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Pricing Strategies for Gaming-on-Demand

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Abstract

In the last few years, social gaming has resulted in significant upheavals in the traditional video gaming industry. However, an even bigger threat looms large in the horizon. The advent of cloud gaming or gaming-on-demand is expected to disrupt the traditional gaming industry. However, the success of the gaming-on-demand model would depend to a large extent on the availability of broadband services. As the quality of broadband services is not uniform across the different geographies, pricing of cloud gaming services must take this non-uniformity into account. The paper provides managerial guidelines for cloud game providers on pricing their offerings. We develop a pricing schedule for a typical cloud game provider by modeling the non-uniformity of broadband availability, and a gamer's propensity to engage in gaming. We explore two pricing plans: usage based and fixed fee plan. We determine the conditions under which gamers would select one plan over another, and discuss the significance of these conditions for cloud gaming providers.

keywords:Gaming-on-Demand, **Pricing**, **Price Discrimination**, **Cloud Gaming**

1 Introduction

Cloud gaming or the gaming-on-demand model is well poised to disrupt the traditional video gaming industry. In the gaming-on-demand model, the games are run on huge server farms, and the gaming company allows a gamer to stream the game for a specified fee. The gamers access the game by using a device which does not require a very powerful processor, for example, smart TVs, tablets, PCs, etc., and an internet connection to connect to the gaming company's server. Therefore, gamers do not need to invest on sophisticated and expensive hardware like gaming consoles to play a game. At the same time, gamers get the option to pay according to their usage instead of being compelled to purchase a game. The advantages of this model for the gaming companies are primarily threefold. First, the company can upgrade its games without worrying about the hardware compatibility of its customers. Second, it expands the market to include gamers who are ready to pay a small usage based fee to access online games, but cannot afford to make large upfront payments to buy expensive ones. Third, this model opens up a new revenue stream for outmoded games which are difficult to sell; there may be gamers who will pay to play such a game for a few hours. As the marginal cost of offering a game on demand is very low, it is indeed a profitable model for the gaming company. Moreover, the gaming-on-demand model can help in reducing piracy.

The gaming-on-demand market has already become fiercely competitive. In January 2012, Gaikai Inc., which started gaming-on-demand in 2010, announced a strategic partnership with LG Electronics to launch an integrated Smart TV cloud gaming service [1]. This deal allows LG to leverage the cloud platform of Gaikai to offer a broad range of games to its customers. Games are offered through the game portal service operating within the LG Smart TV. OnLive Inc., Gaikai's main competitor, reacted to this strategic alliance by demonstrating their OnLive Game Service on the next generation LG Smart TV with Google TV (G2 series) in the month of June, 2012 [2, 7]. A few days later Gaikai announced its partnership with Samsung which will offer cloud gaming using Samsung Smart TVs [3]. As competitor rivalries propel the industry towards a greater state of flux, traditional gaming companies will be forced to respond to this challenge emanating from the gaming-on-demand model. One other factor that is expected to favor the shift towards cloud gaming is the increasing popularity of tablets and smart phones [4]. Reports indicate that tablets will become the most important computing device in the future. As more and more people start accessing the Internet using devices with low computing power, gaming companies will increasingly feel the need to offer games that can be played using such devices, in effect, offer games as a service. The recent acquisition of Gaikai by Sony, signifies that this transformation is not just inevitable, it is also imminent [20].

However, there is an obstacle to wide scale adoption of cloud gaming or gaming-on-demand - the quality of broadband services. As the game is delivered as a service via the Internet, the user experience of gaming-on-demand will definitely be poorer if she does not have access to high quality broadband service. Sony, according to reports, seriously considered the idea of coming up with a download only version of its gaming console, Play Station 4. It ultimately rejected that idea keeping in mind the inconsistencies in broadband speeds around the world and instead decided to keep an optical drive as a part of the console [19]. For the same reason, Microsoft has also decided to include an optical drive as part of its next Xbox [26]. However, experts believe that as Internet speeds go up, it is a matter of time before console manufacturers include cloud gaming features in their consoles. Although standardization of broadband speeds around the world seems improbable in the near future, it is unlikely to dampen the efforts of cloud gaming providers like Gaikai (now Sony) or OnLive from venturing into the markets of Eastern Europe or South-East Asia where Internet speeds are considerably slower. Therefore, cloud gaming companies will not be able to offer a similar experience to users across the world even when the users are playing the same game. This implies that these companies have to consider the quality of broadband services when they price their offerings in different countries. This adds to the complexity of the pricing mechanism of cloud gaming services. Moreover, the design of pricing plans which can correctly factor in the effect of the quality of broadband service assumes enormous significance in determining the success of the gaming-on-demand business model.

The current literature on cloud gaming essentially studies cloud computing with a focus on the developments in the cloud gaming industry [16]. However, the video games industry has been extensively studied. Vogel [23] discusses the video games industry in the context of the general entertainment industry of the United States (see Williams [24] for a lucid introduction to the industry). Gallagher and Park [10] had looked at the competitive dynamics of industries which are standards based, with a focus on the US home video game market. Most of the studies on the industry have focused on network effects and the effect of vertical integration on the pricing of games and consoles. Shankar and Bayus [17] conducted an empirical study on the importance of network size and network strength in the video games industry. Clements and Ohashi [6] conducted a more extensive study with data from 1994 to 2002. Their findings indicate that introductory pricing and software variety (game variety) play important roles in the diffusion of game systems; whereas introductory pricing proves more effective in the initial stages, the effect of game variety dominates in the later. Hong

Ju Liu [13] developed a framework to study pricing strategies under network effects, consumer heterogeneity and oligopolistic competition. The work discusses alternate strategies of video games console manufacturers through policy simulations, and critically examines the pricing strategies of Nintendo. The paper reports the findings of an empirical investigation on console pricing and studies the effect of vertical integration on the sales performance of video games. Multiple research findings which indicate that vertical integration leads to lower prices and increased competition in the case of video games console. However, this leads to an increase in the number of units of consoles sold, and a higher demand for video games which leads to higher profits [8]. A study by Gill and Warzynski [11] finds that vertically integrated games sell more and at higher prices compared to non-vertically integrated games, which validates the findings of Derdenger [8]. However, results indicate that there is no effect of vertical integration on the quality of the games. The problem of optimal pricing over time for a firm selling a durable good to forward looking consumers [15] has been studied in the context of the video games industry. Results show that the behavior of forward looking consumers has a significant impact on pricing.

Varian and Shapiro [18] defines information good as ”anything that can be digitized - encoded as a stream of bits”. In line with this definition, here we categorize games delivered as-a-service as information good. Pricing information goods generally involves non-linear price structures [18, 5, 25, 22]. It is also quite common to observe multi part tariffs like flat rate pricing, two part tariffs, etc. in the pricing of information goods [12, 21]. If we consider gaming-on-demand usage to be the number of hours a game is played, then non-linear usage based pricing can be used to price gaming-on-demand services. Software vendors who provide software-as-a-service have adopted this pricing model, for example, Salesforce.com. Firms like Amazon.com which provide computing infrastructure-as-a-service have adopted non-linear pricing which has a combination of usage based and fixed fee components. As with the case of other information goods, gaming-as-a-service exhibits zero marginal cost as the cost of providing an additional unit of a game for an unit time is essentially zero. However, cloud gaming providers have to incur a cost to keep track of the usage of the consumer, which is best expressed as transaction cost. Non-linear pricing theory states that the optimal pricing policy of a monopolist should always be based on usage [14, 25]. In a generalized discussion on non-linear pricing of information goods, Sundararajan [21] has shown that if we consider the near zero marginal costs of information goods along with the costs of administering a usage based pricing schedule it is possible to explain the profitability of fixed fee pricing for information goods. In this paper, we make use of the above observation to consider fixed-fee pricing (independent of usage) as well as usage based pricing as possible pricing strategies for gaming-on-demand providers.

In this paper we are interested in determining the optimal pricing structure for gaming-on-demand providers by taking in to account the quality of broadband services available to the gamers. We model heterogeneous gamers characterized by their propensity to engage in gaming, and the quality of their broadband services, in order to develop a pricing schedule. This work contributes to the literature on pricing of cloud gaming services and provides guidelines for cloud gaming providers on pricing their offerings.

In Section 2, we introduce the basic notations and assumptions used in rest of the paper. Pricing plans considered in this paper are elaborated in Section 3. We explain the selection problem of a gamer in Section 4. In Sections 5 to 7, we identify optimal usage based and fixed fee pricing plans for both gamer and cloud game service provider. We take an example in Section 8 to show the applicability of the closed form expressions derived for optimal pricing plans in the previous sections. In Section 9, we conclude after highlighting the application areas and the key contributions of this paper.

2 Model

In this paper, a monopoly cloud gaming provider offers gaming as a service to consumers (gamers). We assume that the variable cost of offering an additional unit of a game as a service for a unit time is zero. We also assume a cost of administering usage based fee incurred by cloud gaming providers, and call it transaction costs. From the perspective of a cloud gaming provider, gamers are characterized using two parameters: gamer type (ρ) and broadband non-uniformity(σ). Gamer type (ρ) indicates propensity of a gamer toward gaming, whereas broadband non-uniformity is defined as the change in data rate of the broadband connection in unit time, and serves as the measure for quality of the broadband service. Mathematically, σ is represented as $|\Delta\phi|/\Delta t$, the absolute value of the change in the data rate ($\Delta\phi$) from time period t to time period $t+\Delta t$. The gamers are heterogeneous, and we index them by their type $\rho \in [\underline{\rho},\ \overline{\rho}]$ and broadband non-uniformity $\sigma \in [\underline{\sigma},\ \overline{\sigma}]$. The utility function of a gamer with type ρ and broadband non-uniformity σ is represented by $U(q(\rho,\sigma),\rho,\sigma)$. From the functional form of utility function, utility gained by a gamer is dependent on its identifiable characteristics, i.e. σ and ρ, and q, the quantity of games played (consumed) from her cloud gaming provider. As discussed earlier, the quantity consumed is the number of hours a game is played. Corresponding net utility for the gamer is expressed as $U(q(\rho,\sigma),\rho,\sigma)-\tau$ where τ is the price paid by the gamer for the quantity of games consumed. In the following discussion, numbered subscripts to functions denote the partial derivatives with respect to the corresponding arguments. For example, $U_1(q(\rho,\sigma),\rho,\sigma)$ is the first order partial derivative of utility function U with respect to the first argument, q, while $U_{11}(q(\rho,\sigma),\rho,\sigma)$ is the second order derivative with respect to q. $U_{12}(q(\rho,\sigma),\rho,\sigma)$ represents the cross partial derivative of utility function U with respect to the first and the second arguments. We consider the following properties of the utility function:

(i) $U(0,\rho,\sigma)=0$; $U_1(q(\rho,\sigma),\rho,\sigma)\geq 0$; $U_{11}(q(\rho,\sigma),\rho,\sigma)<0\ \forall\ q>0$

(ii) $U_3(q(\rho,\sigma),\rho,\sigma)\leq 0$; $U_2(q(\rho,\sigma),\rho,\sigma)>0$

(iii) $U_{12}(q(\rho,\sigma),\rho,\sigma)>0$; $U_{13}(q(\rho,\sigma),\rho,\sigma)<0\ \forall\ q>0$

(iv) $\lim_{q\to\infty} U(q(\rho,\sigma),\rho,\sigma)=V(\rho,\sigma)\ <\ \infty$

According to property (i), utility of a gamer increases with an increase in consumption of games, however, it increases at a decreasing rate. Property (ii) states that gamers of higher types get higher utility, while those with higher values of broadband non-uniformity get less utility. Property (iii) implies that a gamer of higher type will get a higher increase in utility than a gamer of lower type for the same increase in consumption of games q. At the same time, gamers with higher broadband non-uniformity will get a lower increase in utility for the same increase in consumption. Property (iv) assumes an upper bound on the utility that a gamer can get from unlimited consumption.

We further assume that on the lower limit of broadband non-uniformity, i.e., $\sigma\to 0$, utility function is defined as $\lim_{\sigma\to 0} U(q(\rho,\sigma),\rho,\sigma)=U^L(q(\rho),\rho)<\infty$ and on the upper side, it is defined as $\lim_{\sigma\to\infty} U(q(\rho,\sigma),\rho,\sigma)=U^H(q(\rho),\rho)>0$. Utility functions of gamers follow the similar properties for the limiting conditions listed below:

(i) $U^L(0,\rho) = U^H(0,\rho) = 0$; $U_1^L(q(\rho),\rho) \geq 0, U_1^H(q(\rho),\rho) \geq 0$; $U_{11}^L(q(\rho),\rho) < 0$, $U_{11}^H(q(\rho),\rho) < 0 \;\forall\; q > 0$

(ii) $U^L(q(\rho),\rho) \geq U^H(q(\rho),\rho) \;\forall \rho \in \left[\underline{\rho},\ \overline{\rho}\right]$

(iii) $U_2^L(q(\rho),\rho) > 0, U_2^H(q(\rho),\rho) > 0; U_{12}^L(q(\rho),\rho) > 0, U_{12}^H(q(\rho),\rho) > 0 \;\forall\; q > 0$

(iv) $\lim_{q\to\infty} U^L(q(\rho),\rho) = V^L(\rho) \;<\; \infty, \lim_{q\to\infty} U^H(q(\rho),\rho) = V^H(\rho) \;<\; \infty$

3 Pricing Plans

In this paper, we consider two different pricing plans offered by cloud gaming providers: fixed fee and usage based fee.

(i) Fixed fee: A gamer pays a pre-specified fixed amount T for unlimited consumption of games for a specific time period.

(ii) Usage based fee: In this plan, there is a price for each unit time period of game played, and the entire schedule of quantity price pairs is available to the gamers. From the revelation principle [9] we can assume that a gamer will choose a unique price-quantity combination that maximizes her net utility. Therefore the usage based pricing plan can be represented by a menu of quantity-price pairs offered by cloud gaming provider satisfying the following two constraints:

*Incentive Compatibility **[IC]**:* For each gamer characterized by ρ and σ,

$U(q(\rho,\sigma),\rho,\sigma) - \tau(\rho,\sigma) \geq U(q(x,y),\rho,\sigma) - \tau(x,y) \;\forall\; x \in \left[\underline{\rho},\ \overline{\rho}\right]$ and $\forall\; y \in [\underline{\sigma},\ \overline{\sigma}]$

*Individual Rationality **[IR]**:* For each gamer characterized by ρ and σ, $U(q(\rho,\sigma),\rho,\sigma) - \tau(\rho,\sigma) \geq 0$. If the two conditions are met, then the gamer will choose the pair, $[q(\rho,\sigma),\tau(\rho,\sigma)]$.

Here, $\tau(\rho,\sigma)$ is the price function for gamers of type ρ and broadband non-uniformity σ in usage based contract.

These two conditions are valid for limiting conditions of non-uniformity of broadband connection as well.

4 Selection Problem of a Gamer

In this section we determine the conditions under which gamers adopt a fixed fee plan in the presence of an incentive compatible usage based plan. We first establish some initial results related to usage based plan. Unless otherwise stated, all proofs are presented in the appendix.

Lemma 1. *If $q(\rho, \sigma)$ denotes the consumption of a gamer (characterized by gamer type ρ and broadband broadband disruption σ) who has opted for an incentive compatible plan, then:*

(a) $q_1(\rho, \sigma) \geq 0$.

(b) $q_2(\rho, \sigma) \leq 0$.

Lemma 2. *If preference function of a gamer is defined as*

$F(q(\rho, \sigma), \rho, \sigma) = U(q(\rho, \sigma), \rho, \sigma) - \tau(\rho, \sigma)$, *then:*

(a) $F(q(\rho, \sigma), \rho, \sigma)$ *is strictly increasing in* ρ.

(b) $F(q(\rho, \sigma), \rho, \sigma)$ *is non-increasing in* σ.

For limiting cases of broadband non-uniformity, preference functions are defined as $F^L(q(\rho), \rho)$ (for $\sigma \to 0$) and as $F^H(q(\rho), \rho)$ (for $\sigma \to \infty$). Accordingly Lemma 2 is modified as follows:

Lemma 3. *For limiting cases of user variability, if preference functions are defined as* $F^L(q(\rho), \rho) = U^L(q(\rho), \rho) - \tau(\rho)$ *and* $F^H(q(\rho), \rho) = U^H(q(\rho), \rho) - \tau(\rho)$, *then*

(a) $F^L(q(\rho), \rho)$ *is strictly increasing in* ρ.

(b) $F^H(q(\rho), \rho)$ *is strictly increasing in* ρ.

4.1 Presence of a Fixed Fee Plan

Let us suppose that a gamer has the option of a fixed fee plan in addition to a usage based plan. Given this choice, a gamer will opt for fixed fee plan only if,

$$V(\rho, \sigma) - T \geq U(q(\rho, \sigma), \rho, \sigma) - \tau(\rho, \sigma) \tag{1}$$

$$V(\rho, \sigma) - U(q(\rho, \sigma), \rho, \sigma) + \tau(\rho, \sigma) \geq T \tag{2}$$

Here, we assume that the gamer who is indifferent will opt for the fixed fee plan. The left hand side of Equation 2 is defined as *Fixed Fee Surplus.* If the fixed fee surplus is more than or equal to the fixed fee T, then the gamer opts for fixed fee plan instead of usage based plan.

Lemma 4. *If the fixed fee surplus is defined as* $X(q(\rho,\sigma),\rho,\sigma) = V(\rho,\sigma) - U(q(\rho,\sigma),\rho,\sigma) + \tau(\rho,\sigma)$*, then the following results are established.*

(a) $X(q(\rho,\sigma),\rho,\sigma)$ *is strictly increasing in* ρ

(b) $X(q(\rho,\sigma),\rho,\sigma)$ *is strictly decreasing in* σ

Lemmas 2 and 4 lead to the following proposition:

Proposition 1. *Given an option to choose between two plans: usage based and fixed fee, a gamer's choice will follow the conditions given below:*

(a) If $V(\underline{\rho},\sigma) - T \geq U(q(\underline{\rho},\sigma),\underline{\rho},\sigma) - \tau(\underline{\rho},\sigma)\ \forall\ \sigma$ *or* $V(\rho,\overline{\sigma}) - T \geq U(q(\rho,\overline{\sigma}),\rho,\overline{\sigma}) - \tau(\rho,\overline{\sigma})\ \forall\ \rho$*, then all gamers opt for fixed fee plan.*

(b) If $V(\overline{\rho},\sigma) - T < U(q(\overline{\rho},\sigma),\overline{\rho},\sigma) - \tau(\overline{\rho},\sigma)\ \forall\ \sigma$ *or* $V(\rho,\underline{\sigma}) - T < U(q(\rho,\underline{\sigma}),\rho,\underline{\sigma}) - \tau(\rho,\underline{\sigma})\ \forall\ \rho$*, then all gamers opt for usage based plan.*

(c) As fixed fee surplus strictly increases with ρ *and strictly decreases with* σ*, there is no unique* ρ *or* σ *to choose between fixed fee plan and usage based plan when,* $\rho,\sigma \neq 0$ *and* $\rho,\sigma < \infty$*.*

Proof of Part (a). Using results found in Lemma 4, fixed fee surplus increases with increasing ρ. Hence, if a gamer of type $\underline{\rho}$ adopts fixed fee plan, then all gamers (with any $\rho \geq \underline{\rho}$) and with same broadband non-uniformity σ will adopt fixed fee plan because of higher fixed fee surplus. As this argument is valid for gamers of all σ, all gamers will opt for fixed fee. This concludes the proof for part (a) of Proposition 1. □

Proof of Part (b). Using results from Lemma 4 with fixed fee surplus strictly increasing with ρ, if a gamer of type $\overline{\rho}$ opts for usage based plan, all gamers with same broadband non-uniformity σ will opt for the same because of decreasing fixed fee surplus. As this argument is valid for gamers of all σ, all gamers will opt for fixed fee. This concludes proof for part (b). □

4.1.1 Limiting Cases

In this section we look at the decision problem of a gamer in the limiting cases of broadband non-uniformity.We first establish the following lemma to define the characteristics of fixed fee surplus in limiting cases.

Lemma 5. *For limiting cases of broadband non-uniformity, if fixed fee surpluses are defined as*

$X^L(q(\rho),\rho) = V^L(\rho) - U^L(q(\rho),\rho) + \tau(\rho)$ *and*

$X^H(q(\rho),\rho) = V^H(\rho) - U^H(q(\rho),\rho) + \tau(\rho)$*, then*

(a) $X^L(q(\rho),\rho)$ *is strictly increasing in* ρ

(b) $X^H(q(\rho),\rho)$ *is strictly increasing in* ρ

Lemmas 3 and 5 lead to Proposition 2. We use the following definitions of gamer types for limiting cases in Proposition 2.

Gamer types ρ_U^L and ρ_U^H are defined as:

$q^*(\rho) = 0 \ \forall \rho < \rho_U^L$ when $\sigma \to 0$ and $q^*(\rho) = 0 \ \forall \rho < \rho_U^H$ when $\sigma \to \infty$.

Types ρ_S^L and ρ_S^H are defined as:

$\rho_S^L = Min\{\rho : V^L(\rho) - U^L(q(\rho),\rho) + \tau(\rho) = T\}$ and

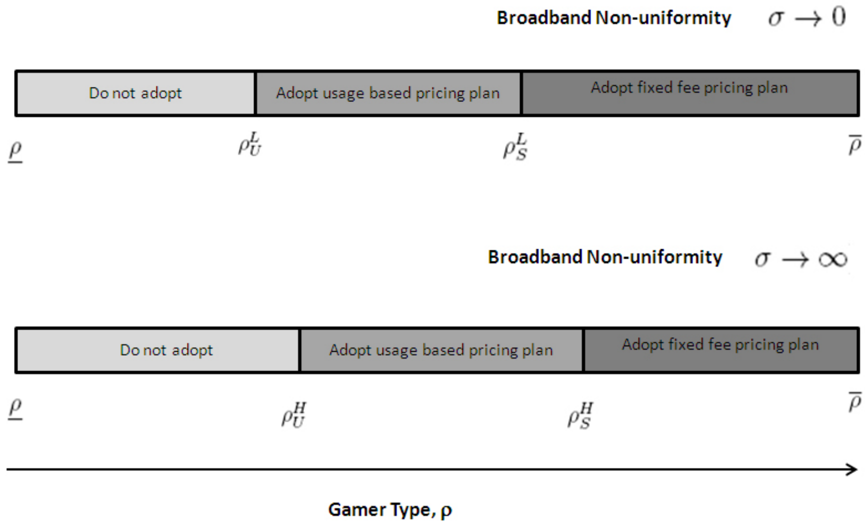
$\rho_S^H = Min\{\rho : V^H(\rho) - U^H(q(\rho),\rho) + \tau(\rho) = T\}$

Proposition 2. *Given an option to choose between two plans: usage based and fixed fee, a gamer's choice will follow the conditions given below:*

(a) If $V^L(\underline{\rho}) - T \geq U^L(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$ *or* $V^H(\underline{\rho}) - T \geq U^H(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$*, then gamers with* $\rho \in [\rho_U^L, \overline{\rho}]$ *and* $\rho \in [\rho_U^H, \overline{\rho}]$ *opt for fixed fee plan.*

(b) If $V^L(\overline{\rho}) - T < U^L(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$ *or* $V^H(\overline{\rho}) - T < U^H(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$*, then gamers with* $\rho \in [\rho_U^L, \overline{\rho}]$ *and* $\rho \in [\rho_U^H, \overline{\rho}]$ *opt for usage based plan.*

(c) If $V^L(\underline{\rho}) - T < U^L(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$ *and* $V^L(\overline{\rho}) - T \geq U^L(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$*, then gamers of type* $[\rho_U^L, \rho_S^L]$ *will continue with usage based plan whereas the gamers of type* $\rho \in [\rho_S^L, \overline{\rho}]$ *switch to the fixed fee plan.*



If $V^H(\underline{\rho}) - T < U^H(q(\underline{\rho}), \underline{\rho}) - \tau(\underline{\rho})$ and $V^H(\overline{\rho}) - T \geq U^H(q(\overline{\rho}), \overline{\rho}) - \tau(\overline{\rho})$, then gamers of type $[\rho_U^H, \rho_S^H]$ will continue with usage based plan whereas gamers with $\rho \in [\rho_S^H, \overline{\rho}]$ switch to the fixed fee plan.

(d) $\rho_U^H \geq \rho_U^L$

(e) $\rho_S^H \geq \rho_S^L$

[h]

Figure 1 represents the behavior of the gamers for the limiting cases of broadband non-uniformity. Proposition 2(d) states that for a very high values of broadband non-uniformity, we can expect cloud gaming adoption at higher values of gamer type. This result is expected as very high non-uniformity reduces the utility of the gamers to such an extent that the net utility essentially becomes less than zero, and therefore adoption occurs at higher values of gamer type. The significant take-away for cloud gaming providers is that the first adopters of their usage based pricing plans will be gamers who have a relatively stable broadband connection (low broadband non-uniformity).

Proposition 2(e) states that for very high values of broadband non-uniformity, we can expect a shift from usage based plan to fixed fee plan at higher values of gamer type. The result is expected as gamers who have to deal with high non-uniformity will find the fixed fee plan attractive only if their propensity to play games is high (high gamer type). This is because fixed fee plan entails a payment independent of usage and can lead to a reduction in net utility if the data rate of the broadband connection is non-uniform in nature. On the other hand, a gamer who has access to a steady broadband connection will opt for a fixed fee plan even if her gamer type is relatively low.

Propositions 2(d) and 2(e) taken together imply that for a high level of broadband non-uniformity we can expect both adoption of cloud gaming and the shift from usage based plan to fixed fee plan to occur at higher values of gamer type. Gamers with a relatively better quality broadband connection (low broadband non-uniformity) will not only adopt cloud gaming at lower values of gamer type but also shift to the fixed fee plan from the usage based plan earlier.

5 Optimal Usage Based Pricing Plan by Cloud Gaming Providers

In this section we determine the pricing plan offered by the cloud gaming providers which maximizes its profits. We first look at the scenario where the gaming provider offers a usage based plan only. We also assume that the transaction cost is in the form $c(q(\rho,\sigma))$. In the absence of any fixed fee plan, if $q^*(\rho,\sigma)$ is the optimal quantity of games consumed and $\tau(q^*(\rho,\sigma))$ is the price charged by the gaming provider, then the following proposition determines the optimal price-quantity combination.

Proposition 3. *We define ρ_U and σ_H as follows:*

$$q^*(\rho,\sigma) = 0 \ \forall \ \rho < \rho_U \tag{3}$$

$$q^*(\rho,\sigma) = 0 \ \forall \ \sigma > \sigma_H \tag{4}$$

ρ_U is the value of gamer type below which the quantity of games consumed, q is zero, irrespective of the value of σ. Similarly, σ_H is the value of broadband non-uniformity above which the quantity of games consumed q is zero, irrespective of the value of ρ. Using this definition of ρ_U and σ_H, optimal quantity of game consumed by a gamer of type ρ and having broadband

non-uniformity σ is calculated by solving the following unconstrained optimization problem:

$$\max_{q(.,.)} \quad \int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\bar{\sigma}}^{\sigma_H} [U(q(\rho,\sigma),\rho,\sigma) - c(q(\rho,\sigma))] g(\sigma) d\sigma d\rho +$$
$$G(\underline{\sigma}) \int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\rho_U}^{\rho} \int_{\bar{\sigma}}^{\sigma_H} [U_{12}(q(x,y),x,y).q_1(x,y)] dy dx d\rho +$$
$$G(\underline{\sigma}) \int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\rho_U}^{\rho} \int_{\bar{\sigma}}^{\sigma_H} U_{23}(q(x,y),x,y) dy dx d\rho -$$
$$\int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\underline{\sigma}}^{\sigma_H} G(\sigma) \int_{\rho_U}^{\rho} [U_{12}(q(x,\sigma),x,\sigma).q_1(x,\sigma)] dx d\sigma d\rho -$$
$$\int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\underline{\sigma}}^{\sigma_H} G(\sigma) \int_{\rho_U}^{\rho} U_{23}(q(x,\sigma),x,\sigma) dx d\sigma d\rho$$

Optimal pricing plan for optimal quantity of game consumed ($q^*(\rho,\sigma)$) is defined by the following expression:

$$\tau(q^*(\rho,\sigma)) = U(q^*(\rho,\sigma),\rho,\sigma) - \int_{\rho_U}^{\rho} \int_{\sigma}^{\sigma_H} [U_{12}(q(x,y),x,y).q_1(x,y) + U_{23}(q(x,y),x,y)] dy dx \tag{5}$$

where $h(\rho)$ and $g(\sigma)$ charaterize the density functions of gamer type and broadband non-uniformity for different gamers. Individual rationality condition is satisfied in following ranges of ρ and σ: $\rho \in [\rho_U, \bar{\rho}]$ and $\sigma \in [\underline{\sigma}, \sigma_H]$.

A cloud gaming provider will offer cloud games at a price which is greater than or at least equal to the transaction cost that the it incurs in offering that service. Therefore, if the transaction cost is high for the cloud gaming provider, then the price charged is higher too which results in a decrease in the number of adopters.

6 Utility of Fixed Fee Plan for Cloud Gaming providers

In Section 4.1, we established some results to prove the willingness of gamers to opt for a fixed fee plan under certain conditions. In this section, we see the incentive of the cloud gaming provider in offering a fixed fee plan. In the absence of fixed fee, we define the optimal quantity of games consumed by a gamer with ρ and σ as $q^*(\rho,\sigma)$. In usage based plan, a cloud gaming provider incurs a transaction cost by monitoring quantity of games consumed per unit time and we denote this transaction cost as $c(q^*(\rho,\sigma))$. In this paper, we assume zero transaction cost for fixed fee plan. Based on this set of conditions, we establish the following proposition.

Proposition 4. *If the transaction cost of monitoring the usage of games is non-zero, then it* *is always profit improving for the cloud gaming provider to offer fixed fee plan along with usage based plan.*

Proof. In absence of any fixed fee plan, $q^*(\rho, \sigma)$ is the optimal quantity of games consumed by a gamer (characterized by ρ and σ) and corresponding cost incurred by the gamer is $\tau^*(\rho, \sigma)$. To see the impact of fixed fee plan for a cloud gaming provider, we consider two situations: one with $q^*(\overline{\rho}, \underline{\sigma}) = 0$ and second with $q^*(\overline{\rho}, \underline{\sigma}) > 0$.

In the first situation, as $q^*(\overline{\rho}, \underline{\sigma}) = 0$, Proposition 1 establishes that $q^*(\rho, \sigma) = 0 \; \forall \rho, \sigma$ and hence the profit of a cloud gaming provider is zero. Fixed fee of $T = V(\underline{\rho}, \overline{\sigma})$ strictly increases the profit of cloud gaming provider in this situation.

In the second situation, $q^*(\overline{\rho}, \underline{\sigma}) > 0$ which signifies the acceptance of the usage based plan by the gamers. Now, always a fixed fee T can be chosen such that $\tau^*(\overline{\rho}, \underline{\sigma}) - c(q^*(\overline{\rho}, \underline{\sigma})) < T < \tau^*(\overline{\rho}, \underline{\sigma})$, which increases the profit of the cloud gaming provider. From the perspective of a gamer, $V(\overline{\rho}, \underline{\sigma}) - T > U(q^*(\overline{\rho}, \underline{\sigma}), \overline{\rho}, \underline{\sigma}) - \tau^*(\overline{\rho}, \underline{\sigma})$. It indicates some gamers who have subscribed to usage based plan will now switch to fixed fee plan. □

7 Optimal Pricing Structure in the Presence of Fixed Fee

In the previous section, we determined the optimal pricing structure for a cloud gaming provider in the absence of fixed fee. In this section we extend our model to determine the optimal pricing structure in presence of both usage based and fixed fee. From Proposition 1 we know that it is not possible to find out a unique pair of values for ρ and σ for which a gamer will shift from usage based plan to fixed fee plan. This creates problem in determining optimal pricing structure as distinct gamer sets cannot be identified for fixed fee and usage based fee and hence a closed form expression to calculate the total profit is absent. In order to determine the optimal fixed fee that a cloud gaming provider should charge, we assume a gamer who has broadband non-uniformity of σ_M. Sundararajan [21] has shown that the optimal usage based pricing schedule in the presence of a fixed fee is independent of the value of the fixed fee. This property greatly reduces the problem of finding the optimal combination of fixed fee and usage based fee. The following proposition describes the solution to the problem.

Proposition 5. *For a gamer with broadband non-uniformity σ_M, if we assume that the gamer shifts to fixed fee plan from the usage based plan at a gamer type ρ_M, where $\rho_M \in [\rho_S^L, \rho_S^H]$, then the optimal combination of usage based fee and fixed fee can be determined as follows.*

(a) $U_1(q^*(\rho,\sigma_M),\rho,\sigma) = c_1(q^*(\rho,\sigma_M)) + \frac{1-F(\rho)}{f(\rho)} U_{12}(q^*(\rho,\sigma_M)$

(b) $\rho_S^{M*} = argmax_{\rho_S^M} \int_{\rho_U}^{\rho_S^M} [\tau^*(\rho,\sigma_M) - c(q^*(\rho,\sigma_M))] f(\rho) d\rho + [1 - F(\rho_S^M)]$
$[V(\rho_S^M,\sigma_M) - U(q^*(\rho_S^{M*},\sigma_M),\rho_S^M,\sigma_{M*}) + \tau^*(\rho_S^{M*},\sigma_M)]$

(c) $T^* = V(\rho_S^{M*},\sigma_M) - U(q^*(\rho_S^{M*},\sigma_M),\rho_S^{M*},\sigma_M) + \tau^*(\rho_S^{M*},\sigma_M)$ *Here $q^*(\rho,\sigma)$ is the optimal quantity and $\tau^*(\rho,\sigma)$ is the corresponding price (Proposition 3).*

Proposition 3 and Proposition 5 together helps us determine the optimal pricing schedule that a cloud gaming provider should publish for a gamer with a broadband non-uniformity σ_M. Therefore, if the gaming provider can segment the gamer market into different categories of broadband non-uniformity, for example, high, medium and low, it will be possible for the cloud gaming provider to draw up an optimal plan for each of the different market segments. Furthermore, from Proposition 4 we can infer that the introduction of a fixed fee is beneficial for the gamer. As the optimal fixed fee does not affect the optimal usage based fee, therefore, the consumer surplus can only increase if a fixed fee is introduced. We have already seen the benefit of introducing a fixed fee in the case of cloud gmaing providers. Therefore, the introduction of a fixed fee is beneficial for both cloud gaming providers as well as gamers.

8 Example

In this section, we take an example and first derive the results for optimal usage and price in the absence of fixed fee. We assume the utility function of a gamer as:

$$U = q(\rho-\sigma) - \frac{1}{2}q^2 \; for \; q \leq (\rho-\sigma)$$
$$U = \frac{(\rho-\sigma)^2}{2} \; for \; q \geq (\rho-\sigma)$$

We also assume $c(q) = cq$; $h(\rho) = \alpha e^{-\alpha\rho}$; $g(\sigma) = \beta e^{-\beta\sigma}$ Therefore, $U_1 = \rho - \sigma - q$; $U_{12} = 1$; $U_2 = q$; $U_{23} = 0$; $G(y) = 1 - e^{-\beta y}$

The value of optimal usage q^* is determined by using Proposition 3. The unconstrained optimization problem for calculating the optimal quantity can be expressed in the form $\int_{\rho_U}^{\bar{\rho}} \int_{\underline{\sigma}}^{\sigma_H} I dy d\rho$ where integrand I is defined as:

$$\begin{aligned} I = U(q(\rho, y), \rho, y)g(y)h(\rho) - c(q(\rho, y))g(y)h(\rho) - \\ h(\rho) \int_{\rho_U}^{\rho} [U_{12}(q(x, y), x, y)q_1(x, y) - U_{23}(q(x, y), x, y)][G(y) - G(\underline{\sigma})]dx \end{aligned} \quad (6)$$

To determine q^* we differentiate integrand I with respect to q and equate it to 0 using the first order condition.

$$\frac{dI}{dq} = ((\rho - \sigma) - q - c)\alpha\beta e^{(-(\alpha\rho + \beta y))} - \alpha e^{-\alpha\rho} e^{-\beta\underline{\sigma}} - e^{-\beta y} \frac{d}{dq} \int_{\rho_U}^{\rho} q_1(x, y)dx$$

Simplifying this equation we get,

$$q^* = \rho - \sigma - c - \frac{(e^{-\beta\underline{\sigma}} - e^{-\beta\sigma})}{\beta e^{-\beta\sigma}}$$

Using the value of q^* to calculate τ^* from Proposition 3,

$$\tau^*(\rho, \sigma) = U(q^*(\rho, \sigma), \rho, \sigma) - \int_{\rho_U}^{\rho} \int_{\sigma}^{\sigma_H} [U_{12}(q(x, y), x, y).q_1(x, y) + U_{23}(q(x, y), x, y)]dydx$$

For the utility function in the example, $U_{12} = 1$ and $U_{23} = 0$. Therefore,we get

$$\begin{aligned} \tau^*(\rho, \sigma) = U(q^*(\rho, \sigma), \rho, \sigma) - \int_{\sigma}^{\sigma_H} q(\rho, y)dy \\ \tau^*(\rho, \sigma) = U(q^*(\rho, \sigma), \rho, \sigma) - (\rho - c + \frac{1}{\beta})\sigma_H - (\rho - c + \frac{1}{\beta})\sigma + \frac{1}{2}(\sigma_H^2 - \sigma^2) - \\ e^{\beta\underline{\sigma}}(e^{\beta\sigma_H} - e^{\beta\sigma}/) \end{aligned} \quad (7)$$

To calculate the optimal fixed fee, we have to assume a value for σ and then calculate the value of ρ_S^{M*}. From Proposition 5, the value of ρ_S^{M*} can be determined as the roots of the equation,

$$\begin{aligned} (\rho_S^M)^2(\frac{1}{\alpha} - \frac{1}{2}) + (\rho_S^M)(q^* + \sigma_M + c + \frac{1}{\alpha} - \frac{\sigma_M}{\alpha}) + \\ (-q^*\sigma_M - \frac{1}{2}q^{*2} + \frac{\rho_U^2}{2} - \sigma_M\rho_U - c\rho_U - \frac{\rho_U}{\alpha} - k - cq^*) = 0 \end{aligned}$$

We can then use the value of ρ_S^{M*} to find out the optimal value of T^*. The derivation is algebraically cumbersome, so we omit the rest of it.

9 Concluding Discussions

This work is an attempt to address the issue of pricing cloud gaming offerings. The pricing decision is modeled by introducing two important factors - gamer type or propensity of a gamer to engage in cloud gaming, and non-uniformity of the gamer's broadband connection - which affect the utility that a gamer derives from cloud gaming offerings. We first determine the conditions under which gamers opt for different pricing plans, viz., usage based plan and fixed fee plan. From our analysis we can expect that for gamers who suffer from high broadband non-uniformity, both adoption of cloud gaming as well as a shift from usage based plan to fixed fee plan will be observed for gamers who have a high propensity to engage in gaming. We also show that the introduction of a fixed fee plan improves profits for the cloud gaming providers. Therefore, for cloud gaming offerings, for which the marginal cost is zero, and where there is a transaction cost involved in administering usage based fee, only usage based pricing plans is not optimal. Interestingly, the pricing plans of companies which offer computing infrastructure as a service (IaaS) like Amazon, Rackspace exhibit plans which are in accordance with our findings in this paper. Neither of the two IaaS providers offer pure usage based plans. The pricing plans of these two companies are primarily of two types: a pure usage based plan and a combined fee plan. The combined fee pricing model is characterized by a combination of a usage based component and a fixed fee component. The usage based fee of a combined fee plan is generally much lower compared to the fee of a pure usage based plan, and therefore this lends to higher usage of computing resources by the customers.

This work provides a guideline for cloud gaming providers on pricing their offerings to gamers. It provides an understanding of pricing related issues for both incumbent and new entrants in this field, and offers a mechanism for designing optimal tariff structures for gamers. The fact that we consider both gamer type and the quality of broadband connectivity makes our contribution more relevant as cloud gaming providers who do not take into account broadband non-uniformity will find it difficult to offer relevant pricing plans for gamers. Our findings on the effect of broadband non-uniformity on the fraction of adopters of usage based and fixed fee plan will help cloud gaming providers segment gamers based on the broadband non-uniformity, and target them with appropriate offers. This work also highlights the importance of transaction costs for cloud gaming providers. As it becomes possible for these companies to reduce the transaction costs, they should also adjust the usage based pricing to increase the fraction of adopters.

In future, we plan to analyze the effect of combined fee plan on the decision of gamers. In some cases of combined fee plans of IaaS providers, the availability of computing resources is guaranteed (Amazon EC2) [http://aws.amazon.com/ec2/purchasing-options/]; for some other cases these contracts are bundled with a specialized consulting service (Rackspace) [http://www.rackspace.com/cloud/cloud_hosting_products/servers/pricing]. These special bundle offers are generally given as incentives so that users shift to the combined fee model from the usage based pricing plan. Analogous incentives for cloud gaming could be newsletters on the latest games, exclusive reviews of latest releases, extra points or credits that can be used to play games, etc. In future, we plan to incorporate these special incentives in our model to attempt a deeper analysis of pricing of cloud gaming. In this paper, we have assumed the set of all games offered by the cloud gaming provider to be homogeneous. This may not hold in the real world as gamers may have different valuations for the various games offered by a gaming company. It is possible to adapt our model to accommodate a set of heterogeneous games by giving a weightage to each game. So, the different games offered by the gaming company can be arranged in a quality scale and the quantity consumed for a game which ranks higher in the scale is considered to be more than a lower ranked game. In our future work, however, we plan to modify the model to account for variations in the games offered by a cloud gaming company.

References

[1] Lg electronics & gaikai announce partnership to deliver digital tv cloud gaming. Press Release, January 2012.

[2] Onlive cloud gaming demonstrated on lg smart tvs with google tv. Press Release, June 2012.

[3] Samsung and gaikai to stream aaa-quality video games directly to samsung smart tvs without the need of additional hardware. Press Release, June 2012.

[4] C Arthur. How tablets are eating the pc's future but might save the desktop computer, April 2012.

[5] A Bagh and H.K Bhargava. How to price discriminate when tariff size matters. *Marketing Science*, Articles in Advance:1 – 16, 2012.

[6] M.T Clements and H Ohashi. Indirect network effects and the product cycle: Video games in the u.s., 1994-2002. *The Journal of Industrial Economics*, 53(4):515–542, 2005.

[7] David Crookes. Consoles in the cloud: Could games hardware soon be a thing of the past?, June 2012.

[8] Timothy Derdenger. *Vertical Integration and Two-Sided Market Pricing: Evidence from the Video Game Industry*. PhD thesis, University of Southern California, 2008.

[9] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press (Cambridge, MA), 1991.

[10] S Gallagher and S. H Park. Innovation and competition in standard-based industries: A historical analysis of the u.s. home video game market. *IEEE Transactions on Engineering Management*, 49(1):67 – 82, 2002.

[11] R. Gil and F. Warzynski. Vertical integration, exclusivity and game sales performance in the u.s. video game industry. *MRPA, University of Munich Library*, MPRA Paper No. 21049:1–36, 2009.

[12] S Jain and P.K Kannan. Pricing of infrmation products on online servers: Issues, models and analysis. *Management Science*, 48 (9):1123 – 1142, 2002.

[13] Hong Ju Liu. Dynamics of pricing in the video game console market: Skimming or penetration? *Journal of Marketing Research*, 47:428–443, 2010.

[14] E. Maskin and J. Riley. Monopoly with incomplete information. *Rand Journal of Economics*, 15:171–196, 1984.

[15] H Nair. Intertemporal price discrimination with forward-looking consumers: Application to the us market for console video-games. *Quantitative Marketing & Economics*, 5(3):239–292, 2007.

[16] Ojala and Tyrvainen. Developing cloud business models: A case study on cloud gaming. *IEEE Software*, 28(4):42 – 47, 2011.

[17] V Shankar and B.L Bayus. Network effects and competition: An empirical analysis of the home video game industry. *Strategic Management Journal*, 24:375 – 384, 2003.

[18] C Shapiro and H.R Varian. *Information Rules.* Harvard Business Review Press (Boston, Massachusetts), 1999.

[19] I Sherr and D Wakabayashi. Sony rejects web-based playstation console, May 2012.

[20] K. Stuart. Sony buys cloud gaming company gaikai for $380m, July 2012.

[21] A. Sundararajan. Nonlinear pricing of information goods. *Management Science*, 50:12:1660–1673, 2004.

[22] Varian. Pricing information goods. In *Proc. Scholarship New Inform. Environment Sympos. Harvard Law School, Cambridge, MA*, 1995.

[23] H. L Vogel. *Entertainment Industry Economics.* Cambridge University Press, 2011.

[24] D Williams. Structure and competition in the u.s. home video game industry. *The International Journal on Media Management*, 4(1):41 – 54, 2002.

[25] R. Wilson. *Nonlinear Pricing.* Oxford University Press (New York), 1993.

[26] W Yin-Poole. Microsoft: Internet bandwidth issues make cloud gaming a ”challenge”, June 2012.

Appendix

1 Selection Problem of a Gamer

Lemma 1. *If $q(\rho, \sigma)$ is the quantity of games consumed by a gamer who has opted for an incentive compatibility plan, then:*

(a) $q_1(\rho, \sigma) \geqslant 0$.

(b) $q_2(\rho, \sigma) \leqslant 0$.

Proof of part (a). Let us assume $q_1(\rho, \sigma) < 0$. Therefore $q(\rho, \sigma) > q(\rho + \epsilon, \sigma)$ for $\epsilon > 0$. As the plan is incentive compatible, from condition [IC]

$$U(q(\rho, \sigma), \rho, \sigma) - \tau(\rho, \sigma) \geqslant U(q(\rho + \epsilon, \sigma), \rho, \sigma) - \tau(\rho + \epsilon, \sigma) \quad (1)$$

From condition [IC] for a gamer with type $\rho + \epsilon$ and broadband non-uniformity σ,

$$U(q(\rho + \epsilon, \sigma), \rho + \epsilon, \sigma) - \tau(\rho + \epsilon, \sigma) \geqslant U(q(\rho, \sigma), \rho + \epsilon, \sigma) - \tau(\rho, \sigma) \quad (2)$$

Adding up Inequalities 1 and 2 yields the following inequality:

$$U(q(\rho + \epsilon, \sigma), \rho + \epsilon, \sigma) - U(q(\rho, \sigma), \rho + \epsilon, \sigma) \geqslant U(q(\rho + \epsilon, \sigma), \rho, \sigma) - U(q(\rho, \sigma), \rho, \sigma) \quad (3)$$

Inequation 3 implies that $U_2(q(\rho, \sigma), \rho, \sigma) \leqslant 0$ as $q(\rho, \sigma) > q(\rho + \epsilon, \sigma)$ - a contradiction of utility function property, which completes the proof. □

Proof of part (b). Let us assume $q_2(\rho, \sigma) > 0$, Therefore $q(\rho, \sigma + \epsilon) > q(\rho, \sigma)$ for $\epsilon > 0$. Using condition [IC],

$$U(q(\rho, \sigma), \rho, \sigma) - \tau(\rho, \sigma) \geqslant U(q(\rho, \sigma + \epsilon), \rho, \sigma) - \tau(\rho, \sigma + \epsilon) \quad (4)$$

Following condition [IC] for a gamer with type ρ and broadband non-uniformity $\sigma + \epsilon$,

$$U(q(\rho, \sigma + \epsilon), \rho, \sigma + \epsilon) - \tau(\rho, \sigma + \epsilon) \geqslant U(q(\rho, \sigma), \rho, \sigma + \epsilon) - \tau(\rho, \sigma) \quad (5)$$

Adding up Inequalities 4 and 5 yields the following inequation:

$$U(q(\rho, \sigma + \epsilon), \rho, \sigma + \epsilon) - U(q(\rho, \sigma), \rho, \sigma + \epsilon) \geqslant U(q(\rho, \sigma + \epsilon), \rho, \sigma) - U(q(\rho, \sigma), \rho, \sigma) \quad (6)$$

Inequation 6 implies that $U_3(q(\rho, \sigma), \rho, \sigma) \geqslant 0$ as $q(\rho, \sigma + \epsilon) > q(\rho, \sigma)$ - a contradiction of utility function property, which completes the proof. □

1.1 Selection of usage based pricing Plan

Lemma 2. *If preference function of a gamer is defined as* $F(q(\rho, \sigma), \rho, \sigma) = U(q(\rho, \sigma), \rho, \sigma) - \tau(\rho, \sigma)$, *then:*

(a) $F(q(\rho, \sigma), \rho, \sigma)$ *is strictly increasing in* ρ.

(b) $F(q(\rho, \sigma), \rho, \sigma)$ *is non-increasing in* σ.

Proof. Applying first order condition to satisfy condition [IC],

$$U_1(q(\rho,\sigma),\rho,\sigma).q_1(\rho,\sigma) - \tau_1(\rho,\sigma) = 0 \tag{7}$$

$$U_1(q(\rho,\sigma),\rho,\sigma).q_2(\rho,\sigma) - \tau_2(\rho,\sigma) = 0 \tag{8}$$

Differentiating $F(q(\rho,\sigma),\rho,\sigma)$ with respect to ρ yields:

$$F_2(q(\rho,\sigma),\rho,\sigma) = U_1(q(\rho,\sigma),\rho,\sigma).q_1(\rho,\sigma) + U_2(q(\rho,\sigma),\rho,\sigma) - \tau_1(\rho,\sigma) \tag{9}$$

Using the results obtained in Equation 7, Equation 9 results into:

$$F_2(q(\rho,\sigma),\rho,\sigma) = U_2(q(\rho,\sigma),\rho,\sigma) \tag{10}$$

As $U_2(q(\rho,\sigma),\rho,\sigma) > 0$, $F_2(q(\rho,\sigma),\rho,\sigma) > 0$

To prove part (b), differentiating $F(q(\rho,\sigma),\rho,\sigma)$ with respect to σ yields:

$$F_3(q(\rho,\sigma),\rho,\sigma) = U_1(q(\rho,\sigma),\rho,\sigma).q_2(\rho,\sigma) + U_3(q(\rho,\sigma),\rho,\sigma) - \tau_2(\rho,\sigma) \tag{11}$$

Using the results obtained in Equation 8, Equation 10 results into:

$$F_3(q(\rho,\sigma),\rho,\sigma) = U_3(q(\rho,\sigma),\rho,\sigma) \tag{12}$$

As $U_3(q(\rho,\sigma),\rho,\sigma) \leqslant 0$, it shows that $F_3(q(\rho,\sigma),\rho,\sigma) \leqslant 0$

□

Lemma 3. *For limiting cases of user variability, if preference functions are defined as $F^L(q(\rho),\rho) = U^L(q(\rho),\rho) - \tau(\rho)$ and $F^H(q(\rho),\rho) = U^H(q(\rho),\rho) - \tau(\rho)$, then*

(a) $F^L(q(\rho),\rho)$ is strictly increasing in ρ.

(b) $F^H(q(\rho),\rho)$ is strictly increasing in ρ.

Proof. Applying first order condition to satisfy condition [IC],

$$U_1^L(q(\rho),\rho).q_1(\rho) - \tau_1(\rho) = 0 \tag{13}$$

Differentiating $F^L(q(\rho),\rho)$ with respect to ρ yields:

$$F_2^L(q(\rho),\rho) = U_1^L(q(\rho),\rho).q_1(\rho) + U_2^L(q(\rho),\rho) - \tau_1(\rho) \tag{14}$$

Using Equation 13, Equation 14 can be rewritten as:

$$F_2^L(q(\rho),\rho) = U_2^L(q(\rho),\rho) \tag{15}$$

As $U_2^L(q(\rho),\rho) > 0$, it shows that $F_2^L(q(\rho),\rho) > 0$ □

Part (b) can be established analogously.

1.2 Introduction of fixed fee pricing plan

Lemma 4. *If the fixed fee surplus is defined as $X(q(\rho,\sigma),\rho,\sigma) = V(\rho,\sigma) - U(q(\rho,\sigma),\rho,\sigma) + \tau(\rho,\sigma)$, then*

(a) $X(q(\rho,\sigma),\rho,\sigma)$ is strictly increasing in ρ

(b) $X(q(\rho,\sigma),\rho,\sigma)$ is strictly decreasing in σ

Proof of Part (a). Differentiating $X(q(\rho,\sigma),\rho,\sigma)$ w.r.t ρ

$$X_2(q(\rho,\sigma),\rho,\sigma) = V_1(\rho,\sigma) - U_1(q(\rho,\sigma),\rho,\sigma).q_1(\rho,\sigma) - U_2(q(\rho,\sigma),\rho,\sigma) + \tau_1(\rho,\sigma) \tag{16}$$

From Equation 7, Equation 16 simplifies to:

$$X_2(q(\rho,\sigma),\rho,\sigma) = V_1(\rho,\sigma) - U_2(q(\rho,\sigma),\rho,\sigma) \tag{17}$$

$$X_2(q(\rho,\sigma),\rho,\sigma) = \lim_{q\to\infty} U_2(q(\rho,\sigma),\rho,\sigma) - U_2(q(\rho,\sigma),\rho,\sigma) \tag{18}$$

As $U_{12}(q(\rho,\sigma),\rho,\sigma) > 0$, it proves that $X_2(q(\rho,\sigma),\rho,\sigma) > 0$ □

Proof of Part (b). Differentiating $X(q(\rho,\sigma),\rho,\sigma)$ w.r.t σ

$$X_3(q(\rho,\sigma),\rho,\sigma) = V_2(\rho,\sigma) - U_1(q(\rho,\sigma),\rho,\sigma).q_2(\rho,\sigma) - U_3(q(\rho,\sigma),\rho,\sigma) + \tau_2(\rho,\sigma) \tag{19}$$

From Equation 8, Equation 19 simplifies to:

$$X_3(q(\rho,\sigma),\rho,\sigma) = V_2(\rho,\sigma) - U_3(q(\rho,\sigma),\rho,\sigma) \tag{20}$$

$$X_3(q(\rho,\sigma),\rho,\sigma) = \lim_{q\to\infty} U_3(q(\rho,\sigma),\rho,\sigma) - U_3(q(\rho,\sigma),\rho,\sigma) \tag{21}$$

As $U_{13}(q(\rho,\sigma),\rho,\sigma) < 0$, $X_3(q(\rho,\sigma),\rho,\sigma) < 0$ □

1.2.1 Limiting Cases

Lemma 5. *For limiting cases of broadband non-uniformity, if fixed fee surpluses are defined as*
$X^L(q(\rho),\rho) = V^L(\rho) - U^L(q(\rho),\rho) + \tau(\rho)$ *and*
$X^H(q(\rho),\rho) = V^H(\rho) - U^H(q(\rho),\rho) + \tau(\rho)$, *then*

(a) $X^L(q(\rho),\rho)$ *is strictly increasing in* ρ

(b) $X^H(q(\rho),\rho)$ *is strictly increasing in* ρ

Proof of Part (a). Differentiating $X^L(q(\rho),\rho)$ w.r.t ρ

$$X_2^L(q(\rho),\rho) = V_1^L(\rho) - U_1^L(q(\rho),\rho).q_1(\rho) - U_2^L(q(\rho),\rho) + \tau_1(\rho) \tag{22}$$

From Equation 13, Equation 22 simplifies to:

$$X_2^L(q(\rho),\rho) = V_1^L(\rho) - U_2^L(q(\rho),\rho) \tag{23}$$

$$X_2^L(q(\rho),\rho) = \lim_{q\to\infty} U_2^L(q(\rho),\rho) - U_2(q(\rho),\rho) \tag{24}$$

As $U_{12}^L(q(\rho),\rho) > 0$, it proves that $X_2^L(q(\rho),\rho) > 0$ □

Part (b) can be established analogously.

Proposition 2. *Given an option to choose between two plans: usage based and fixed fee, a gamer's choice will follow the conditions given below:*

(a) If $V^L(\underline{\rho}) - T \geqslant U^L(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$ *or* $V^H(\underline{\rho}) - T \geqslant U^H(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$, *then all gamers opt for fixed fee plan.*

(b) If $V^L(\overline{\rho}) - T < U^L(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$ *or* $V^H(\overline{\rho}) - T < U^H(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$, *then all gamers opt for usage based plan.*

(c) If $V^L(\underline{\rho}) - T < U^L(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$ *and* $V^L(\overline{\rho}) - T \geqslant U^L(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$, *then gamers of type* $[\underline{\rho},\rho_S^L)$ *will continue with usage based plan whereas the gamers with* $\rho \in [\rho_S^L,\overline{\rho}]$ *switch to the fixed fee plan.*

If $V^H(\underline{\rho}) - T < U^H(q(\underline{\rho}),\underline{\rho}) - \tau(\underline{\rho})$ *and* $V^H(\overline{\rho}) - T \geqslant U^H(q(\overline{\rho}),\overline{\rho}) - \tau(\overline{\rho})$, *then gamers of type* $[\underline{\rho},\rho_S^H)$ *will continue with usage based plan whereas the gamers of type* $\rho \in [\rho_S^H,\overline{\rho}]$ *switch to the fixed fee plan.*

(d) $\rho_U^H \geqslant \rho_U^L$

(e) $\rho_S^H \geqslant \rho_S^L$

Revenue responses ρ_U^L *and* ρ_U^H *are defined as:*
$q^*(\rho) = 0\ \forall \rho < \rho_U^L$ *when* $\sigma \to 0$ *and* $q^*(\rho) = 0\ \forall \rho < \rho_U^H$ *when* $\sigma \to \infty$.
Revenue responses ρ_S^L *and* ρ_S^H *are defined as:*
$\rho_S^L = Min\{\rho : V^L(\rho) - U^L(q(\rho), \rho) + \tau(\rho) = T\}$ *and*
$\rho_S^H = Min\{\rho : V^H(\rho) - U^H(q(\rho), \rho) + \tau(\rho) = T\}$

Proof of Part (a). Using results found in Lemma 5, fixed fee surplus increases with increasing ρ for limiting conditions of σ. Hence if a gamer of type $\underline{\rho}$ adopts fixed fee plan, then all gamers (with any $\rho \geqslant \underline{\rho}$) will adopt fixed fee plan because of higher fixed fee surplus. This concludes the proof for part (a) of Proposition 2. □

Proof of Part (b). Using results from Lemma 5, if a gamer of type $\overline{\rho}$ opts for usage based plan, all gamers will opt for the same plan because of decreasing fixed fee surplus with decreasing ρ. This argument is valid for both limiting conditions of broadband non-uniformity. This concludes the proof. □

Proof of Part (c). From Lemma 5, $X^L(q(\rho), \rho)$ and $X^H(q(\rho), \rho)$ are increasing with $\rho \in [\underline{\rho}], \overline{\rho}$. As gamers of type $\underline{\rho}$ opt for usage based plan, there will be a particular ρ at which $X^L(q(\rho), \rho) = T$. Because ρ_S^L and ρ_S^H are the lowest such values of gamer type for which $X^L(q(\rho), \rho) = T$ and $X^H(q(\rho), \rho) = T$, all gamers with ρ greater than these will opt for fixed fee plan and the rest will opt for usage based plan. These concludes part(c) of Proposition 2. □

Proof of part (d). Proof by Contradiction: Lets assume that $\rho_U^H < \rho_U^L$.
From the definition of ρ_U^H,

$$U^H(q(\rho_U^H), \rho_U^H) - \tau(\rho_U^H) = 0 \tag{25}$$

Using the property of the utility functions for the limiting cases, e.g. $U^L(q(\rho), \rho) \geqslant U^H(q(\rho), \rho)$, we can write from Equation 25 that:

$$U^L(q(\rho_U^H), \rho_U^H) - \tau(\rho_U^H) \geqslant 0 \tag{26}$$

From the definition of ρ_U^L it follows that:

$$U^L(q(\rho_U^L), \rho_U^L) - \tau(\rho_U^L) = 0 \tag{27}$$

As $\rho_U^H < \rho_U^L$ and $F^L(q(\rho), \rho)$ is strictly increasing with ρ, Equations 26 and 27 contradicts each other. Hence by contradiction it is proved that $\rho_U^H \geqslant \rho_U^L$. □

Proof of part (e). Lets assume that $\rho_S^H < \rho_S^L$. From the property of ρ_S^L,

$$V^L(\rho_S^L) - T \geqslant U^L(q(\rho_S^L), \rho_S^L) - \tau(q(\rho_S^L)) \tag{28}$$

$$V^L(\rho_S^L) - U^L(q(\rho_S^L), \rho_S^L) \geqslant T - \tau(q(\rho_S^L)) \tag{29}$$

As $U_{13}(q(\rho, \sigma), \rho, \sigma) < 0$, Equation 29 can be expressed as:

$$V^H(\rho_S^L) - U^H(q(\rho_S^L), \rho_S^L) < T - \tau(q(\rho_S^L)) \tag{30}$$

As $\rho_S^H < \rho_S^L$,

$$V^H(\rho_S^H) - U^H(q(\rho_S^H), \rho_S^H) < T - \tau(q(\rho_S^H)) \tag{31}$$

Equation 31 contradicts the definition of ρ_S^H, hence $\rho_S^H \geqslant \rho_S^L$. □

2 Optimal Usage Based Pricing Plan by Cloud Gaming Providers

Proposition 3. *The optimal quantity ($q^*(\rho,\sigma)$) and corresponding price ($\tau(q^*(\rho,\sigma))$) is determined by the following set of expressions:*

$$q^*(\rho,\sigma) = 0 \; \forall \; \rho < \rho_U \tag{32}$$

$$q^*(\rho,\sigma) = 0 \; \forall \; \sigma > \sigma_H \tag{33}$$

ρ_U is the value of gamer type below which the quantity of games consumed, q is zero, irrespective of the value of σ. Similarly, σ_H is the value of broadband non-uniformity above which the quantity of games consumed q is zero, irrespective of the value of ρ.

Optimal quantity of games consumed is calculated by solving the following unconstrained optimization problem:

$$\begin{gathered}\max_{q(.,.)} \quad \int_{\rho_U}^{\bar\rho} h(\rho) \int_{\bar\sigma}^{\sigma_H} [U(q(\rho,\sigma),\rho,\sigma) - c(q(\rho,\sigma))]g(\sigma)d\sigma d\rho + \\ G(\underline{\sigma}) \int_{\rho_U}^{\bar\rho} h(\rho) \int_{\rho_U}^{\rho} \int_{\bar\sigma}^{\sigma_H} [U_{12}(q(x,y),x,y).q_1(x,y)]dydxd\rho + \\ G(\underline{\sigma}) \int_{\rho_U}^{\bar\rho} h(\rho) \int_{\rho_U}^{\rho} \int_{\bar\sigma}^{\sigma_H} U_{23}(q(x,y),x,y)dydxd\rho - \\ \int_{\rho_U}^{\bar\rho} h(\rho) \int_{\underline{\sigma}}^{\sigma_H} G(\sigma) \int_{\rho_U}^{\rho} [U_{12}(q(x,\sigma),x,\sigma).q_1(x,\sigma)]dxd\sigma d\rho - \\ \int_{\rho_U}^{\bar\rho} h(\rho) \int_{\underline{\sigma}}^{\sigma_H} G(\sigma) \int_{\rho_U}^{\rho} U_{23}(q(x,\sigma),x,\sigma)dxd\sigma d\rho\end{gathered}$$

Optimal pricing plan for optimal quantity of games consumed ($q^*(\rho,\sigma)$) is defined by the following expression:

$$\tau^*(\rho,\sigma) = U(q^*(\rho,\sigma),\rho,\sigma) - \int_{\rho_U}^{\rho} \int_{\sigma}^{\sigma_H} [U_{12}(q^*(x,y),x,y).q_1(x,y) + U_{23}(q^*(x,y),x,y)]dydx \tag{34}$$

Density functions of gamer type and broadband non-uniformity for different gamers are characterized as $h(\rho)$ and $g(\sigma)$. Individual rationality condition is satisfied in following ranges of ρ and σ: $\rho \in [\rho_U, \bar\rho]$ and $\sigma \in [\underline{\sigma}, \sigma_H]$.

Proof. **Optimality conditions for incentive compatibility:**
Incentive Compatibility [IC] condition is satisfied when ρ, σ are solutions to the following maximization problem:

$$\max_{x\in[\underline{\rho},\bar\rho]; y\in[\underline{\sigma},\bar\sigma]} U(q(x,y),\rho,\sigma) - \tau(x,y) \tag{35}$$

From the first order conditions:

$$U_1(q(x,y),\rho,\sigma)q_1(x,y) - \tau_1(x,y) = 0 \tag{36}$$

$$U_1(q(x,y),\rho,\sigma)q_2(x,y) - \tau_2(x,y) = 0 \tag{37}$$

Every gamer with $\rho \in [\underline{\rho},\bar\rho]$ and $\sigma \in [\underline{\sigma},\bar\sigma]$ will follow the above two expressions and it leads to the following set of equations:

$$U_1(q(\rho,\sigma),\rho,\sigma)q_1(\rho,\sigma) - \tau_1(\rho,\sigma) = 0 \tag{38}$$

$$U_1(q(\rho,\sigma),\rho,\sigma)q_2(\rho,\sigma) - \tau_2(\rho,\sigma) = 0 \tag{39}$$

From the second order conditions, $H_{xx} < 0$, $H_{yy} < 0$ and determinant of Hessian matrix should be non-negative. H_{xx} and H_{yy} are determined by differentiating equations 36 and 37 with respect to x and y.

$$H_{xx} = U_{11}(q(x,y),\rho,\sigma)(q_1(x,y))^2 + U_1(q(x,y),\rho,\sigma)q_{11}(x,y) - \tau_{11}(x,y) \tag{40}$$

$$H_{yy} = U_{11}(q(x,y),\rho,\sigma)(q_2(x,y))^2 + U_1(q(x,y),\rho,\sigma)q_{22}(x,y) - \tau_{22}(x,y) \tag{41}$$

$$H_{xy} = H_{yx} = U_{11}(q(x,y),\rho,\sigma)q_1(x,y)q_2(x,y) + U_1(q(x,y),\rho,\sigma)q_{12}(x,y) - \tau_{12}(x,y) \tag{42}$$

Substituting ρ and σ as solutions in Equation 40,

$$U_{11}(q(\rho,\sigma),\rho,\sigma)(q_1(\rho,\sigma))^2 + U_1(q(\rho,\sigma),\rho,\sigma)q_{11}(\rho,\sigma) - \tau_{11}(\rho,\sigma) < 0 \tag{43}$$

Differentiating Equation 38 with respect to ρ and substituting the expression of $\tau_{11}(\rho,\sigma)$ into Equation 43 gives:

$$-U_{12}(q(\rho,\sigma),\rho,\sigma)q_1(\rho,\sigma) < 0 \tag{44}$$

As $U_{12}(q(\rho,\sigma),\rho,\sigma) > 0$, Equation 44 gives $q_1(\rho,\sigma) > 0$, which is established in Lemma 1. Substituting ρ and σ as solutions in Equation 41,

$$U_{11}(q(\rho,\sigma),\rho,\sigma)(q_2(\rho,\sigma))^2 + U_1(q(\rho,\sigma),\rho,\sigma)q_{22}(\rho,\sigma) - \tau_{22}(\rho,\sigma) < 0 \tag{45}$$

Differentiating Equation 39 with respect to σ and substituting the expression of $\tau_{22}(\rho,\sigma)$ into Equation 45 gives:

$$-U_{13}(q(\rho,\sigma),\rho,\sigma)q_2(\rho,\sigma) < 0 \tag{46}$$

As $U_{13}(q(\rho,\sigma),\rho,\sigma) < 0$, Equation 45 gives $q_2(\rho,\sigma) < 0$, which is established in the Lemma 1.

We substitute ρ and σ while calculating the determinant of the Hessian matrix. $\tau_{12}(\rho,\sigma)$ and $\tau_{21}(\rho,\sigma)$ are calculated by differentiating Equation 38 with respect to σ and by differentiating Equation 39 with respect to ρ and corresponding expressions are substituted in Hessian matrix. The determinant of the Hessian matrix comes to zero. Equating $\tau_{12}(\rho,\sigma)$ and $\tau_{21}(\rho,\sigma)$, we get the following condition of optimality,

$$U_{12}(q(\rho,\sigma),\rho,\sigma)q_2(\rho,\sigma) = U_{13}(q(\rho,\sigma),\rho,\sigma)q_1(\rho,\sigma) \tag{47}$$

Redefining the objective function of cloud gaming provider:

Informational rent of a gamer with ρ and σ is defined as:

$$s(\rho,\sigma) = U(q(\rho,\sigma),\rho,\sigma) - \tau(\rho,\sigma) \tag{48}$$

Differentiating Equation 48 with respect to ρ,

$$s_1(\rho,\sigma) = U_1(q(\rho,\sigma),\rho,\sigma)q_1(\rho,\sigma) + U_2(q(\rho,\sigma),\rho,\sigma) - \tau_1(\rho,\sigma) \tag{49}$$

Using Equation 38, Equation 49 is rewritten as,

$$s_1(\rho,\sigma) = U_2(q(\rho,\sigma),\rho,\sigma) \tag{50}$$

Differentiating Equation 48 with respect to σ,

$$s_2(\rho,\sigma) = U_1(q(\rho,\sigma),\rho,\sigma)q_2(\rho,\sigma) + U_3(q(\rho,\sigma),\rho,\sigma) - \tau_2(\rho,\sigma) \tag{51}$$

Using Equation 39, Equation 51 is rewritten as,

$$s_2(\rho,\sigma) = U_3(q(\rho,\sigma),\rho,\sigma) \tag{52}$$

Differentiating Equation 50 with respect to σ and Equation 52 with respect to ρ yields:

$$\begin{aligned} s_{12}(\rho,\sigma) = s_{21}(\rho,\sigma) &= U_{12}(q(\rho,\sigma),\rho,\sigma)q_1(\rho,\sigma) + U_{23}(q(\rho,\sigma),\rho,\sigma) && (53)\\ &= U_{13}(q(\rho,\sigma),\rho,\sigma)q_2(\rho,\sigma) + U_{23}(q(\rho,\sigma),\rho,\sigma) && (54) \end{aligned}$$

The second equation follows from the optimality condition given in equation 47 We take the individual rationality [IR] satisfied for $\rho \in [\rho_U, \bar{\rho}]$ and $\sigma \in [\underline{\sigma}, \sigma_H]$. Using this condition,

$$s(\rho,\sigma) = \int_{\rho_U}^{\rho} \int_{\sigma}^{\sigma_H} [U_{12}(q(x,y),x,y)q_1(x,y) + U_{23}(q(x,y),x,y)]dydx \tag{55}$$

From Equation 48,

$$\tau(\rho,\sigma) = U(q(\rho,\sigma),\rho,\sigma) - \int_{\rho_U}^{\rho}\int_{\sigma}^{\sigma_H}[U_{12}(q(x,y),x,y)q_1(x,y) + U_{23}(q(x,y),x,y)]dydx \tag{56}$$

Objective function of cloud gaming provider:
Objective function of cloud gaming provider to maximize profit,

$$\max_{q(.,.),\tau(.,.)} \int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}[\tau(\rho,\sigma) - c(q(\rho,\sigma))]f(\rho,\sigma)d\rho d\sigma \tag{57}$$

Substituting the expression of $\tau(\rho,\sigma)$ from Equation 56,

$$\max_{q(.,.)} \int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}[U(q(\rho,\sigma),\rho,\sigma) - \int_{\rho_U}^{\rho}\int_{\sigma}^{\sigma_H}[U_{12}(q(x,y),x,y)q_1(x,y) + U_{23}(q(x,y),x,y)]dydx$$
$$-c(q(\rho,\sigma))]f(\rho,\sigma)d\rho d\sigma \tag{58}$$

Assuming independence of density functions, i.e. $f(\rho,\sigma) = h(\rho)g(\sigma)$, objective function becomes,

$$\max_{q(.,.)} \int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}[U(q(\rho,\sigma),\rho,\sigma) - \int_{\rho_U}^{\rho}\int_{\sigma}^{\sigma_H}[U_{12}(q(x,y),x,y)q_1(x,y) + U_{23}(q(x,y),x,y)]dydx$$
$$-c(q(\rho,\sigma))]h(\rho)g(\sigma)d\rho d\sigma \tag{59}$$
$$= \max_{q(.,.)} \int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}[U(q(\rho,\sigma),\rho,\sigma) - c(q(\rho,\sigma))]h(\rho)g(\sigma)d\rho d\sigma - \int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}E(\rho,\sigma)h(\rho)g(\sigma)d\rho d\sigma \tag{60}$$

with $E(\rho,\sigma)$ is defined as:

$$E(\rho,\sigma) = \int_{\rho_U}^{\rho}\int_{\sigma}^{\sigma_H}[U_{12}(q(x,y),x,y)q_1(x,y) + U_{23}(q(x,y),x,y)]dydx \tag{61}$$

Hence,

$$\frac{dE(\rho,\sigma)}{d\sigma}.d\sigma = dE(\rho,\sigma) = \frac{d}{d\sigma}[\int_{\rho_U}^{\rho}\int_{\sigma}^{\sigma_H}[U_{12}(q(x,y),x,y)q_1(x,y) + U_{23}(q(x,y),x,y)]dydx]d\sigma$$
$$= [-\int_{\rho_U}^{\rho}[U_{12}(q(x,\sigma),x,\sigma)q_1(x,\sigma)dx - \int_{\rho_U}^{\rho}U_{23}(q(x,\sigma),x,\sigma)]dx]d\sigma \tag{62}$$

Expanding the second part of the integral defined in Equation 60 and using integration by parts, $\int udv = uv - \int vdu$, with $u = E(\rho,\sigma)$ and $v = G(\sigma)$,

$$\int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}E(\rho,\sigma)h(\rho)g(\sigma)d\rho d\sigma = \int_{\rho_U}^{\bar{\rho}}\{[E(\rho,\sigma).G(\sigma)]_{\underline{\sigma}}^{\sigma_H} - \int_{\underline{\sigma}}^{\sigma_H}G(\sigma)dE(\rho,\sigma)\}h(\rho)d\rho$$
$$= \int_{\rho_U}^{\bar{\rho}}\{[-E(\rho,\underline{\sigma}).G(\underline{\sigma})] - \int_{\underline{\sigma}}^{\sigma_H}G(\sigma)dE(\rho,\sigma)\}h(\rho)d\rho \tag{63}$$

Substituting expression of $dE(\rho,\sigma)$ from Equation 62, Equation 63 can be rewritten as:

$$\int_{\rho_U}^{\bar{\rho}} -E(\rho,\underline{\sigma}).G(\underline{\sigma})h(\rho)d\rho -$$
$$\int_{\rho_U}^{\bar{\rho}}\int_{\underline{\sigma}}^{\sigma_H}G(\sigma)[-\int_{\rho_U}^{\rho}U_{12}(q(x,\sigma),x,\sigma)q_1(x,\sigma)dx - \int_{\rho_U}^{\rho}U_{23}(q(x,\sigma),x,\sigma)dx]d\sigma h(\rho)d(\rho) \tag{64}$$

$$= \int_{\rho_U}^{\bar{\rho}} -E(\rho,\underline{\sigma}).G(\underline{\sigma})h(\rho)d\rho + \int_{\rho_U}^{\bar{\rho}}h(\rho)\int_{\underline{\sigma}}^{\sigma_H}G(\sigma)\int_{\rho_U}^{\rho}U_{12}(q(x,\sigma),x,\sigma)q_1(x,\sigma)dxd\sigma d\rho$$
$$+ \int_{\rho_U}^{\bar{\rho}}h(\rho)\int_{\underline{\sigma}}^{\sigma_H}G(\sigma)\int_{\rho_U}^{\rho}U_{23}(q(x,\sigma),x,\sigma)dxd\sigma d\rho \tag{65}$$

□

Inserting the expression of integral into Equation 60, objective function becomes:

$$\max_{q(.,.)} \int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\bar{\sigma}}^{\sigma_H} [U(q(\rho,\sigma),\rho,\sigma) - c(q(\rho,\sigma))]g(\sigma)d\sigma d\rho +$$
$$G(\underline{\sigma}) \int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\rho_U}^{\rho} \int_{\bar{\sigma}}^{\sigma_H} [U_{12}(q(x,y),x,y).q_1(x,y)]dydxd\rho +$$
$$G(\underline{\sigma}) \int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\rho_U}^{\rho} \int_{\bar{\sigma}}^{\sigma_H} U_{23}(q(x,y),x,y)dydxd\rho -$$
$$\int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\underline{\sigma}}^{\sigma_H} G(\sigma) \int_{\rho_U}^{\rho} [U_{12}(q(x,\sigma),x,\sigma).q_1(x,\sigma)]dxd\sigma d\rho -$$
$$\int_{\rho_U}^{\bar{\rho}} h(\rho) \int_{\underline{\sigma}}^{\sigma_H} G(\sigma) \int_{\rho_U}^{\rho} U_{23}(q(x,\sigma),x,\sigma)dxd\sigma d\rho \tag{66}$$

3 Optimal Pricing Structure in the Presence of Fixed Fee

Proposition 5. *For a gamer with broadband non-uniformity σ_M, if we assume that the gamer shifts to fixed fee plan from the usage based plan at a gamer type ρ_M, where $\rho_M \in [\rho_S^L, \rho_S^H]$, then the optimal combination of usage based fee and fixed fee can be determined as follows.*

(a) $U_1(q^*(\rho,\sigma_M),\rho,\sigma) = c_1(q^*(\rho,\sigma_M)) + \frac{1-F(\rho)}{f(\rho)} U_{12}(q^*(\rho,\sigma_M)$

(b) $\rho_S^{M*} = argmax_{\rho_S^M} \int_{\rho_U}^{\rho_S^M} [\tau^*(\rho,\sigma_M) - c(q^*(\rho,\sigma_M))]f(\rho)d\rho + [1 - F(\rho_S^M)]$
$[V(\rho_S^M,\sigma_M) - U(q(\rho_S^M,\sigma_M),\rho_S^M,\sigma_M) + \tau(\rho_S^M,\sigma_M)]$

(c) $T^* = V(\rho_S^{M*},\sigma_M) - U(q^*(\rho_S^{M*},\sigma_M),\rho_S^{M*},\sigma_M) + \tau^*(\rho_S^{M*},\sigma_M)$ *Here $q^*(\rho,\sigma)$ is the optimal quantity and $\tau(q^*(\rho,\sigma))$ is the corresponding price (Proposition 3).*

Proof. Informational rent of gamers with ρ and σ_M is defined as:

$$s(\rho,\sigma_M) = U(q(\rho,\sigma_M),\rho,\sigma_M) - \tau(\rho,\sigma_M) \tag{67}$$

Differentiating with respect to ρ yields

$$s_1(\rho,\sigma_M) = U_1(q(\rho,\sigma_M),\rho,\sigma_M))q_1(\rho,\sigma_M) + U_2(q(\rho,\sigma_M),\rho,\sigma_M)) - \tau_1(\rho,\sigma_M) \tag{68}$$

Using Equation 38, Equation 68 can be written as,

$$s_1(\rho,\sigma_M) = U_2(q(\rho,\sigma_M),\rho,\sigma_M) \tag{69}$$

We consider a type $\rho_U^M \in [\rho_U^L, \rho_U^H]$ at which the gamer adopts the usage based plan. Below ρ_U^M, the gamer would not be interested in adopting a usage based plan. Hence,

$$s(\rho,\sigma_M) = \int_{\rho_U^M}^{\rho} U_2(q(x,\sigma_M),x,\sigma_M)dx \tag{70}$$

Hence,

$$\tau(\rho,\sigma_M) = U(q(\rho,\sigma_M),\rho,\sigma_M) - \int_{\rho_U^M}^{\rho} U_2(q(x,\sigma_M),x,\sigma_M)dx \tag{71}$$

The objective function of the cloud gaming provider becomes

$$\max_{q(.)} \int_{\rho_U^M}^{\rho_S^M} [\tau(\rho,\sigma_M) - c(q(\rho,\sigma_M))]f(\rho)d\rho \tag{72}$$

$$= \max_{q(.)} \int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M) - \int_{\rho_U^M}^{\rho} U_2(q(x,\sigma_M),x,\sigma_M)dx - c(q(\rho,\sigma_M))]f(\rho)d\rho \tag{73}$$

$$= \max_{q(.)} \int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M) - c(q(\rho,\sigma_M))]f(\rho)d\rho - \int_{\rho_U^M}^{\rho_S^M} [\int_{\rho_U^M}^{\rho} U_2(q(x,\sigma_M),x,\sigma_M)dx]f(\rho)d\rho \quad (74)$$

Defining

$$G(\rho) = \int_{\rho_U^M}^{\rho} U_2(q(x,\sigma_M),x,\sigma_M)dx \quad (75)$$

Equation 74 can be written as,

$$= \max_{q(.)} \int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M)) - c(q(\rho,\sigma_M))]f(\rho)d\rho - \int_{\rho_U^M}^{\rho_S^M} G(\rho)f(\rho)d\rho \quad (76)$$

Using integration by parts, $\int udv = uv - \int vdu$ where $u = G(\rho), v = F(\rho), dv = f(\rho)d\rho$ we get,

$$\int_{\rho_U^M}^{\rho_S^M} G(\rho)f(\rho)d\rho = G(\rho_S^M).F(\rho_S^M) - \int_{\rho_U^M}^{\rho_S^M} F(\rho)dG(\rho)) \quad (77)$$

$$\frac{dG(\rho)}{d\rho}.d\rho = \frac{d}{d\rho}[\int_{\rho_U^M}^{\rho} U_2(q(x,\sigma_M),x,\sigma_M)dx]d\rho \quad (78)$$

Applying Leibnitz rule,

$$\frac{dG(\rho)}{d\rho}.d\rho = [U_2(q(\rho,\sigma_M),\rho,\sigma_M)]d\rho \quad (79)$$

$$\int_{\rho_U^M}^{\rho_S^M} G(\rho)f(\rho)d\rho = G(\rho_S^M).F(\rho_S^M) - \int_{\rho_U^M}^{\rho_S^M} F(\rho)U_2(q(\rho,\sigma_M),\rho,\sigma_M)d\rho \quad (80)$$

$$\int_{\rho_U^M}^{\rho_S^M} G(\rho)f(\rho)d\rho = \int_{\rho_U^M}^{\rho_S^M} U_2(q(x,\sigma_M),x,\sigma_M)F(\rho_S^M)dx - \int_{\rho_U^M}^{\rho_S^M} F(\rho).U_2(q(\rho,\sigma_M),\rho,\sigma_M)d\rho \quad (81)$$

$$\int_{\rho_U^M}^{\rho_S^M} G(\rho)f(\rho)d\rho = \int_{\rho_U^M}^{\rho_S^M} [F(\rho_S^M) - F(\rho)]U_2(q(\rho,\sigma_M),\rho,\sigma_M)d\rho \quad (82)$$

Therefore, the objective function becomes,

$$\max_{q(.)} \int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M) - c(q(\rho,\sigma_M))]f(\rho)d\rho - \int_{\rho_U^M}^{\rho_S^M} [F(\rho_S^M) - F(\rho)]U_2(q(\rho,\sigma_M),\rho,\sigma_M)d\rho \quad (83)$$

$$= \max_{q(.)} \int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M) - c(q(\rho,\sigma_M)) - \frac{F(\rho_S^M) - F(\rho)}{f(\rho)}.U_2(q(\rho,\sigma_M),\rho,\sigma_M)]f(\rho)d\rho \quad (84)$$

Now, if we introduce fixed fee T, the objective function becomes,

$$= \max_{q(.),T} \int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M)) - c(q(\rho,\sigma_M)) - \frac{F(\rho_S^M) - F(\rho)}{f(\rho)}.U_2(q(\rho,\sigma_M),\rho,\sigma_M)]f(\rho)d\rho + T(1 - F(\rho_S^M)) \quad (85)$$

subject to

$$\int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M)d\rho - [V(\rho_S^M,\sigma_M) - T] = 0 \quad (86)$$

The constrained optimization problem is represented as Lagrangian problem: $L(q(.), T, \lambda)$

$$\int_{\rho_U^M}^{\rho_S^M} [U(q(\rho,\sigma_M),\rho,\sigma_M)) - c(q(\rho,\sigma_M)) - \frac{F(\rho_S^M) - F(\rho)}{f(\rho)}.U_2(q(\rho,\sigma_M),\rho,\sigma_M)]f(\rho)d\rho + T(1 - F(\rho_S^M))+ \tag{87}$$

$$\lambda[\int_{\rho_U^M}^{\rho_S^M} U(q(\rho,\sigma_M),\rho,\sigma_M)d\rho - V(\rho_S^M,\sigma_M) + T]$$

FOC of the Lagrangian problem:

$$[\frac{\partial L}{\partial q} = 0] : [U_1(q(\rho,\sigma_M),\rho,\sigma_M)) - c_1(q(\rho,\sigma_M)) - [\frac{F(\rho_S^M) - F(\rho) - \lambda}{f(\rho)}].U_{12}(q(\rho,\sigma_M)]f(\rho) = 0 \tag{88}$$

$$[\frac{\partial L}{\partial T} = 0] : (1 - F(\rho_S * M)) + \lambda = 0 \tag{89}$$

$$[\frac{\partial L}{\partial \lambda} = 0] : \int_{\rho_U^M}^{\rho_S^M} U(q(\rho,\sigma_M),\rho,\sigma_M)d\rho - V(\rho_S^M,\sigma_M) + T = 0 \tag{90}$$

From Equation 89,

$$\lambda = F(\rho_S^M)) - 1 \tag{91}$$

Substituting the value of λ in Equation 88 gives

$$[U_1(q(\rho,\sigma_M),\rho,\sigma_M)) - c_1(q(\rho,\sigma_M)) - [\frac{1 - F(\rho)}{f(\rho)}].U_{12}(q(\rho,\sigma_M)]f(\rho) = 0 \tag{92}$$

or

$$U_1(q(\rho,\sigma_M),\rho,\sigma) = c_1(q(\rho,\sigma_M)) + \frac{1 - F(\rho)}{f(\rho)}U_{12}(q(\rho,\sigma_M) \tag{93}$$

This expression shows that optimal usage based quantity $q^*(\rho,\sigma_M)$ in independent of ρ_U^M,ρ_S^M and T.

Corresponding ρ_S^{M*} value is

$$\rho_S^{M*} = argmax_{\rho_S^M} \int_{\rho_U}^{\rho_S^M} [\tau^*(\rho,\sigma_M) - c(q^*(\rho,\sigma_M))]f(\rho)d\rho + [1 - F(\rho_S^M)][V(\rho_S^M,\sigma_M) - U(q(\rho_S^M,\sigma_M),\rho_S^M,\sigma_M) + \tau(\rho_S^M,\sigma_M)] \tag{94}$$

The optimal fixed fee T^* is given by

$$T^* = V(\rho_S^{M*},\sigma_M) - U(q^*(\rho_S^{M*},\sigma_M),\rho_S^{M*},\sigma_M) + \tau^*(\rho_S^{M*},\sigma_M) \tag{95}$$

□