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Static vs. Dynamic Policies for Vehicle Routing Problems with Backhauling and Dynamically Arising Customer Demands

by

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Abstract

Dynamic vehicle routing problems (VRP) have attracted more attention than dynamic vehicle routing problems with backhauling (VRPB) in the relevant literature. Dynamic VRPB are more complex than dynamic VRP, and since VRP are a special case of VRPB, models and algorithms for dynamic VRPB can easily be adapted for dynamic VRP. In this paper, we compare static vs. dynamic policies for solving dynamic VRPB with dynamically arising customer delivery and pickup demands. We develop MILP formulations and search algorithms for small-to-medium-sized problems under static and dynamic policies. Although dynamic policies are always at least as good as static policies, we observe from numerical experimentations that static policies perform relatively well for small-sized problems and low degrees of dynamism (*dod*). On the other hand, dynamic policies are expected to perform significantly better than static policies for large-sized problems, high degrees of dynamism (*dod*) and early availabilities of dynamic customer delivery and pickup demand information. We conclude the paper by providing directions for future research on dynamic VRPB.

Keywords: Vehicle routing problem; Backhauling; Split deliveries and pickups; Dynamic demand; Degree of dynamism; Static vs. dynamic policy

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1 Introduction

In the vehicle routing problem (VRP), a depot has to serve the linehaul demands of a set of customers with a fleet of vehicles. The objective is to determine the vehicle routes in order to minimize the total travelling time/cost of the vehicles. The problem is known to be NP-Hard. A number of variants of the basic VRP exist such as multiple depots, multiple time periods, time windows, service times, maximum route time limitations on vehicles and so on. An even more difficult problem is the vehicle routing problem with backhauling (VRPB) where a customer has both linehaul and backhaul demands. It is intuitive that the VRP is a special case of the VRPB where the backhaul demands of customers are zero. Since both problems are NP-Hard, the solution approach in the literature so far has been the development of exact or mixed integer linear programming (MILP) formulations for solving small-sized problems and approximate or heuristic algorithms for solving medium to large-sized problems. For the different variants of the VRP/VRPB and their solution methodologies, readers are referred to *Laporte et al.* [8], *Ropke and Pisinger* [18] and *Mitra* [14].

The basic VRP/VRPB is static in the sense that all the information on customers, their demands, distances between locations, travelling times/costs and so on are known to the decision maker in the beginning of the planning period so that the solution to the problem, i.e. the vehicle routes remain static until the end of the planning period. However, in practice, much of the information may not be available at the time of decision-making, which may be revealed dynamically over time such as customer demands, travelling times and so on. This set of problems is referred to as the dynamic VRP/VRPB, which has produced a significant amount of research since about the last one-and-a-half decades. The dynamic VRP may consist of customers with delivery-only or pickup-only demands, dynamic customer requests and/or dynamic travel/service times, real-time diversions of vehicles en route to a customer or replanning of vehicle routes only when vehicles reach their next destinations, capacitated or uncapacitated vehicles, single time period or multiple time periods, single objective or multiple objectives, and so on. The solution approach for the different variants of the dynamic VRP has been re-optimization, i.e. the updation of vehicle routes as and when new information becomes available. *Larsen et al.* [9] defined an index called the degree of dynamism (*dod*) depending on the number of dynamic customer requests in comparison to the total number of customer requests for a setting with delivery-only customers, unknown service times, single objective function and updation of vehicle routes only at customer locations, and compared the performances of several routing policies with respect to the *dod*. *Angelelli et al.* [1, 2] compared the performances of static vs. dynamic policies and collaborative vs. individual dynamic transportation policies for a dynamic VRP with pickup-only and postponable or unpostponable customer requests, uncapacitated vehicles, multiple time periods and multiple objective functions allowing real-time diversions of vehicles en route to a customer. *Wen et al.* [19] developed an MILP formulation and a three-phase heuristic for a multi-period dynamic VRP with delivery-only customer requests, capacitated vehicles, service time and maximum route time restrictions, and multiple objectives. The problem was solved on a rolling plan basis, and no real-time diversions of vehicles or updation of vehicle routes were allowed. Sensitivity analyses were carried out for the different parameters of the problem under consideration. *Lorini et al.* [11] extended an earlier model (*Potvin et al.* [15]) for a dynamic VRP with pickup-only customer requests, dynamic demands and travel times, uncapacitated vehicles, real-time diversions of vehicles and multiple objectives. They showed that real-time diversions of vehicles produced better results than the situation when updation of vehicle routes can only take place at customer locations (*Potvin et al.* [15]). For a detailed understanding of the different issues of the dynamic VRP and more comprehensive reviews of the related literature, readers may refer to *Psaraftis* [16, 17], *Gendreau and Potvin* [5], *Ichoua et al.* [7], *Ghiani et al.* [6] and *Larsen et al.* [10].

In contrast to the dynamic VRP, the dynamic VRPB has attracted less attention. The only related literature found was on dynamic pickup and delivery problems where vehicles pick up shipments from a set of customers to be delivered to another set of customers, which is clearly different from the setup of the dynamic VRPB. Interested readers may refer to *Berbeglia et al.* [4] for a literature review. In this paper, we consider the single-period dynamic VRPB with dynamically arising customer delivery and pickup demands, capacitated vehicles, updation of vehicle routes only at customer locations and a single objective function of minimizing the total travelling distance of the vehicles. As with the static VRP/VRPB, the dynamic VRPB is more complex than the dynamic VRP. To quote from *Angelelli et al.* [2], “In fact, it is assumed that delivery requests are not consistent with this dynamic setting, because if a delivery request is issued during the day, then the vehicle cannot be deviated to serve the new customer. Moreover, if a vehicle leaves the depot with the load to be delivered to a customer, the service of that customer cannot be later assigned to a different vehicle. In the case that the company has to face both pickup and delivery requests, the assumption is that the fleet is divided into two parts, a part dedicated to the delivery service and the other part dedicated to the pickup service. The part dedicated to the delivery service works as traditionally in a static context where the vehicles follow during the day the plan assigned to them at the beginning of the day. The part dedicated to the pickup service is managed dynamically.” However, in this paper, we relax this assumption by making no distinction between delivery and pickup vehicles. The same vehicle can cater to both delivery and pickup demands, and the route of the vehicle is managed dynamically as and when new customer requests arise. Since the dynamic characteristic of the problem may give rise to numerous possibilities, we had to narrow down the boundary of the problem. For example, since the vehicles are capacitated and the delivery and pickup requests from all the customers are not available at the beginning of the period, it is not known in advance how many vehicles would be needed to satisfy customer demands. Similarly, it is also not known in advance at how many points in time during the period new customer requests might arise dynamically. In this paper, we assume that a single vehicle would have sufficed had all customer delivery and pickup demands, known or unknown, been available at the beginning of the period, and there is only one another point in time during the period, other than the beginning of the period, when new customer demands might arise dynamically so that at most two vehicles would be needed to satisfy the delivery and pickup demands of all the customers. The purpose of these two assumptions is only to limit the boundary of the problem and facilitate numerical experimentations. It may be noted that the models and algorithms developed in this paper are applicable in practice to handle more dynamic situations. The major objective of this paper is to compare static vs. dynamic policies for such a problem setup. In the static policy, the route of a vehicle, once planned, is not changed thereafter, whereas in the dynamic policy, the route of a vehicle may change dynamically as and when new customer requests arise (It should be clear from the definitions of the policies that the models and algorithms for the static policy can easily be extended even if we relax the assumptions on the number of vehicles and the number of points in time when demands arise dynamically. To do the same for the dynamic policy, we need to keep track of the locations of the vehicles in real time, i.e. we need additional information on vehicle speeds/travelling times between locations and so on, which will lead us to make further assumptions. However, in practice, since all information is revealed dynamically over time, the models and algorithms may be suitably adapted and repeatedly run without any prejudice). The advantage of the static policy is reduced planning effort whereas the advantage of the dynamic policy is reduced travelling distance. *Lund et al.* [12] observed that static policies still performed relatively well for low degrees of dynamism (*dod*). *Angelelli et al.* ([2], p. 692, Table 4) showed for different problem scenarios that although the dynamic policy was superior to the static policy, the savings in terms of the travelling distance were not significant, which is why we wish to explore static vs. dynamic policies in our problem setup. In particular, we wish to identify the situations when the advantage of the dynamic policy over the static policy is not significant so that the decision-maker may stick to the static policy, and the situations when the dynamic policy is far superior to the static policy so that the decision-maker must switch over to the dynamic policy.

The paper is organized as follows. Section 2 gives the problem description. In Section 3, we develop the MILP formulations and a greedy search algorithm of worst-case-complexity of $O(n^2)$ where n is the number of locations, in line with the Nearest Neighbour (NN) routing policy as given in *Larsen et al.* [9], for solving the dynamic VRPB under static and dynamic policies. *Larsen et al.* [9] observed that the NN routing policy outperformed other routing policies for moderate or strong degrees of dynamism (*dod*). Section 4 provides numerical examples demonstrating the implementation of static and dynamic policies for 6 customers. Experimentations with 6- and 20-customer problem instances are also carried out, followed by a discussion on the results obtained. We stick to small-to-medium-sized problems since, as explained before, we assume that only one vehicle would have served all the customer requests had the information on all the delivery and pickup demands been known right at the beginning of the period. As noted before, our models and algorithms can easily be extended for larger problems. Finally, Section 5 presents the concluding remarks and directions for future research.

2 Problem description

Consider a supplier (depot), who has to serve daily the delivery and pickup demands of a number of grocery stores (customers) with at most two vehicles of equal capacity. A customer may have both delivery and pickup demands. The cumulative delivery and pickup demands of all customers individually do not exceed the vehicle capacity so that if the demands of all customers were known in advance at the time of planning vehicle routing, one vehicle would have been sufficient to deliver finished goods to and pick up returns from all customers. However, complete demand information from all customers is not available at the time of planning vehicle routing. Therefore, the route of the first vehicle is planned based on the partial information on customer demands available so far. At a later point in time, when the first vehicle is still under way serving the customers on its route, demand information from customers that was not visible earlier becomes available and requires to be served. At this point, the decision maker has two choices – she may follow a static policy, i.e. she may not disturb the route of the first vehicle as originally planned and she may decide the route of the second vehicle to serve the customers whose demands originated later, or she may follow a dynamic policy that may alter the route of the first vehicle as originally planned, and simultaneously decides the routes of both the vehicles to serve the remaining customers planned to be served by the first vehicle and the new customers whose demands originated later. As per the descriptions of the policies, it is apparent that while in the static policy the routes of the vehicles are non-overlapping, in the dynamic policy there may be overlaps of the vehicle routes.

Both vehicles leave the depot with delivery loads (finished goods) only once and upon serving the customers on their respective routes return to the depot with pickup loads (returns). Split deliveries and pickups are allowed [13,14], i.e. each customer may be served by more than one vehicle and more than once by the same vehicle (For a recent review on vehicle routing problems with split deliveries, readers are referred to Archetti and Speranza [3]). Delivery and pickup at a customer location may or may not be simultaneous, and in case they are not simultaneous, they can be in any sequence. This is a single period problem where all customer demands are satisfied within the period. There are no restrictions on time windows and the maximum distance travelled by a vehicle. The cost of travel between any pair of locations is proportional to the distance between the locations and is independent of the load of the vehicle. The objective of the problem is to determine the routes of the two vehicles for both the static and dynamic policies, which minimize the total distance travelled by both the vehicles.

3 Model formulation and heuristic development

In this section, we first provide the MILP formulations of the problem and then develop a greedy search algorithm for both the static and dynamic policies.

3.1 MILP formulation for the static policy

Since the routes of the vehicles are non-overlapping, the following formulation has to be solved twice to determine the routes of the two vehicles – once in the beginning to determine the route of the first vehicle when partial information on customer demands is available, and then at a later point in time to determine the route of the second vehicle when demand information from the rest of the customers becomes available.

The following notations have been used in the formulation.

Index

i, j	set of depot and customers (0, 1, 2… where 0 represents the depot and 1, 2… represent the customers)

Data

v	vehicle capacity
d_j	delivery demand of customer j
r_j	pickup demand of customer j
c_{ij}	distance between customer i and customer j ($c_{ii} = \infty$)

Decision variables

x_{ij}	number of times the vehicle moves from customer i to customer j
y_{ij}	quantity of delivery loads moved from customer i to customer j
z_{ij}	quantity of pickup loads moved from customer i to customer j

The MILP formulation is as follows.

$$
\begin{aligned}
&\textit{Minimize} \quad \sum_i \sum_j c_{ij} x_{ij} \\
&\textit{Subject to} \quad \sum_i y_{ij} - \sum_i y_{ji} = d_j \qquad \forall j \qquad (1.1) \\
&\sum_i z_{ji} - \sum_i z_{ij} = r_j \qquad \forall j \qquad (1.2) \\
&\sum_i y_{i0} = 0 \qquad (1.3) \\
&\sum_j z_{0j} = 0 \qquad (1.4) \\
&\sum_i x_{ij} - \sum_i x_{ji} = 0 \qquad \forall j \qquad (1.5) \\
&\sum_j x_{0j} = 1 \qquad (1.6) \\
&y_{ij} + z_{ij} \le v x_{ij} \qquad \forall i, j \qquad (1.7) \\
&y_{ij}, z_{ij} \ge 0 \qquad \forall i, j \\
&x_{ij} = 0, 1, 2 \ldots \qquad \forall i, j
\end{aligned}
$$

In the formulation above, the delivery and pickup demands of the depot are equal to the negative cumulative delivery and pickup demands of all customers, respectively. Once the optimal values of the decision variables are obtained, the route of the vehicle can be logically traced.

The objective of the formulation is to minimize the total distance travelled by the vehicle. Constraints (1.1) and (1.2) ensure that the delivery and pickup demands of all customers are satisfied. Constraints (1.3) and (1.4) ensure that no finished goods return to the depot and no returns leave the depot, respectively. That the vehicle must leave a customer location after visiting it is ensured by Constraint (1.5). Constraint (1.6) states that the vehicle leaves the depot only once, and Constraint (1.7) ensures that the vehicle capacity on a route is never exceeded.

3.2 MILP formulation for the dynamic policy

In this case, based on the demand information available in the beginning, the MILP formulation given in Section 3.1 is solved to determine the route of the first vehicle. At a later point of time, when the demand information from the rest of the customers becomes available, the first vehicle is still in service and has customers on its route, who are yet to be served. Suppose, at that moment, the first vehicle is situated at customer location *J*, yet to be served, with delivery and pickup loads of *FG* and *RT*, respectively. The delivery and pickup demands of the customers already served by the first vehicle are set to zero. The MILP formulation given below, solved at this particular point of time, simultaneously determines the routes of both the vehicles to serve the remaining customers. It may be noted that while the first vehicle leaves customer location *J* and returns to the depot, the second vehicle leaves from and returns to the depot.

In order to utilize the MILP formulation given above, which requires all routes to begin from and end at the depot, a fictitious route is constructed from the depot to customer location *J* for the first vehicle with delivery and pickup loads of *FG* and *RT*, respectively. This ensures that the first vehicle indeed leaves from customer location *J* where it is currently situated. The objective function is suitably modified by subtracting the fictitious distance travelled by the first vehicle from the depot to customer location *J*.

In addition to the notations introduced earlier, the following notations have been added/modified.

Index

k	set of vehicles: {1, 2}

Data

J	current customer location of the first vehicle
FG	current delivery load of the first vehicle
RT	current pickup load of the first vehicle

Decision variable

x_{ijk}	number of times vehicle k moves from customer i to customer j

The MILP formulation is given below.

$$\text{Minimize} \quad \sum_i \sum_j \sum_k c_{ij} x_{ijk} - c_{0J} x_{0J1}$$

$$\text{Subject to} \quad \sum_i y_{ij} - \sum_i y_{ji} = d_j \qquad \forall j \qquad (2.1)$$

$$\sum_i z_{ji} - \sum_i z_{ij} = r_j \qquad \forall j \qquad (2.2)$$

$$y_{0J} = FG \qquad (2.3)$$

$$z_{0J} = RT \qquad (2.4)$$

$$\sum_i y_{i0} = 0 \qquad (2.5)$$

$$\sum_j z_{0j} = RT \qquad (2.6)$$

$$\sum_i x_{ijk} - \sum_i x_{jik} = 0 \qquad \forall j,k \qquad (2.7)$$

$$\sum_j x_{0jk} = 1 \qquad \forall k \qquad (2.8)$$

$$x_{0J1} = 1 \qquad (2.9)$$

$$y_{ij} + z_{ij} \le v \sum_k x_{ijk} \qquad \forall i,j \qquad (2.10)$$

$$y_{ij}, z_{ij} \ge 0 \qquad \forall i,j$$

$$x_{ijk} = 0,1,2... \qquad \forall i,j,k$$

As already noted, the distance from the depot to customer location *J* is subtracted from the objective function to eliminate the effect of the fictitious route. This, of course, being a constant, would not have affected the routes of the two vehicles. Constraints (2.1), (2.2), (2.5), (2.7), (2.8) and (2.10) are analogous to Constraints (1.1), (1.2), (1.3), (1.5), (1.6) and (1.7), respectively, of the previous MILP formulation. Constraints (2.3) and (2.4) ensure that *FG* units of finished goods and *RT* units of returns are moved on the fictitious route from the depot to customer location *J* where the first vehicle is currently situated. Constraint (2.6) ensures that exactly *RT* units of returns leave the depot. Finally, Constraint (2.9) represents the fictitious route of the first vehicle from the depot to customer location *J*. We note here that Constraint (2.10) represents the aggregate capacity constraint on multiple visits of the vehicles, which for trivial problems, say with one depot and one customer, may not ensure that the vehicle capacity is not exceeded on individual visits. However, for more general problems with more number of customers with delivery and pickup demands, we have found that Constraint (2.10) along with Constraints (2.1) and (2.2) does ensure that the vehicle capacity is not exceeded on individual visits.

Once the above formulation is solved, the distance travelled by the vehicles is added to the distance already travelled by the first vehicle to obtain the total distance travelled by both the vehicles.

3.3 A greedy search algorithm for the static policy

Similar to the MILP formulation for the static policy, the algorithm (in line with the algorithm proposed in [13]) has to be run twice to determine the routes of the two vehicles. In addition to the previous notations, the following are also introduced.

no_of_customers	number of customers to be served by the vehicle
DL	delivery load of the vehicle
PL	pickup load of the vehicle
cumul_PL	cumulative pickup load of the vehicle
total_dist	total distance travelled by the vehicle

cumul_PL is the total pickup load on the route of the vehicle. When the vehicle leaves the depot, its *DL* is equal to the cumulative delivery load on its route and its *PL* is zero. On the other hand, when the vehicle returns to the depot, its *DL* is zero and its *PL* is equal to *cumul_PL*. *DL* and *PL* get updated every time the vehicle visits a customer depending on its delivery and pickup demands.

The pseudo-code of the algorithm is given below, followed by explanations.

$i = 0$ (set the initial location of the vehicle as the depot)
Do
  For $j = 1$ to $j \leq$ *no_of_customers* Step 1
    If $d_j > 0$ or ($r_j > 0$ and $DL + PL < v$) then
      If $c_{ij} <$ *min_dist* (minimum distance obtained so far from i to a customer)
        then *next_dest* $= j$ and *min_dist* $= c_{ij}$

(set j as the next destination and c_{ij} as the new minimum distance)

7. Otherwise if $c_{ij} = min\_dist$ then

8. If $DL + PL + r_{next\_dest} - d_{next\_dest} > v$ and $DL + PL + r_j - d_j \leq v$

9. then $next\_dest = j$

10. Next j

11. $total\_dist = total\_dist + c_{i\ next\_dest}$

12. $DL = DL - d_{next\_dest}$, $d_{next\_dest} = 0$

13. If $r_{next\_dest} \leq v - DL - PL$ then

14. $PL = PL + r_{next\_dest}$, $r_{next\_dest} = 0$

15. Otherwise $r_{next\_dest} = r_{next\_dest} - (v - DL - PL)$, $PL = v - DL$

16. $i = next\_dest$ (set i as the current customer location of the vehicle)

17. While $DL > 0$ or $PL < cumul\_PL$

18. $total\_dist = total\_dist + c_{i0}$

(the vehicle returns to the depot and the distance is added to the total distance)

Lines 3-10: The purpose of this part of the pseudo-code is to identify the next customer on the route. The vehicle would visit a customer if either its delivery demand is positive or its pickup demand is positive and the vehicle has unutilized capacity to pick up the whole or a part of the returns. The term "greedy" is attributed to the algorithm since it inserts the next customer on the route which is closest to the current customer location of the vehicle. In case there are more than one customer that are equidistant from the current vehicle location, the customer whose pickup demand can be completely satisfied by the vehicle while the same cannot be achieved by the vehicle for the other customers, is selected as the next customer on the route.

Lines 11-15: The vehicle moves to the next customer location and the corresponding distance is added to the total distance travelled so far by the vehicle. *DL*, *PL* and the delivery and pickup demands of the next customer are adjusted according to their current values. It is apparent that the delivery demand of the next customer would be reduced to zero. However, its remaining pickup demand would depend on the space available on the vehicle.

Lines 2-17: The Do-While loop is repeated until the delivery load of the vehicle is zero and the pickup load is equal to *cumul_PL*.

3.4 A greedy search algorithm for the dynamic policy

Similar to the MILP formulation for the dynamic policy, the algorithm given in Section 3.3 is first run for the first vehicle to determine its current customer location, and delivery and pickup loads (after serving the demands of the current location) when demand information on the delivery and pickup demands of the rest of the customers becomes available. Subsequently, the following algorithm is run to determine the routes of both the vehicles to serve the remaining customers (The first part of the algorithm is reproduced from Section 3.3 to highlight he necessary modifications while the second part is identical to the algorithm given in Section 3.3). It is assumed that in case a customer is visited by both the vehicles, it is first visited by the first vehicle, followed by the second vehicle. The notations used in the algorithm bear the same meanings as described earlier.

/* Determine the route of the first vehicle */

1. $i = J, DL = FG, PL = RT$
   (set the initial location of the first vehicle as its current customer location, and assign its delivery and pickup loads based on their current values)
2. Do
3.   For $j = 1$ to $j \leq no\_of\_customers$ Step 1
4.   If $d_j > 0$ or ($r_j > 0$ and $DL + PL < v$) then
5.   If $c_{ij} < min\_dist$ (minimum distance obtained so far from i to a customer)
6.   then $next\_dest = j$ and $min\_dist = c_{ij}$
   (set j as the next destination and c_{ij} as the new minimum distance)
7.   Otherwise if $c_{ij} = min\_dist$ then
8.   If $DL + PL + r_{next\_dest} - d_{next\_dest} > v$ and $DL + PL + r_j - d_j \leq v$
9.   then $next\_dest = j$
10.   Next j

11. $total\_dist = total\_dist + c_{i\ next\_dest}$

12. If $DL \geq d_{next\_dest}$ then

13. $DL = DL - d_{next\_dest}$, $d_{next\_dest} = 0$

14. Otherwise $d_{next\_dest} = d_{next\_dest} - DL$, $DL = 0$

15. If $r_{next\_dest} \leq v - DL - PL$ then

16. $PL = PL + r_{next\_dest}$, $r_{next\_dest} = 0$

17. Otherwise $r_{next\_dest} = r_{next\_dest} - (v - DL - PL)$, $PL = v - DL$

18. $i = next\_dest$ (set i as the current customer location of the first vehicle)

19. While $DL > 0$

20. $total\_dist = total\_dist + c_{i0}$

(the first vehicle returns to the depot and the distance is added to the total distance)

/* Determine the route of the second vehicle */

[The pseudo-code is the same as that given in Section 3.3]

The logic of selection of the next customer on the route of a vehicle is the same as described earlier for the static policy. In fact, the second part of the pseudo-code, which determines the route of the second vehicle, is identical to the pseudo-code of the algorithm for the static policy. There are some minor changes in the first part of the pseudo-code (Lines 1-20), which are highlighted below.

Line 1: The current customer location of the first vehicle, and its current delivery and pickup loads are inputs to the algorithm.

Lines 12-14: The first vehicle may not be able to satisfy the delivery demands of some of the customers entirely, which will be served by the second vehicle at a later stage. Accordingly, the delivery loads of the first vehicle and the delivery demands of these customers have to be adjusted based on their current values.

Lines 2-19: The Do-While loop is repeated until the delivery load of the first vehicle is zero. The first vehicle returns to the depot as soon as its delivery load falls to zero even if it may have some unutilized space to pick up some more returns. Since the second vehicle is anyway on its route, it is expected to serve the delivery and pickup demands of the rest of the customers.

4 Numerical examples and discussions

We demonstrate the application of the formulations and algorithms by means of a six-customer problem set. The details of generation of locations of customers and their delivery and pickup demands are explained below.

4.1 Generation of locations of customers

The locations of customers, designated by the numbers 1-6, are randomly generated on a Euclidean plane. It is assumed that the depot, designated by the number 0, is located at the origin – (0, 0), and the customers are located within 25 km on either side of the depot on the x and y axes. The x and y coordinates of a customer are generated from the Uniform distribution – Uniform(-25, 25), and the Euclidean distance between any pair of locations is computed. Table 1 shows the Euclidean distances between all pairs of locations. Since distances are symmetric, only the upper triangular matrix is shown. Also, in the inputs to the formulations and algorithms, prohibitive distances are entered for the diagonal elements of the matrix to avoid self-looping.

Table 1 Euclidean distances (km) between all pairs of locations

Depot/ Customer	0	1	2	3	4	5	6
0	-	5.47	21.27	26.61	17.15	23.96	19.82
1		-	25.24	29.62	13.44	21.57	14.37
2			-	7.68	38.32	45.06	37.09
3				-	43.04	50.55	39.48
4					-	9.27	13.77
5						-	22.45
6							-

4.2 Generation of customers' delivery and pickup demands

Without loss of generality, the vehicle capacity is assumed to be 1 unit. The delivery and pickup demands of customers are generated as fractions of the vehicle capacity. We use the Rand() function to generate the Uniform(0, 1) delivery and pickup demands of customers. Subsequently, the delivery and pickup demands are proportionately adjusted so that one vehicle would suffice to satisfy the demands of all customers, as already mentioned in the problem description. Table 2 shows the delivery and pickup demands of all six customers.

Table 2 Delivery and pickup demands of customers as fractions of vehicle capacity (Set 1)

Customer	Delivery demand	Pickup demand
1	0.05	0.22
2	0.16	0.10
3	0.07	0.21
4	0.36	0.13
5	0.05	0.30
6	0.31	0.04

4.3 Experimental set-up and results

Although the delivery and pickup demands of all six customers are generated simultaneously, not all demand information may be available in the beginning for planning vehicle routing. The demand information on the remaining customers becomes available when the first vehicle is still on its way serving the initial customers on its route. Suppose, in our example with six customers, the delivery and pickup demands of four customers are available in the beginning, and the delivery and pickup demands of the remaining two customers are known when the first vehicle has already served the first two customers on its route, and is currently situated at the third customer location. While the static policy independently determines the routes of the two vehicles, the dynamic policy takes into account the current position of the first vehicle and simultaneously determines the routes of the two vehicles, which may have the potential to alter the route of the first vehicle.

Instance 1

To demonstrate, suppose the delivery and pickup demands of Customers 1-4 are available in the beginning, and the same information on Customers 5 and 6 are available at a later point of time. Then applying the MILP formulation for the static policy, the routes of the two vehicles are obtained as shown in Fig. 1. Table 3 shows the delivery and pickup loads of the vehicles on their respective routes.

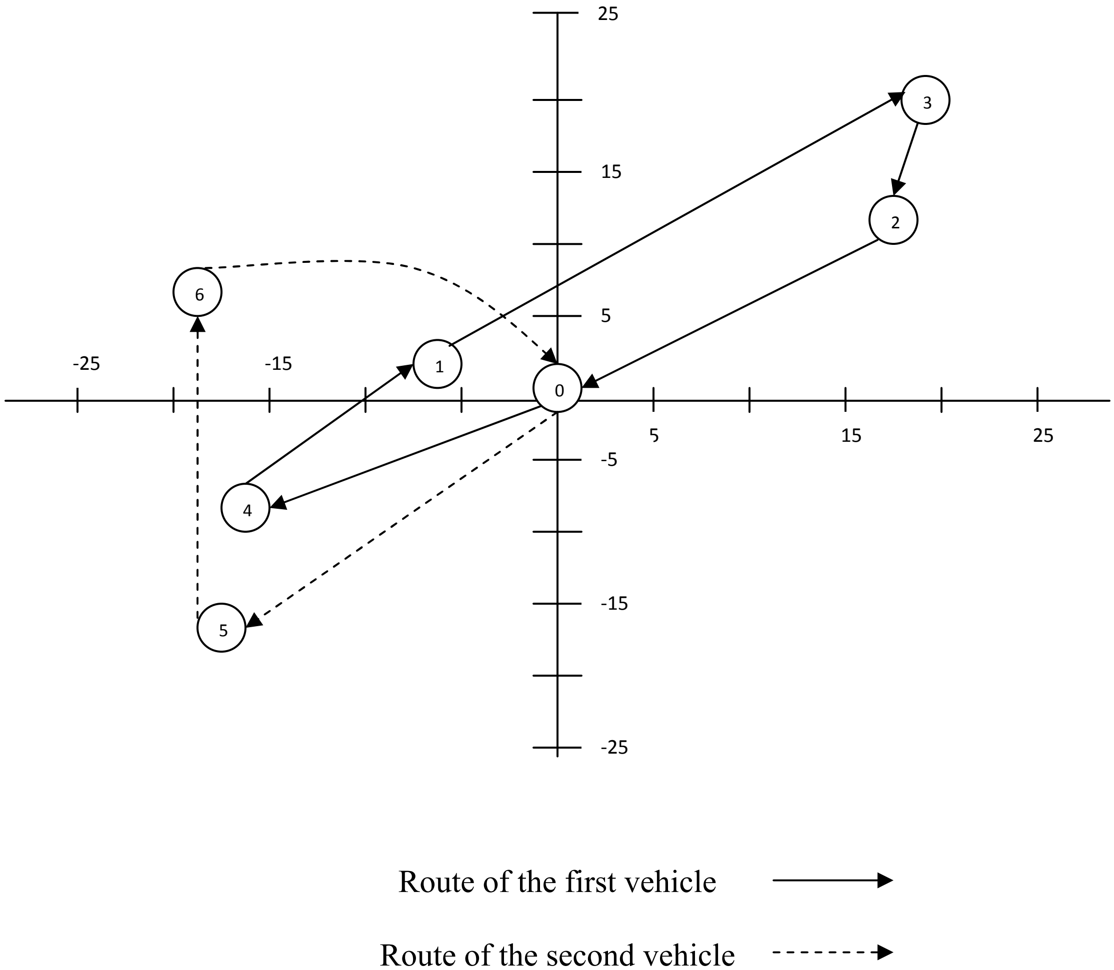

Fig. 1 Vehicle routes as obtained by the MILP formulation for the static policy (Instance 1)

Table 3 Delivery and pickup loads of the vehicles as obtained by the MILP formulation for the static policy (Instance 1)

Vehicle	Route	Delivery load	Pickup load
1	0 – 4	0.64	0
	4 – 1	0.28	0.13
	1 – 3	0.23	0.35
	3 – 2	0.16	0.56
	2 – 0	0	0.66
2	0 – 5	0.36	0
	5 – 6	0.31	0.30
	6 – 0	0	0.34

The total distance travelled by the two vehicles is 155.39 km obtained by the MILP formulation for the static policy. Next, the proposed search algorithm is applied to the data, which gives the routes of the vehicles and their delivery and pickup loads, as shown by Fig. 2 and Table 4, respectively.

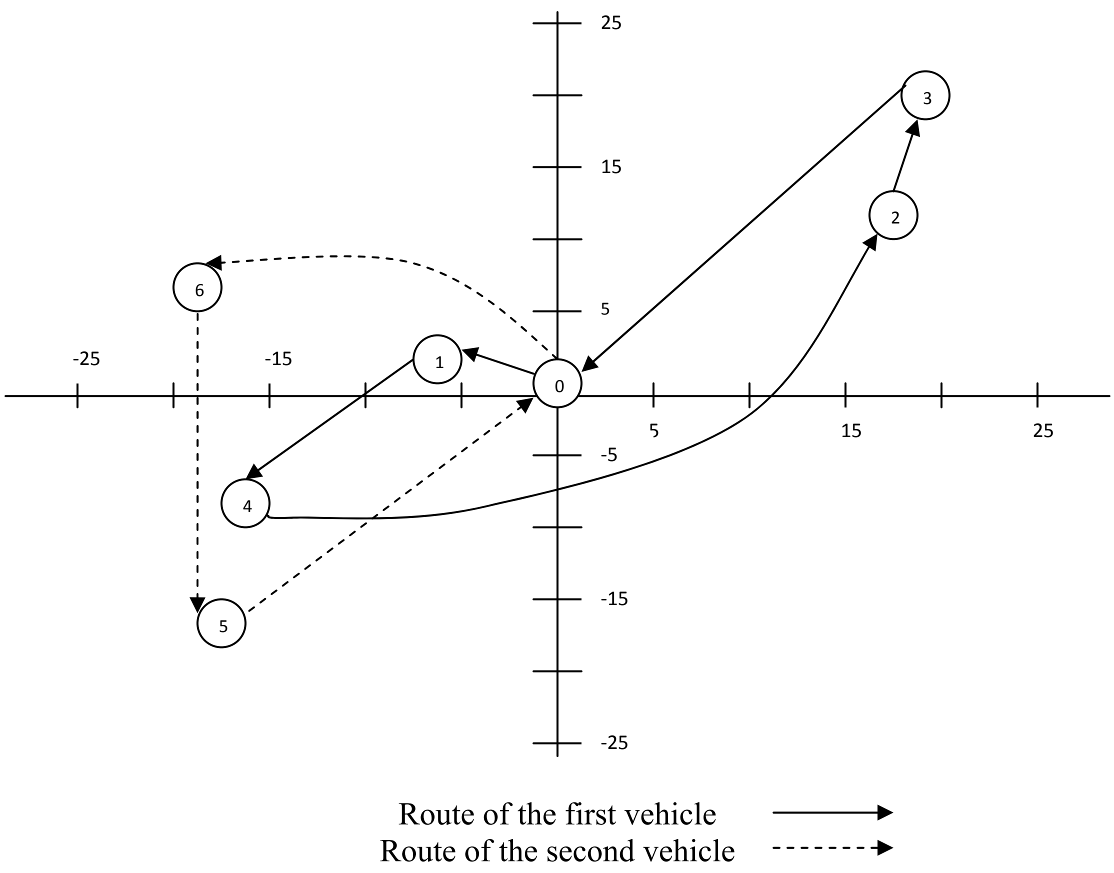

Fig. 2 Vehicle routes as obtained by the search algorithm for the static policy (Instance 1)

Table 4 Delivery and pickup loads of the vehicles as obtained by the search algorithm for the static policy (Instance 1)

Vehicle	Route	Delivery load	Pickup load
1	0 – 1	0.64	0
	1 – 4	0.59	0.22
	4 – 2	0.23	0.35
	2 – 3	0.07	0.45
	3 – 0	0	0.66
2	0 – 6	0.36	0
	6 – 5	0.05	0.04
	5 – 0	0	0.34

The total distance travelled by the two vehicles is 157.75 km obtained by the search algorithm for the static policy, which compares well with the total distance travelled by the two vehicles obtained by the MILP formulation for the static policy.

To apply the dynamic policy, suppose the first vehicle has already served Customers 4 and 1, and is currently located at Customer 3 with delivery and pickup loads of 0.23 and 0.35, respectively, (Fig. 1 and Table 3) when demand information from Customers 5 and 6 become available. Then applying the MILP formulation for the dynamic policy, we observe that the solution obtained is no different from the solution to the static policy obtained earlier. The same is also true for the search algorithm for the dynamic policy (Fig. 2 and Table 4). Therefore, for this particular instance, there is no difference between the static and dynamic policies.

Instance 2

Suppose, in another instance, the delivery and pickup demands of Customers 3-6 are available in the beginning, and the demand information on Customers 1 and 2 become available at a later point of time. Upon application of the MILP formulation for the static policy, we obtain the vehicle routes and the delivery and pickup loads carried by the vehicles, as shown by Fig. 3 and Table 5, respectively.

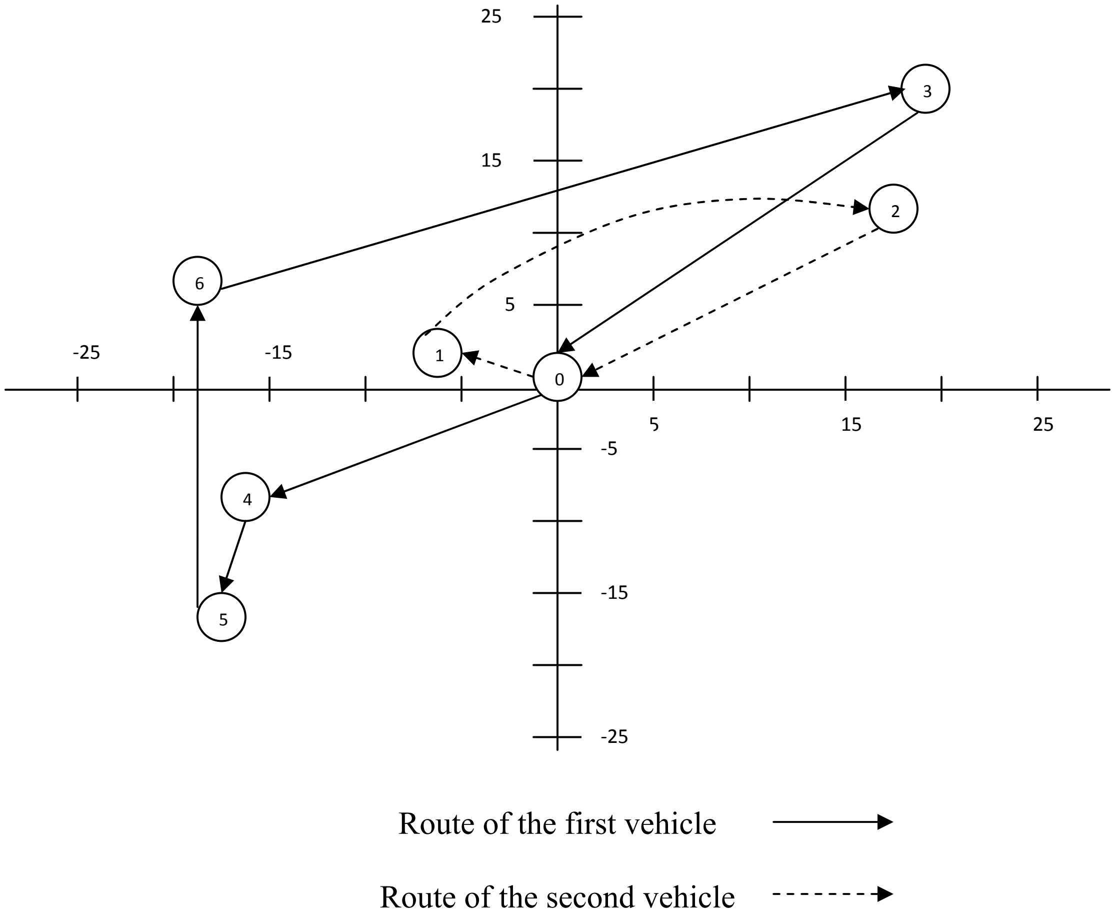

Fig. 3 Vehicle routes as obtained by the MILP formulation for the static policy (Instance 2)

Table 5 Delivery and pickup loads of the vehicles as obtained by the MILP formulation for the static policy (Instance 2)

Vehicle	Route	Delivery load	Pickup load
1	0 – 4	0.79	0
	4 – 5	0.43	0.13
	5 – 6	0.38	0.43
	6 – 3	0.07	0.47
	3 – 0	0	0.68
2	0 – 1	0.21	0
	1 – 2	0.16	0.22
	2 – 0	0	0.32

The total distance travelled by the two vehicles is 166.94 km obtained by the MILP formulation for the static policy. The application of the search algorithm also gives the same solution.

Now, for the dynamic policy, suppose the first vehicle has already served Customers 4 and 5, and is currently located at Customer 6 with delivery and pickup loads of 0.38 and 0.43, respectively, when demand information from Customers 1 and 2 become available. By applying the MILP formulation for the dynamic policy, we obtain the vehicle routes and the delivery and pickup loads carried by the vehicles, as shown by Fig. 4 and Table 6, respectively.

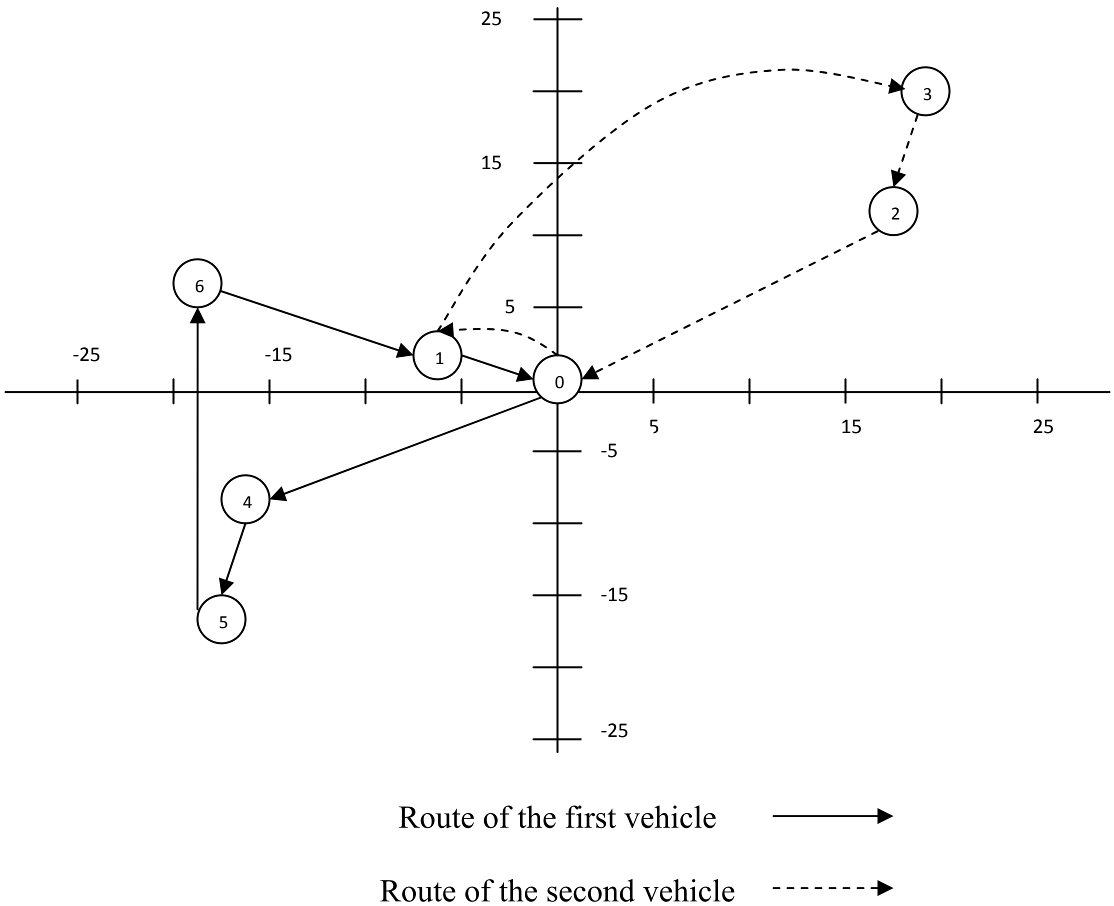

Fig. 4 Vehicle routes as obtained by the MILP formulation for the dynamic policy (Instance 2)

Table 6 Delivery and pickup loads of the vehicles as obtained by the MILP formulation for the dynamic policy (Instance 2)

Vehicle	Route	Delivery load	Pickup load
1	0 – 4	0.79	0
	4 – 5	0.43	0.13
	5 – 6	0.38	0.43
	6 – 1	0.07	0.47
	1 – 0	0	0.69
2	0 – 1	0.21	0
	1 – 3	0.23	0
	3 – 2	0.16	0.21
	2 – 0	0	0.31

Note that in this case there is an overlap at Customer 1, i.e. both vehicles serve Customer 1. The total distance travelled by the vehicles is 132.75 km obtained by the MILP formulation for the dynamic policy, which is lower than that obtained by the formulation for the static policy.

When the search algorithm for the dynamic policy is applied to the same data, we get the vehicle routes and the delivery and pickup loads of the vehicles, as shown by Fig. 5 and Table 7, respectively.

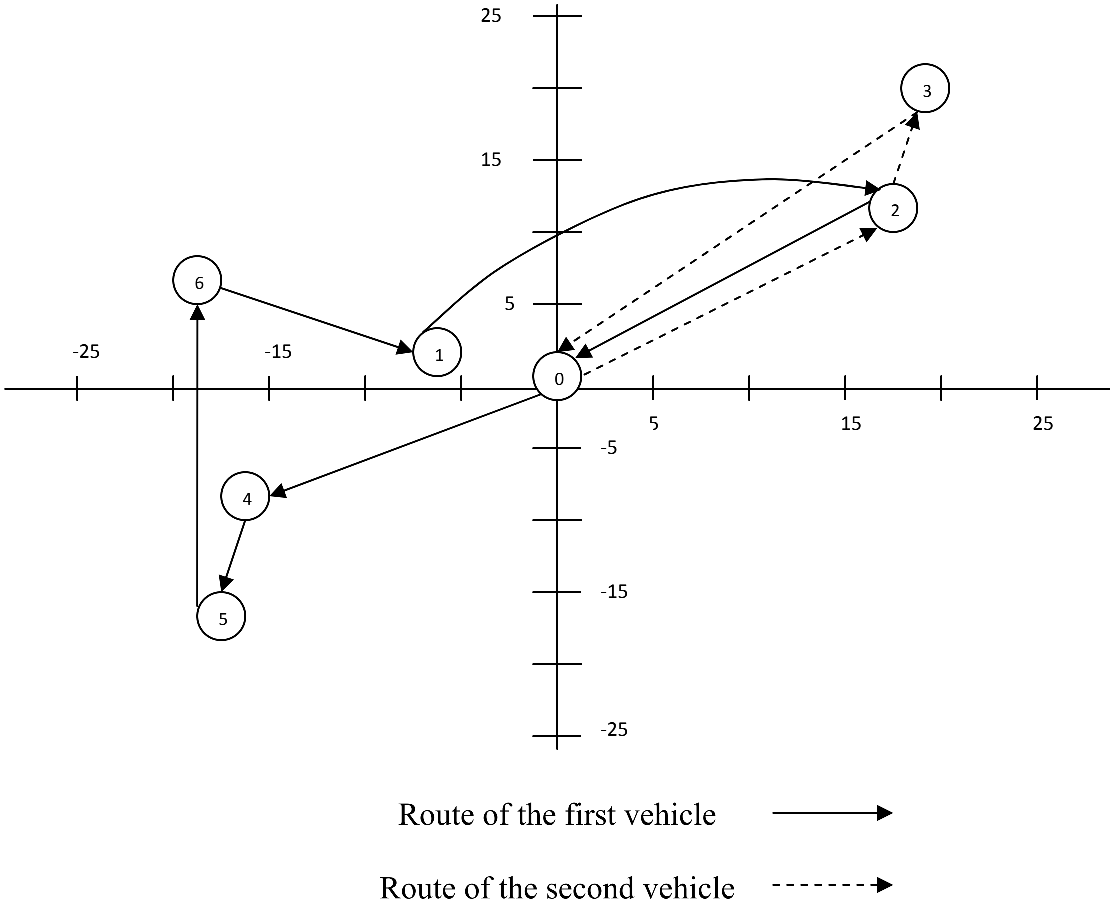

Fig. 5 Vehicle routes as obtained by the search algorithm for the dynamic policy (Instance 2)

Table 7 Delivery and pickup loads of the vehicles as obtained by the search algorithm for the dynamic policy (Instance 2)

Vehicle	Route	Delivery load	Pickup load
1	0 – 4	0.79	0
	4 – 5	0.43	0.13
	5 – 6	0.38	0.43
	6 – 1	0.07	0.47
	1 – 2	0.02	0.69
	2 – 0	0	0.79
2	0 – 2	0.21	0
	2 – 3	0.07	0
	3 – 0	0	0.21

In this case also, there is an overlap at Customer 2, i.e. both vehicles serve Customer 2. The total distance travelled by the vehicles is 165.31 km obtained by the search algorithm for the dynamic policy. Although, the total distance travelled for the dynamic policy is slightly lower than that for the static policy, the saving is not comparable with the saving obtained by the MILP formulation. This is due to the structural differences between the MILP formulation and the search algorithm. The MILP formulation is less constrained than the search algorithm. It may be noted from Table 6 that the first vehicle actually leaves 0.02 units of delivery load at Customer 1, which is later picked up by the second vehicle. However, to make such a provision in the search algorithm, a more complicated logic has to be built, which in its present form has been kept as simple as possible with the shortest distance being the primary criterion for the selection of the next customer on the route.

4.3.1 Experimentation with 6 customers

Since the problem is known to be NP-Hard, the MILP formulation fails to produce optimal solutions to problems with a larger number of customers in a reasonable amount of time. Also, because the search algorithm performs reasonably well vis-à-vis the MILP formulation for small-to-medium-sized problems [13], henceforth we use this algorithm for solving vehicle routing problems with backhauling and dynamically arising customer demands.

We experimented with different combinations of customer locations and their delivery and pickup demands. Since the results are similar, we present here the results of one experimental set-up with customer locations given in Table 1.

Given the experimental set-up of the amount of demand information available in the beginning (from 4 customers) and later (from the remaining 2 customers), we can create a total of 15 problem instances for one set of data on delivery and pickup demands, as given in Table 2, which we refer to as Set 1. We generate another set of data on delivery and pickup demands, as given in Table 8, which we refer to as Set 2. Combining Set 1 and Set 2, we can altogether generate a total of 30 problem instances (15 for each data set) for each which the total distances travelled by the vehicles as per the search algorithm for the static and dynamic policies are shown in Table 9.

Table 8 Delivery and pickup demands of customers as fractions of vehicle capacity (Set 2)

Customer	Delivery demand	Pickup demand
1	0.06	0.20
2	0.30	0.11
3	0.08	0.21
4	0.16	0.14
5	0.36	0.21
6	0.04	0.13

Table 9 Total distances (km) travelled by the vehicles for 2 data sets for the static and dynamic policies

Problem instance	Demand information available first from customers	Demand information available later from customers	Total distance (km) travelled by the vehicles					
			Data Set 1 (Table 2)			Data Set 2 (Table 8)		
			Static	Dynamic	Dynamic/ Static	Static	Dynamic	Dynamic/ Static
1	1, 2, 3, 4	5, 6	157.75	157.75	1	190.74	190.74	1
2	2, 3, 4, 5	1, 6	145.43	145.43	1	181.97	187.35	**1.03**
3	1, 3, 4, 5	2, 6	183.52	161.65	0.88	168.02	168.02	1
4	1, 2, 4, 5	3, 6	180.42	161.65	0.90	180.82	187.35	**1.04**
5	1, 2, 3, 5	4, 6	157.13	157.13	1	190.24	190.24	1
6	2, 3, 4, 6	1, 5	153.30	153.30	1	221.29	181.85	0.82
7	1, 3, 4, 6	2, 5	189.06	177.02	0.94	178.98	172.85	0.97
8	1, 2, 4, 6	3, 5	206.20	191.06	0.93	203.78	210.73	**1.03**
9	1, 2, 3, 6	4, 5	141.60	141.60	1	157.27	172.85	**1.10**
10	3, 4, 5, 6	1, 2	166.94	165.31	0.99	193.60	193.60	1
11	2, 4, 5, 6	1, 3	168.93	165.31	0.98	161.20	161.20	1
12	2, 3, 5, 6	1, 4	157.68	157.68	1	227.56	219.95	0.97
13	1, 4, 5, 6	2, 3	126.01	126.01	1	196.83	196.83	1
14	1, 3, 5, 6	2, 4	196.19	158.47	0.81	172.44	172.44	1
15	1, 2, 5, 6	3, 4	195.42	158.47	0.81	210.32	210.32	1

The column 'Dynamic/Static' gives the ratios of the total distances travelled by the dynamic policy by those by the static policy. It may be observed from Table 9 that for Set 1 the dynamic policy performs at least as good as the static policy whereas for Set 2 there are 4 instances where the dynamic policy is outperformed by the static policy. This may be treated as an aberration as dynamic policies are indeed at least as good as static policies, and can be attributed to the limitation of the search algorithm of not being able to ensure the same. In case such an aberration happens, we simply revert to the static policy.

We observe from Table 9 that the dynamic policy outperforms the static policy in only 11 of the 30 problem instances (36.67%), and also out of these 11 instances, there are only a few where the savings could be considered substantial. A t-test also confirmed that the null hypothesis that there is no difference between the static and dynamic policies could not be rejected at 1% level of significance.

4.3.2 Experimentation with 20 customers

The experimentation with 6 customers can only indicate the relative performances of the static and dynamic policies. However, the quantum of demand information available in the beginning and how early or late the demand information from the remaining customers becomes available when the first vehicle is still in service might also have impacted the performances of the static and dynamic policies, which could not be tested with only 6 customers in the problem sets. Therefore, we consider a problem with 20 customers for which the locations and the delivery and pickup demands are generated in the same way as before, and are represented by Table A.1 and Table A.2, respectively, in Appendix A. We restrict to 20 customers because of two reasons. First, we consider only small-to-medium-sized problems, and second, as already mentioned, the performance of the search algorithm compared well with that of the MILP formulation for problem sizes of up to 20 customers [13] beyond which the MILP formulation failed to deliver solutions in a reasonable amount of time. Since the performance of the algorithm could not be compared with that of the MILP formulation beyond 20 customers, we limit the problem size to 20 customers in this paper. However, it is possible to extend the search algorithm for solving larger problems.

We, of course, experimented with different combinations of customer locations, their delivery and pickup demands, and dynamic information availability, and obtained similar results. We present the results of one experimental set-up here.

We consider 2 scenarios – a) Demand information from Customers 1-15 are available in the beginning and the same from Customers 16-20 are available later when the first vehicle is still in service, and b) Demand information from Customers 1-10 are available in the beginning and the same from Customers 11-20 are available later when the first vehicle is still in service. For each of these 2 scenarios, we consider that the demand information from the remaining customers becomes available when the first vehicle is located on the 3rd, 6th, 9th and 12th (relevant for the first instance only) customers on its route. Table 10 gives the total distances travelled by the vehicles as obtained by the search algorithm for the above-mentioned instances under the static and dynamic policies.

Table 10 Total distances (km) travelled by the vehicles for the static and dynamic policies (20 customers)

Demand information available later from the remaining customers when the first vehicle is located on its route at the	Total distance (km) travelled by the vehicles					
	Demand information available in the beginning from Customers 1-15			Demand information available in the beginning from Customers 1-10		
	Static	Dynamic	Dynamic/ Static	Static	Dynamic	Dynamic/ Static
3rd customer	349.42	289.37	0.83	337.28	276.38	0.82
6th customer	349.42	343.04	0.98	337.28	270.75	0.80
9th customer	349.42	343.04	0.98	337.28	316.54	0.94
12th customer	349.42	344.85	0.99	-	-	-

From Table 10, we note the following 3 observations – (a) When the demand information is available from a less number of customers in the beginning, the total distance travelled by the vehicles by either policy is also lower, (b) For a given scenario, the dynamic policy seems to perform better than the static policy when demand information from the remaining customers becomes available earlier on the route of the first vehicle, and as the same demand information becomes available quite late on the route of the first vehicle, the advantage of the dynamic policy over the static policy seems to taper off quickly, and (c) Between the 2 scenarios, given the same point of time when demand information from the remaining customers becomes available, the advantage of the dynamic policy over the static policy seems to be more pronounced for the scenario for which the demand information is available from a less number of customers in the beginning.

4.4 Discussions

From the numerical experimentations, we derive the following observations:

a) For a small number of customers, it may not be worthwhile to deploy the dynamic policy because its cost advantage over the static policy may not be able to outweigh the additional effort to be put in for its deployment.
b) Both static and dynamic policies perform better when less demand information is available at the beginning of the period and more is revealed at a later point of time during the period.
c) The advantage of the dynamic policy over the static policy is more significant when new customer requests arise at an earlier point of time during the period.
d) Given the same point of time of the arrival of new customer requests, the advantage of the dynamic policy over the static policy is more pronounced for the situation when less demand information is available at the beginning of the period. This corroborates the observation in *Lund et al.* [12] that static policies still perform relatively well for low degrees of dynamism (*dod*).

The above-mentioned observations are based on specific problem sizes (small-to-medium) and instances, and hence cannot be generalized. They are at best conjectures, the validity of which needs to be established by means of extensive numerical experimentations with larger problem sizes. Nevertheless, we hope that the results obtained in this paper through limited numerical experimentations would provide a platform to explore the trade-off between static and dynamic policies in further detail.

5 Conclusions and directions for future research

Although the dynamic VRP has attracted the attention of many researchers, there has been limited attention to the dynamic VRPB in the literature so far. The objective of this paper has been to compare between static and dynamic policies for the dynamic VRPB. We have developed the MILP formulations and a search algorithm for the dynamic VRPB under consideration. Experimentations with a small-to-medium number of customer locations have shown that the static policy may perform reasonably well for a small number of customers and low degrees of dynamism (*dod*). On the other hand, the dynamic policy may be the better option for a larger number of customers, higher degrees of dynamism (*dod*) and earlier availabilities of dynamic customer requests. However, since the scope of the numerical experimentations has been limited, the above-mentioned observations are only conjectures and cannot be generalized. A larger experimental setup needs to be designed to generalize the observations in this paper for which the assumptions that only one vehicle would suffice to satisfy the delivery and pickup demands of all the customers for the static version of the dynamic VRPB and there is only one another point in time besides at the beginning of the period when new customer requests may arise, need to be relaxed. Moreover, the efficacy of the search algorithm for low degrees of dynamism (*dod*) needs to be examined as *Larsen et al.* [9] conjectured that for low *dod*, insertion-based heuristics might outperform the NN routing policy. In this paper, we have considered a single-period problem, dynamism only in terms of new customer requests and a single objective of minimizing the total travelling distance. However, future research on the dynamic VRPB should also include multiple periods, dynamism in terms of travelling times and customer service times, besides new customer requests, and multiple objectives such as minimizing customer waiting times and the number of customers not served and lost to competitors, besides minimizing the total travelling distance/time/cost.

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Appendix A

Table A.1 Euclidean distances (km) between all pairs of locations (20 customers)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	-	30.36	20.92	24.82	19.04	26.43	14.58	24.53	14.70	30.30	19.13	20.00	14.03	11.25	9.42	20.84	7.87	15.30	5.01	24.03	18.89
1		-	40.94	40.62	11.40	56.46	24.74	16.13	17.16	43.13	18.16	25.11	43.60	34.28	34.28	14.94	34.99	44.84	34.91	41.49	38.25
2			-	45.02	33.23	27.77	16.21	42.43	24.38	50.70	23.08	40.39	17.54	9.70	29.19	26.35	26.79	17.77	18.85	43.89	2.71
3				-	30.76	35.89	38.14	25.64	33.94	5.78	39.94	15.66	31.21	35.72	15.83	40.02	18.26	32.02	26.19	2.09	43.33
4					-	45.46	17.71	10.98	9.18	34.13	13.74	16.20	32.92	25.13	22.91	11.84	23.60	34.18	24.01	31.33	30.55
5						-	36.21	49.39	40.25	41.12	43.01	41.82	13.32	25.68	25.67	45.57	23.60	12.24	21.55	33.91	28.49
6							-	27.94	8.54	43.18	7.30	28.64	22.96	10.83	23.77	10.43	22.45	23.99	16.94	37.72	13.51
7								-	19.68	27.38	24.70	10.10	38.29	33.42	24.12	22.76	25.86	39.55	29.35	26.86	39.83
8									-	38.35	6.07	21.98	27.05	17.12	21.95	6.42	21.58	28.25	18.97	33.95	21.68
9										-	44.41	18.13	36.97	41.33	21.52	44.19	23.99	37.77	31.90	7.41	48.96
10											-	27.95	29.68	18.03	27.31	3.28	26.60	30.78	22.64	39.89	20.45
11												-	32.28	30.74	16.17	27.10	18.44	33.46	24.03	16.79	38.06
12													-	12.73	16.84	32.26	14.23	1.27	9.04	29.63	17.30
13														-	19.92	21.00	17.69	13.60	9.96	34.73	7.65
14															-	28.38	2.65	17.90	10.40	14.82	27.50
15																-	27.95	33.39	24.76	40.14	23.71
16																	-	15.32	7.94	17.11	25.19
17																		-	10.30	30.40	17.72
18																			-	25.04	17.32
19																				-	42.29
20																					-

Table A.2 Delivery and pickup demands of customers as fractions of vehicle capacity (20 Customers)

Customer	Delivery demand	Pickup demand
1	0.11	0.07
2	0.06	0.04
3	0.01	0.01
4	0.10	0.10
5	0.04	0.10
6	0.00	0.01
7	0.11	0.01
8	0.00	0.09
9	0.09	0.00
10	0.00	0.08
11	0.02	0.01
12	0.06	0.09
13	0.06	0.05
14	0.12	0.02
15	0.05	0.06
16	0.08	0.06
17	0.03	0.04
18	0.03	0.05
19	0.03	0.01
20	0.00	0.08