theta(\alpha-\beta)) + (\frac{\%1}{2b-\gamma} - c - K(\theta_0-\theta))((\alpha-\beta) - \frac{(b-\gamma)(\alpha-\beta-bK)}{2b-\gamma}) - 2I\theta$$

$$where \qquad \%1 = a + \theta(\alpha-\beta) + b(c + K(\theta_0-\theta))$$

The second order condition gives :

$$2b\frac{(\alpha-\beta+K(b-\gamma))^2}{(2b-\gamma)^2} - 2I$$

which is strictly less than zero when $I > \dfrac{b(\alpha-\beta+K(b-\gamma))^2}{(2b-\gamma)^2}$. Thus equating the first order condition to zero and solving for θ gives

$$\theta^F = \frac{[(\alpha-\beta)+K(\beta-\gamma)]b[a-(b-\gamma)(c+K\theta_0)]}{bK^2\gamma(2b-\gamma)+(b-\gamma)(4Ib-(\alpha-\beta)2bK)-b((\alpha-\beta)^2+(bK)^2)+I\gamma^2}$$

This is written as:

$$\theta^F = \frac{N_3 b N_2}{N_1}$$

Substituting the optimum greening level(θ^F) into the profit function of Firm i gives:

$$\Pi_i^F = [N_5 - c - N_4][a - (b-\gamma)N_5 + \frac{N_3 b N_2(\alpha-\beta)}{N_1}] - \frac{I N_3^2 b^2 N_2^2}{N_1^2} + F$$

Substituting the optimum greening level(θ^F) into the profit function of Firm j gives:

$$\Pi_j^F = [N_5 - c - N_4][a - (b-\gamma)N_5 + \frac{N_3 b N_2(\alpha-\beta)}{N_1}] - F$$

where
$N_1 = I(4b(b-\gamma)+\gamma^2) - b((\alpha-\beta)+K(b-\gamma))^2$
$N_2 = a-(b-\gamma)(c+K\theta_0)$
$N_3 = \alpha-\beta+K(b-\gamma)$
$N_4 = K(\theta_0 - \dfrac{N_3 b N_2}{N_1})$
$N_5 = \dfrac{a + \dfrac{N_3 b N_2(\alpha-\beta)}{N_1} + b(c+N_4)}{2b-\gamma}$
Now, $\theta^F < \theta_0$ which gives the condition

$$\textbf{Condition} : I > \frac{bN_3[N_2+\theta_0 N_3]}{\theta_0(4b(b-\gamma)+\gamma^2)}$$

Greening through Revenue Sharing

The profit function of Firm i is given as
$\Pi_i = (p_i - c - K(\theta_0-\theta))q_i - I\theta^2 + (1-\omega)p_j q_j$
The first order condition with respect to p_i gives :

$$\frac{\partial \Pi_i}{\partial p_i} = (a - bp_i + \gamma p_j + \theta(\alpha-\beta)) - b(p_i - c) + Kb(\theta_0-\theta) + (1-\omega)\gamma p_j$$

The second order condition with respect to p_i gives :

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -2b < 0$$

Thus profit function of Firm i is concave in p_i. The profit function of Firm j is given as
$\Pi_j = \omega(p_j q_j) - (c + K(\theta_0-\theta))q_j$
The first order condition with respect to p_j gives :

$$\frac{\partial \Pi_j}{\partial p_j} = \omega(a + \gamma p_i + \theta(\alpha-\beta)) - 2\omega b p_j + b(c + K(\theta_0-\theta))$$

The second order condition with respect to p_j gives :

$$\frac{\partial^2 \Pi_j}{\partial p_j^2} = -\omega 2b < 0$$

Thus profit function of Firm j is concave in p_j. Thus equating the first order conditions to zero and solving the two simultaneous equations we get

$$p_i = \frac{\omega 2b(a+bc) + (\alpha-\beta)\omega\theta(2b+\gamma(2-\omega)) + (\omega a+bc)\gamma(2-\omega) + (\theta_0-\theta)Kb(\gamma(2-\omega)+\omega 2b)}{\omega(4b^2-\gamma^2(2-\omega))}$$

$$p_j = \frac{2b(\omega a+bc) + \omega\gamma(a+bc) + (\theta_0-\theta)Kb(2b+\omega\gamma) + (\alpha-\beta)\omega\theta(2b+\gamma)}{\omega(4b^2-\gamma^2(2-\omega))}$$

We substitute the prices, quantities as a function of level of greening (θ) into the profit function of Π_i and get

$$\Pi_i^{RS} = [\frac{S_2}{\omega S_1} - (c+K(\theta_0-\theta))]S_4 - I_i\theta^2 + \frac{(1-\omega)S_3S_5}{\omega S_1}$$

where $S_1 = 4b^2 - \gamma^2(2-\omega)$
$S_2 = 2b\omega(a+bc) + \gamma(2-\omega)(\omega a+bc) + (\theta_0-\theta)Kb(2\omega b+\gamma(2-\omega)) + (\alpha-\beta)\theta\omega(2b+\gamma(2-\omega))$
$S_3 = 2b(\omega a+bc) + (\theta_0-\theta)Kb(\omega\gamma+2b) + (\alpha-\beta)\omega\theta(\gamma+2b) + \omega\gamma(a+bc)$
$S_4 = (a - \frac{bS_2}{\omega S_1} + \frac{\gamma S_3}{\omega S_1} + \theta(\alpha-\beta))$
$S_5 = (a - \frac{bS_3}{\omega S_1} + \frac{\gamma S_2}{\omega S_1} + \theta(\alpha-\beta))$
The profit function is concave in θ (derived from second order condition w.r.t θ) when

$$I > \frac{(S_8S_6 + (1-\omega)S_9S_7)}{\omega S_1} + KS_6$$

where $S_6 = (\alpha - \beta - \frac{(bS_8 - \gamma S_9)}{\omega S_1})$
$S_7 = (\alpha - \beta - \frac{(bS_9 - \gamma S_8)}{\omega S_1})$
$S_8 = (\alpha-\beta)\omega(2b+\gamma(2-\omega)) - Kb(2b\omega+\gamma(2-\omega))$
$S_9 = 2b\omega(\alpha-\beta) - 2Kb^2 - \omega\gamma(Kb-\alpha)$
Equating the first order condition w.r.t θ, we derive the optimal greening level (θ^{RS}) as

$$\theta^{RS} = (1/2)(\frac{S_{12}}{S_{11}})$$

Substituting the above value of (θ^{RS}) into the profit function of each firm gives

$$\Pi_i^{RS} = (\frac{S_{13}}{\omega S_1} - c)S_{10} - 1/4(\frac{I_iS_{12}^2}{S_{11}^2}) - K(\theta_0 - 1/2(\frac{S_{12}}{S_{11}}))S_{10} + \frac{(1-\omega)S_{14}(a - \frac{bS_{14}}{\omega S_1} + \frac{\gamma S_{13}}{\omega S_1} + 1/2\frac{S_{12}(\alpha-\beta)}{S_{11}})}{\omega S_1}$$

$$\Pi_j^{RS} = \frac{(S_{15}S_{16})}{S_1} - cS_{16} - K(\theta_0 - 1/2(\frac{S_{12}}{S_{11}}))S_{16}$$

where $S_{10} = (a - \frac{bS_{13}}{\omega S_1} + \frac{\gamma S_{14}}{\omega S_1} + 1/2\frac{S_{12}(\alpha-\beta)}{S_{11}})$
$S_{11} = \%1$
$S_{12} = \%2$
$S_{13} = \%4 = (\omega a+bc)\gamma(2-\omega) + 2\omega b(a+bc) + \gamma Kb\theta_0(2-\omega) - \frac{\gamma KbS_{12}}{S_{11}} + 2\omega Kb^2\theta_0 + \frac{\omega S_{12}(\alpha-\beta)(\beta+\gamma)}{S_{11}} - \frac{\omega Kb^2S_{12}}{S_{11}} - (1/2)\frac{\omega^2\gamma S_{12}(\alpha-\beta)}{S_{11}} + (1/2)\frac{\omega\gamma KbS_{12}}{S_{11}}$
$S_{14} = \%5 = \omega 2ab + 2Kb^2\theta_0 + \frac{\omega bS_{12}(\alpha-\beta)}{S_{11}} + 2b^2c + \omega\gamma(a+b(c+K\theta_0)) + (1/2)\frac{\omega\gamma S_{12}(\alpha-\beta)}{S_{11}} - (1/2)\frac{\omega\gamma KbS_{12}}{S_{11}} - \frac{Kb^2S_{12}}{S_{11}}$
$S_{15} = \%4_j = \omega 2ab + 2b^2(c+K\theta_0) + \frac{\omega bS_{12}(\alpha-\beta)}{S_{11}} - (1/2)\frac{\omega\gamma KbS_{12}}{S_{11}} - \frac{Kb^2S_{12}}{S_{11}} + (1/2)\frac{\omega\gamma S_{12}(\alpha-\beta)}{S_{11}} + \omega\gamma(a+b(c+K\theta_0))$
$S_{16} = \%5_j = a - \frac{bS_{15}}{\omega S_1} + \frac{\gamma}{\omega S_1}[2\gamma bc + 2b\omega(a+bc) - (1/2)\frac{\omega^2\gamma S_{12}\alpha}{S_{11}} + \gamma Kb\theta_0(2-\omega) + 2\omega(\gamma a + Kb^2\theta_0) - \frac{\gamma KbS_{12}}{S_{11}} - \omega\gamma(\omega a + bc) + \frac{b\omega S_{12}(\alpha-\beta)}{S_{11}} - \frac{\omega Kb^2S_{12}}{S_{11}} + \frac{\omega\gamma S_{12}(\alpha-\beta)}{S_{11}} + (1/2)(\frac{\omega\gamma S_{12}}{S_{11}})(\omega\beta+Kb)] + (1/2)\frac{S_{12}(\alpha-\beta)}{S_{11}}$

Greening through cost sharing contract

The second order condition gives

$$\frac{\partial^2}{\partial\theta^2}\Pi(\theta) = \frac{4b(S_1-T)(S_2-T) - 2I_cW^2 + 4BKW(S_2-T)}{W^2}$$

which when subjected to the condition of being negative for a global maximum gives the condition

$$\textbf{Condition}: I_c > \frac{2b(S_2 - T)(S_1 - T + KW)}{W^2}$$

When, $\theta^* < \theta_0$, we get the condition:

$$\textbf{Condition}: I_c > \frac{b}{W^2\theta_0}[(S_2 - T)(A_1 - W(c + K\theta_0)) + KWA_2 + (S_1 - T)A_2 + 2\theta_0(S_1 - T + KW)(S_2 - T)]$$

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