INDIAN INSTITUTE OF MANAGEMENT CALCUTTA

WORKING PAPER SERIES

WPS No. 681/ September 2011

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Abstract

Infrastructure-as-a-Service (IaaS) is expected to grow at a rapid pace in the next few years. This is primarily because of the flexibility that it offers to organizations to meet variable demand without any fixed investment in capacity. Our work focuses on a particular customer segment of IaaS - online platform providers. These online businesses experience fluctuations in the number of users of their platforms which indirectly impact their advertising revenues. It is thus necessary for these businesses to support any sudden surge in demand without incurring excessive upfront investments in computing infrastructure. IaaS offers an attractive option for such businesses. We model the fluctuations in the number of users and the impact of these fluctuations on the revenue of the platform providers to develop the underlying logic of selecting a pricing policy. We explore three pricing policies for online platform providers: usage based contract, fixed fee contract and combined fee contract. We determine the conditions under which platform providers selects one contract over another, and discuss the significance of these conditions for IaaS providers. We use our model to determine the impact of the degree of fluctuations in the number of users on the decision problem of the platform providers.

1 Introduction

Cloud-delivered platform and infrastructure services is expected to grow from 964mn in revenue in 2010 to 3.9bn in 2013 - a CAGR of 60% [Hil (2010)]. There are two main reasons behind the high expectations from Infrastructure-as-a-Service (IaaS). First, it allows organizations the flexibility to access and to pay for computing resources as and when required, and thus reduce upfront expenditure on computing infrastructure. Second, organizations can scale up operations quickly as access to computing resources is not a constraint; at the same time organizations can avoid potential losses from capital investments on unused computing infrastructure if market conditions force a scaling down of operations. One of the better known success

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stories of leveraging IaaS for rapid growth is Zynga, the online gaming publisher [Gan (2010)]. Zynga's initial predictions of the success of FarmVille were much less than what they experienced in reality; for the first 26 weeks FarmVille added 1 million net new users per week. Zynga had run out of in-house data center space within a few weeks after the launch of Farmville, and utilized Amazon EC2 to scale up. Today, Zynga's ability to support 10 million active users per day depends primarily on IaaS providers. Social networking sites experience huge surges in users during certain events, for example, natural disasters, political crises, etc. It is difficult for these companies to invest on computing infrastructure to support such spiky demand as the infrastructure would lie idle after the number of users reduces to initial levels. The above examples serve to highlight the importance of IaaS for those online businesses which experience unexpected and massive fluctuations in number of users on a regular basis. These businesses are primarily two sided platforms where one side, more often the consumer side, is subsidized. The revenue side, advertisers, are charged a premium. An increase in the number of users attracts more advertisers since advertisers can now reach a larger audience. This is known as cross side network effects [Parker and Alstyne (2005)]. The advertising fee increases with more advertisers competing for a limited number of slots, and therefore increase in the number of users indirectly results in higher revenues for the online platform providers. Therefore, it is important that such businesses (online platform providers) have enough computing resources to support a sudden spike in consumer demand, and at the same time be flexible enough to scale down when the number of users goes down. IaaS offers a cost effective way of achieving this desired flexibility.

Two major concerns evolve from both the stakeholders in this cloud computing platform: pricing Iaas by Iaas providers and selection of pricing policies by online platform providers. Importance of pricing IaaS for online two sided platform providers is mentioned in papers like Marston et al. (2011). Literature on pricing policies for information goods [Maskin and Riley (1984), Wilson (1993)] talks of pricing based on the usage of the information product. In this paper, we focus on selection of pricing policies by online platform providers. We consider IaaS usage to be the amount of computing resources used per unit time, and non-linear usage based pricing can be used in pricing the service. Software vendors who provide software-as-a-service have adopted this pricing model, for example, salesforce.com. In addition, fixed-fee pricing (independent of usage) as well as a combination of both fixed fee and a usage based pricing can be adopted by IaaS providers. Recently, the problem of pricing cloud services under scarcity of resources have been studied with the objective of helping the providers decide on services that should be rejected or accepted [Anandasivam and Weinhardt (2010)]. In a more generalized discussion on non-linear pricing of information goods, Sundararajan [Sundarajan (2004)] has shown that accounting for the near zero marginal costs of information goods along with the costs of administering a usage based pricing schedule can explain the profitability of fixed fee pricing. In this paper we model the fluctuations in the number of users of an online platform provider and its consequent impact on the revenues in order to develop a deeper understanding of the pricing policies of IaaS providers. This work contributes to the literature on pricing cloud services and provides guidelines for online platform providers on selection of pricing policies offered.

In Section 2, we introduce the basic notations, definitions and functional properties which we use to prove results in subsequent sections. We introduce the pricing policies considered in this paper in Section 3. The selection problem of online platform provider with two available pricing policies: usage based fee and fixed fee, is discussed in Section 4. Section 5 elaborates on the impact of fixed fee contract towards the profit gained by IaaS providers. We show the shift in online platform provider's preferences when combined fee contract is introduced in Section 6. It also discusses the results found in limiting cases of user variability. We conclude this paper by highlighting our key contributions and suggesting some interesting future directions of this research work.

2 Notations and Definitions

A monopoly IaaS provider offers computing infrastructure that may be used by online platform providers in varying quantities. We assume that the variable cost of offering a computing resource for a unit time is zero. We also assume a cost of administering usage based fee, and call it transaction costs. The online platform providers face fluctuations in the number of users over time; we term this user variability .We denote it by σ, and is assumed to vary from 0 to ∞. We represent σ mathematically as $|\Delta\eta|/\Delta t$, the absolute value of the change in number of users from time period t to time period $t+\Delta t$. We also define the term revenue response (ρ) of an online platform provider as $\frac{\Delta R/R}{\Delta\eta/\eta}$, where the numerator is the ratio of the change in revenue, ΔR, to the original revenue R. By revenue, we mean only the revenues coming from advertisement. The denominator is the ratio of the change in number of users, $\Delta\eta$, to the original number of users, η. The online platform providers are heterogeneous, and we index them by their revenue response $\rho \in \left[\underline{\rho},\ \overline{\rho}\right]$. The net utility of an online platform provider with revenue response ρ , and user variability σ is expressed as $U(q(\rho), \rho, \sigma) - p$, where $q(\rho)$ is the quantity of computing resources used and p is the price paid by the online platform provider. We consider a price function $\tau(q(\rho))$ to be paid by online platform providers with revenue response ρ in usage based contract. $U(q(\rho), \rho, \sigma)$ denotes the utility function of the online platform provider with the following properties:

(i) $U(0, \rho, \sigma) = 0$; $\partial U/\partial q \geqslant 0$; $\partial^2 U/\partial^2 q < 0 \; \forall \; q$
(ii) $\partial U/\partial \sigma \leqslant 0$; $\partial U/\partial \rho > 0$
(iii) $\frac{\partial^2 U}{\partial q.\partial \rho} > 0$; $\frac{\partial^2 U}{\partial q.\partial \sigma} < 0 \; \forall \; q$
(iv) $lim_{q(\rho)\to\infty} U(q(\rho), \rho, \sigma) = V(\rho, \sigma) \; < \; \infty$
(v) $\lim_{\sigma\to\infty} U(q(\rho), \rho, \sigma) = U_H(q(\rho), \rho) \; > \; 0$; $\frac{\partial U_H}{\partial q} \geqslant 0$; $\frac{\partial^2 U_H}{\partial q \partial \rho} > 0$
(vi) $\lim_{\sigma\to 0} U(q(\rho), \rho, \sigma) = U_L(q(\rho), \rho) \; < \; \infty$; $\frac{\partial U_L}{\partial q} \geqslant 0$; $\frac{\partial^2 U_L}{\partial q \partial \rho} > 0$

Property (ii) states that online platform providers with higher values of revenue response will get higher utility, and those with higher values of user variability will get less utility. Property (iii) states that online platform providers with higher values of revenue response will get a higher increase in utility than those with lower values of revenue response for the same increase in usage of computing infrastructure. At the same time, platform providers with higher values of user variability will get a lower increase in utility than those with lower user variability for the same increase in usage. Property (iv) simply states that there is a maximum bound on the utility that an online platform provider can get from unlimited usage. Similarly, properties (v) and (vi) put the minimum and the maximum bounds on the utility for very large and very small user variability respectively.

3 Pricing policies

In this paper, we consider the following pricing policies for computing resources:

(i) Fixed fee: The platform provider pays a pre-specified fixed amount T for unlimited use.

(ii) Usage based fee: In this policy, there is a price for each unit of computing resource used, and the entire schedule of quantity price pairs is available to all the customers. From the revelation principle [Fudenberg and Tirole (1991)] we can assume that the platform provider will select the price quantity pair that has been designed for her. The usage based contract is represented by a menu of quantity - price pairs $[q(t), \tau(q(t))]$ where $t \in [\underline{\rho}, \; \overline{\rho}]$. This pricing schedule must satisfy two constraints: *Incentive Compatibility **[IC]**:* For each ρ, $U(q(\rho), \rho, \sigma) - \tau(q(\rho)) \geqslant U(q(t), \rho, \sigma) - \tau(q(t)) \; \forall \; t \in [\underline{\rho}, \; \overline{\rho}]$ and *Individual Rationality **[IR]**:* For each ρ, $U(q(\rho), \rho, \sigma) - \tau(q(\rho)) \geqslant 0$. If the two conditions are met, then a platform provider with revenue response ρ will choose the pair, $[q(\rho), \tau(q(\rho))]$

(iii) Combined fee: This is a common pricing model in the IaaS industry. There is a usage based fee in addition to the fixed fee component. The usage based component is much lower compared to the fee of a pure usage based contract, and therefore the usage will be much higher. However, a buyer will not treat this policy as a fixed fee, infinite usage plan. In some cases the availability of computing resources is guaranteed (Amazon EC2); for some other cases these contracts come with a specialized consulting service (Rackspace). In Section 6, we will look at how the combined fee contract affects the choice of platform providers while deciding the pricing contract.

4 Selection problem of online platform provider

In this section we look at the conditions which dictate the choice of the platform provider when he has two options: to subscribe to the fixed fee or to the usage based fee contract. We first state some initial results associated to usage based contract.

Lemma 1. *If $q(\rho)$ is the capacity booked using incentive compatible contract, then $\partial q/\partial \rho \geqslant 0$.*

Proof. Lets assume $\partial q/\partial \rho < 0$, Therefore $q(\rho) > q(\rho + \epsilon)$ for $\epsilon > 0$. Using condition [IC],

$$U(q(\rho), \rho, \sigma) - \tau(q(\rho)) \geqslant U(q(\rho + \epsilon), \rho, \sigma) - \tau(q(\rho + \epsilon)) \tag{1}$$

When the revenue response is $\rho + \epsilon$, using condition [IC],

$$U(q(\rho + \epsilon), (\rho + \epsilon), \sigma) - \tau(q(\rho + \epsilon)) \geqslant U(q(\rho), (\rho + \epsilon), \sigma) - \tau(q(\rho)) \tag{2}$$

Adding up Equation 1 and Equation 2 yields the following equation:

$$U(q(\rho + \epsilon), (\rho + \epsilon), \sigma) - U(q(\rho), (\rho + \epsilon), \sigma) \geqslant U(q(\rho + \epsilon), \rho, \sigma) - U(q(\rho), \rho, \sigma) \tag{3}$$

From Equation 3, it shows that $\partial U/\partial \rho \leqslant 0$ as $q(\rho) > q(\rho+\epsilon)$ - a contradiction of utility function property, which completes the proof. □

Lemma 2. *If preference function of the online platform provider is defined as $F(q(\rho), \rho, \sigma) = U(q(\rho), \rho, \sigma) - \tau(q(\rho))$, then:*

(a) $\partial \tau/\partial q \geqslant 0$

(b) $F(q(\rho), \rho, \sigma)$ *is strictly non-decreasing in* ρ.

(c) $F(q(\rho), \rho, \sigma)$ *is non-increasing in* σ.

Proof. Applying first order condition to condition [IC],

$$\partial F/\partial q = 0 \tag{4}$$

$$\frac{\partial U}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial \tau}{\partial q}.\frac{\partial q}{\partial \rho} = 0 \tag{5}$$

$$\frac{\partial U}{\partial q}, \frac{\partial q}{\partial \rho} \geqslant 0, \text{Hence } \frac{\partial \tau}{\partial q} \geqslant 0 \tag{6}$$

Equation 6 proves the part (a) of Lemma 2. To prove part (b),

$$\frac{\partial F}{\partial \rho} = \frac{\partial U}{\partial q}.\frac{\partial q}{\partial \rho} + \frac{\partial U}{\partial \rho} - \frac{\partial \tau}{\partial q}.\frac{\partial q}{\partial \rho} \tag{7}$$

Using Equation 5, Equation 7 can be rewritten as $\frac{\partial F}{\partial \rho} = \frac{\partial U}{\partial \rho}$. As $\frac{\partial U}{\partial \rho} > 0$, it proves part (b) of Lemma 2, i.e. $\frac{\partial F}{\partial \rho} > 0$. To prove part (c) of Lemma 2, differentiating $F(q(\rho), \rho, \sigma)$ w.r.t. σ yields

$$\partial F/\partial \sigma = \partial U/\partial \sigma \leqslant 0 \tag{8}$$

Hence the third part of the lemma is proved. □

When the online platform provider is given an option of fixed fee contract along with usage based contract, he will go for fixed fee contract if and only if,

$$V(\rho, \sigma) - T \geqslant U(q(\rho), \rho, \sigma) - \tau(q(\rho)) \tag{9}$$

$$V(\rho, \sigma) - U(q(\rho), \rho, \sigma) + \tau(q(\rho)) \geqslant T \tag{10}$$

The left hand side of Equation 10 is named as Fixed Fee Surplus. If the surplus is more than or equal to the fixed fee rent T, then the online platform provider opts for fixed fee contract instead of usage based.

To see the behavior of fixed fee surplus with the change in exogenous variables, i.e. ρ and σ, we state the following results:

Lemma 3. *(a) $X(q(\rho), \rho, \sigma) = V(\rho, \sigma) - U(q(\rho), \rho, \sigma) + \tau(q(\rho))$ is strictly increasing for ρ*

(b) $X(q(\rho), \rho, \sigma)$ is strictly decreasing for σ

Proof of Part (a).

$$\frac{\partial X}{\partial \rho} = \frac{\partial V}{\partial \rho} - \frac{\partial U}{\partial q}\frac{\partial q}{\partial \rho} - \frac{\partial U}{\partial \rho} + \frac{\partial \tau}{\partial q}\frac{\partial q}{\partial \rho} \tag{11}$$

From Equation 5, Equation 11 simplifies to:

$$\frac{\partial X}{\partial \rho} = \frac{\partial V}{\partial \rho} - \frac{\partial U}{\partial \rho} \tag{12}$$

$$\frac{\partial X}{\partial \rho} = (lim_{q(\rho)\to\infty} \frac{\partial U}{\partial \rho}) - \frac{\partial U}{\partial \rho} \tag{13}$$

As $\frac{\partial^2 U}{\partial q \partial \rho} > 0$ and $\frac{\partial U}{\partial q} \geqslant 0$, $(lim_{q(\rho)\to\infty} \frac{\partial U}{\partial \rho}) > \frac{\partial U}{\partial \rho} \; \forall \; q(\rho) < \infty$
From Equation 13, it proves that $\frac{\partial X}{\partial \rho} > 0$. □

Proof of Part (b).

$$\frac{\partial X}{\partial \sigma} = \frac{\partial V}{\partial \sigma} - \frac{\partial U}{\partial \sigma} \tag{14}$$

$$\frac{\partial X}{\partial \sigma} = (lim_{q(\rho)\to\infty}\frac{\partial U}{\partial \sigma}) - \frac{\partial U}{\partial \sigma} \tag{15}$$

As $\frac{\partial^2 U}{\partial q \partial \sigma} < 0$ and $\frac{\partial U}{\partial \sigma} \leqslant 0$, it shows $\frac{\partial V}{\partial \sigma} < \frac{\partial U}{\partial \sigma}\ \forall\ q(\rho) < \infty$
Using these conditions in Equation 14 gives $\frac{\partial X}{\partial \sigma} < 0$ □

We use results found in Lemmas 2 and 3 to present the following proposition:

Proposition 1. *If the online platform provider can avail either the fixed fee contract or the usage based contract, then online platform provider's choice is dependent on following criteria:*

1. *If $V(\underline{\rho}, \sigma) - T \geqslant U(q(\underline{\rho}), \underline{\rho}, \sigma) - \tau(q(\underline{\rho}))\ \forall\ \sigma$, then all online platform providers take fixed fee contract.*
2. *If $V(\overline{\rho}, \sigma) - T < U(q(\overline{\rho}), \overline{\rho}, \sigma) - \tau(q(\overline{\rho}))\ \forall\ \sigma$, then all online platform providers go for usage based contract.*
3. *As fixed fee surplus $X(q(\rho), \rho, \sigma)$ is strictly increasing in ρ and strictly decreasing in σ, there is no unique ρ or σ to choose between fixed fee contract and usage based contract when, $\rho, \sigma \neq 0$ and $\rho, \sigma < \infty$.*

Proof of Parts (a) and (b). Using results found in Lemma 3, we can say that if an online platform provider of revenue response $\underline{\rho}$ adopts the fixed fee contract, then all platform providers with the same user variability, σ will adopt the fixed fee contract. This argument is valid for all user variability σ and hence all online platform providers will go for fixed fee contract. Similarly, if an online platform provider of revenue response $\overline{\rho}$ does not adopt the fixed fee contract, then no platform provider with the same user variability, σ will adopt the fixed fee contract. This proves points (a) and (b) of Proposition 1. □

4.1 The effect of user variability

In this section, we analyze the situation when the customer variability is in two extremes: $\lim_{\sigma\to\infty}$ and $\lim_{\sigma\to 0}$. In our discussions below, we use the following notations and definitions:

(i) $\lim_{q(\rho)\to\infty} U_L(q(\rho), \rho) = V_L(\rho)$
(ii) $\lim_{q(\rho)\to\infty} U_H(q(\rho), \rho) = V_H(\rho)$

For limiting cases in usage based contract, condition **[IC]** holds and is represented as:

[IC-L]: $U_L(q(\rho), \rho) - \tau(q(\rho)) \geqslant U_L(q(t), \rho) - \tau(q(t)) \ \forall \ t \in [\underline{\rho}, \ \overline{\rho}]$
[IC-H]: $U_H(q(\rho), \rho) - \tau(q(\rho)) \geqslant U_H(q(t), \rho) - \tau(q(t)) \ \forall \ t \in [\underline{\rho}, \ \overline{\rho}]$
Fixed fee surpluses are as follows:
$X_L(q(\rho), \rho) = V_L(\rho) - U_L(q(\rho), \rho) + \tau(q(\rho))$ when $\lim_{\sigma \to 0}$ and
$X_H(q(\rho), \rho) = V_H(\rho) - U_H(q(\rho), \rho) + \tau(q(\rho))$ when $\lim_{\sigma \to \infty}$
In limiting cases, fixed fee surpluses have the following property:

Lemma 4. *(a)* $X_L(q(\rho), \rho)$ *is strictly increasing with* ρ.

(b) $X_H(q(\rho), \rho)$ *is strictly increasing with* ρ.

Proof of part (a). Using the first order condition of **[IC-L]**,

$$\frac{\partial U_L}{\partial q}\frac{\partial q}{\partial \rho} - \frac{\partial \tau}{\partial q}\frac{\partial q}{\partial \rho} = 0 \tag{16}$$

$$\frac{\partial X_L}{\partial \rho} = \frac{\partial V_L}{\partial \rho} - \frac{\partial U_L}{\partial q}\frac{\partial q}{\partial \rho} - \frac{\partial U_L}{\partial \rho} + \frac{\partial \tau}{\partial q}\frac{\partial q}{\partial \rho} \tag{17}$$

Using conditions from Equation 16,

$$\frac{\partial X_L}{\partial \rho} = \frac{\partial V_L}{\partial \rho} - \frac{\partial U_L}{\partial \rho} \tag{18}$$

As $\frac{\partial U_L}{\partial q} \geqslant o$ and $\frac{\partial^2 U_L}{\partial q \partial \rho} > 0$, then $\frac{\partial X_L}{\partial \rho} > 0$ (Hence proved) □

Proof of part (b). Using the first order condition of **[IC-H]**,

$$\frac{\partial U_H}{\partial q}\frac{\partial q}{\partial \rho} - \frac{\partial \tau}{\partial q}\frac{\partial q}{\partial \rho} = 0 \tag{19}$$

$$\frac{\partial X_H}{\partial \rho} = \frac{\partial V_H}{\partial \rho} - \frac{\partial U_H}{\partial q}\frac{\partial q}{\partial \rho} - \frac{\partial U_H}{\partial \rho} + \frac{\partial \tau}{\partial q}\frac{\partial q}{\partial \rho} \tag{20}$$

Using conditions from Equation 19,

$$\frac{\partial X_H}{\partial \rho} = \frac{\partial V_H}{\partial \rho} - \frac{\partial U_H}{\partial \rho} \tag{21}$$

As $\frac{\partial U_H}{\partial q} \geqslant o$ and $\frac{\partial^2 U_H}{\partial q \partial \rho} > 0$, then $\frac{\partial X_H}{\partial \rho} > 0$ (Hence proved) □

Using the results found in Lemma 4, we present the following proposition,

Proposition 2. *For limiting conditions of customer variability, i.e.* $\lim_{\sigma \to 0}$ *and* $\lim_{\sigma \to \infty}$*, revenue responses* ρ_L *and* ρ_H *can be found with the option of choosing within two contracts: fixed fee contract and usage based contract:*

(a) $V_L(\rho)-T \geqslant U_L(q(\rho),\rho)-\tau(q(\rho))\ \forall\ \rho \geqslant \rho_L$ *and* $V_L(\rho)-T < U_L(q(\rho),\rho)-\tau(q(\rho))\ \forall\ \rho < \rho_L$ *where* ρ_L *is defined as* $min\{\rho:\ V_L(\rho) - U_L(q(\rho),\rho) + \tau(q(\rho)) = T\}$

(b) $V_H(\rho)-T \geqslant U_H(q(\rho),\rho)-\tau(q(\rho))\ \forall\ \rho \geqslant \rho_H$ *and* $V_H(\rho)-T < U_H(q(\rho),\rho)-\tau(q(\rho))\ \forall\ \rho < \rho_H$ *where* ρ_H *is defined as* $min\{\rho:\ V_H(\rho) - U_H(q(\rho),\rho) + \tau(q(\rho)) = T\}$

(c) $\rho_H\ \geqslant\ \rho_L$

Proof of parts (a) and (b). For $\lim_{\sigma\to 0}$, online platform providers with revenue response $\rho > \rho_L$ will go for fixed fee contract because of the property described in Lemma 4 [part(a)] and by definition of ρ_L. Similarly for $\lim_{\sigma\to\infty}$, online platform providers with revenue response $\rho > \rho_H$ will go for fixed fee contract because of the property described in Lemma 4 [part(b)] and by definition of ρ_H. □

Proof of part (c). Lets assume that $\rho_H < \rho_L$. From the property of ρ_L,

$$V_L(\rho_L) - T \geqslant U_L(q(\rho_L),\rho_L) - \tau(q(\rho_L)) \tag{22}$$

$$V_L(\rho_L) - U_L(q(\rho_L),\rho_L) \geqslant T - \tau(q(\rho_L)) \tag{23}$$

As $\frac{\partial^2 U}{\partial q \partial \sigma} < 0$, Equation 23 can be expressed as:

$$V_H(\rho_L) - U_H(q(\rho_L),\rho_L) < T - \tau(q(\rho_L)) \tag{24}$$

As $\rho_H < \rho_L$,

$$V_H(\rho_H) - U_H(q(\rho_H),\rho_H) < T - \tau(q(\rho_H)) \tag{25}$$

Equation 25 contradicts the basic property of ρ_H, hence $\rho_H \geqslant \rho_L$. □

Proposition 2 states that for very high values of user variability, online platform providers will opt for the fixed fee contract at a higher value of revenue response compared to platform providers with low user variability. The result is intuitive and expected as platform providers who have to deal with high user variability will find the fixed fee contract attractive only when the revenue response is significantly high. This is because fixed fee contract entails a payment independent of usage and can lead to losses if the user demand is spiky in nature. On the contrary, a platform provider with a steady demand can afford to opt for a fixed fee contract for a relatively lower value of revenue response. This is exactly what we find in the pricing policy of Amazon's EC2 [`http://aws.amazon.com/ec2/purchasing-options/`].

5 Impact of fixed fee

In this section we establish that the profits of an IaaS provider will increase if it introduces a fixed fee contract using the following proposition.

Proposition 3. *If transaction costs for the resource provider are non-zero, i.e.* $C(q) > 0$ *for* $q > 0$*, then the resource provider can always increase profits by offering a fixed fee contract.*

Proof. Let $q^*(\rho)$ and $\tau^*(q^*(\rho))$ be the optimal resources used and cost incurred by service provider in usage based contract with a given customer variability σ and revenue response ρ restricted within the limits $[\underline{\rho}, \overline{\rho}]$. If $q^*(\overline{\rho}) = 0$ for a given σ, then from Lemma 1, $q^*(\rho) = 0 \; \forall \; \rho$ given that σ value. In that case, implementing the fixed fee structure with $T = V(\rho, \sigma)$ increases the profit of resource provider.

In the case where $q^*(\overline{\rho}) > 0$, then $C(q^*(\overline{\rho})) > 0$. Any fixed fee T can be chosen such that

$$[\tau^*(q^*(\overline{\rho})) - C(q^*(\overline{\rho}))] \; < T \; < \tau^*(q^*(\overline{\rho})) \tag{26}$$

As $V(\overline{\rho}, \sigma) \; \geqslant \; U(q(\overline{\rho}), \overline{\rho}, \sigma)$ for all σ, it follows that,

$$V(\overline{\rho}, \sigma) - T \; > \; U(q^*(\overline{\rho}), \overline{\rho}, \sigma) - \tau^*(q^*(\overline{\rho})) \tag{27}$$

From Equation 27 and Proposition 1, this is evident that a fraction of customers will adopt the fixed fee and overall profit margin of resource providers will increase because of this. □

6 Combined fee contract

A platform provider will not treat a combined fee policy as an infinite usage contract. In combined fee contract, online platform providers pay a fixed fee T to get considerable reduction in per unit usage fee. In this scenario, for revenue response ρ, online platform providers use resource $q'(\rho)$ instead of $q(\rho)$ (in usage based contract) and $q'(\rho) > q(\rho)$. So, total amount paid by online platform providers in combined fee contract: $T + \tau'(q'(\rho))$, where $\tau'(q(\rho)) < \tau(q(\rho))$.

In a scenario with two types of contracts, i.e. usage based and combined, one will opt for combined fee contract if and only if,

$$U(q'(\rho), \rho, \sigma) - T - \tau'(q'(\rho)) \geqslant U(q(\rho), \rho, \sigma) - \tau(q(\rho)) \tag{28}$$

$$U(q'(\rho), \rho, \sigma) - U(q(\rho), \rho, \sigma) + \tau(q(\rho)) - \tau'(q'(\rho)) \geqslant T \tag{29}$$

We express the left hand side of Equation 29 as combined fee surplus and denote it by $Y(q'(\rho), q(\rho), \rho, \sigma)$. Also, for combined fee contract, the incentive compatibility condition for accepting a particular quantity-price pair,

$[q'(\rho),\ \tau'(\rho)]$ is:

$$\textbf{[ICC]: } U(q'(\rho),\rho,\sigma)-T-\tau'(q'(\rho)) \geqslant U(q'(t),\rho,\sigma)-T-\tau'(q'(t))\ \forall t\ \in [\underline{\rho},\ \overline{\rho}] \tag{30}$$

Lemma 5. *(a) $Y(q'(\rho),q(\rho),\rho,\sigma)$ is strictly increasing in ρ.*

(b) $Y(q'(\rho),q(\rho),\rho,\sigma)$ is strictly decreasing in σ.

Proof of part (a).

$$\frac{\partial Y}{\partial \rho} = \frac{\partial U'}{\partial q'}.\frac{\partial q'}{\partial \rho} + \frac{\partial U'}{\partial \rho} - \frac{\partial U}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial U}{\partial \rho} + \frac{\partial \tau}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial \tau'}{\partial q'}.\frac{\partial q'}{\partial \rho} \tag{31}$$

[Denoting $U(q(\rho),\rho,\sigma)$ as U and $U(q'(\rho),\rho,\sigma)$ as U' for notational convenience]

From first order condition of Equation 30,

$$\frac{\partial U'}{\partial q'}.\frac{\partial q'}{\partial \rho} - \frac{\partial \tau'}{\partial q'}.\frac{\partial q'}{\partial \rho} = 0 \tag{32}$$

Using Equations 5 and 32, Equation 31 can be rewritten as,

$$\frac{\partial Y}{\partial \rho} = \frac{\partial U'}{\partial \rho} - \frac{\partial U}{\partial \rho} \tag{33}$$

As $\frac{\partial^2 U}{\partial q \partial \rho} > 0$, therefore $\frac{\partial U'}{\partial \rho} > \frac{\partial U}{\partial \rho}$ So, $\frac{\partial Y}{\partial \rho} > 0$ (Hence proved) □

Proof of part (b).

$$\frac{\partial Y}{\partial \sigma} = \frac{\partial U'}{\partial \sigma} - \frac{\partial U}{\partial \sigma} \tag{34}$$

As $\frac{\partial^2 U}{\partial q \partial \sigma} < 0$, therefore $\frac{\partial U'}{\partial \sigma} < \frac{\partial U}{\partial \sigma}$
So, $\frac{\partial Y}{\partial \sigma} < 0$ (Hence proved) □

Proposition 4. *If the online platform provider can avail either the combined fee contract or the usage based contract, then online platform provider's choice is dependent on following criteria:*

(a) If $U(q'(\underline{\rho}),\underline{\rho},\sigma) - T - \tau'(q'(\underline{\rho}),r) \geqslant U(q(\underline{\rho}),\underline{\rho},\sigma) - \tau(q(\underline{\rho}))\ \forall\ \sigma$, then all online platform providers take combined fee contract.

(b) If $U(q'(\overline{\rho}),\overline{\rho},\sigma) - T - \tau'(q'(\overline{\rho}),\rho) < U(q(\overline{\rho}),\overline{\rho},\sigma) - \tau(q(\overline{\rho}))\ \forall\ \sigma$, then all online platform providers go for usage based contract.

(c) As fixed fee surplus $Y(q'(\rho),q(\rho),\rho,\sigma)$ is strictly increasing in ρ and strictly decreasing in σ, there is no unique ρ or σ to choose between combined fee contract and usage based contract when, $\rho,\sigma\ \neq\ 0$ and $\rho,\sigma\ <\ \infty$.

Proof of Parts (a) and (b). Using results found in Lemma 5, we can say that if an online platform provider of revenue response $\underline{\rho}$ adopts the fixed fee contract, then all platform providers with the same user variability, σ will adopt the fixed fee contract. Similarly, if an online platform provider of revenue response $\overline{\rho}$ does not adopt the fixed fee contract, then no platform provider with the same user variability, σ will adopt the fixed fee contract. This proves points (a) and (b) of Proposition 4. □

6.1 Limiting Cases

In this section, we analyze the situation when the customer variability is in two extremes: $\lim_{\sigma \to \infty}$ and $\lim_{\sigma \to 0}$. We use the following notations to prove some related results:

- $\lim_{\sigma \to 0} U(q'(\rho), \rho, \sigma) = U_L(q'(\rho), \rho)$
- $\lim_{\sigma \to \infty} U(q'(\rho), \rho, \sigma) = U_H(q'(\rho), \rho)$

For limiting cases in combined fee contract, constraint **[ICC]** holds and is represented as:

$$\textbf{[ICC-L]:} U_L(q'(\rho), \rho) - \tau'(q'(\rho)) \geqslant U_L(q'(t), \rho) - \tau'(q'(t)) \forall\, t \in [\underline{\rho},\ \overline{\rho}]$$

$$\textbf{[ICC-H]:} U_H(q'(\rho), \rho) - \tau'(q'(\rho), \rho) \geqslant U_H(q'(t), t) - \tau'(q'(t), t) \forall\, t \in [\underline{\rho},\ \overline{\rho}]$$

Combined fee surpluses are as follows:
$Y_L(q'(\rho), q(\rho), \rho) = U_L(q'(\rho), \rho) - U_L(q(\rho), \rho) + \tau(q(\rho)) - \tau'(q'(\rho))$ and
$Y_H(q'(\rho), q(\rho), \rho) = U_H(q'(\rho), \rho) - U_H(q(\rho), \rho) + \tau(q(\rho)) - \tau'(q'(\rho))$

Lemma 6. *(a) $Y_L(q'(\rho), q(\rho), \rho)$ is strictly increasing with ρ.*

(b) $Y_H(q'(\rho), q(\rho), \rho)$ is strictly increasing with ρ.

Proof of part (a).

$$\frac{\partial Y_L}{\partial \rho} = \frac{\partial U_L'}{\partial q'}.\frac{\partial q'}{\partial \rho} + \frac{\partial U_L'}{\partial \rho} - \frac{\partial U_L}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial U_L}{\partial \rho} + \frac{\partial \tau}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial \tau'}{\partial q'}.\frac{\partial q'}{\partial \rho} \tag{35}$$

[Denoting $U_L(q(\rho), \rho, \sigma)$ as U_L and $U_L(q'(\rho), \rho, \sigma)$ as U_L' for notational convenience]

From first order condition of **[ICC-L]**,

$$\frac{\partial U_L'}{\partial q'}.\frac{\partial q'}{\partial \rho} - \frac{\partial \tau'}{\partial q'}.\frac{\partial q'}{\partial \rho} = 0 \tag{36}$$

Using Equations 16 and 36, Equation 35 is reduced to:

$$\frac{\partial Y_L}{\partial \rho} = \frac{\partial U_L'}{\partial \rho} - \frac{\partial U_L}{\partial \rho} \tag{37}$$

As $\frac{\partial^2 U_L}{\partial q \partial \rho} > 0$, so $\frac{\partial U_L'}{\partial \rho} > \frac{\partial U_L}{\partial \rho}$, it proves $\frac{\partial Y_L}{\partial \rho} > 0$ □

Proof of part (b).

$$\frac{\partial Y_H}{\partial \rho} = \frac{\partial U'_H}{\partial q'}.\frac{\partial q'}{\partial \rho} + \frac{\partial U'_H}{\partial \rho} - \frac{\partial U_H}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial U_L}{\partial \rho} + \frac{\partial \tau}{\partial q}.\frac{\partial q}{\partial \rho} - \frac{\partial \tau'}{\partial q'}.\frac{\partial q'}{\partial \rho} \quad (38)$$

[Denoting $U_H(q(\rho), \rho, \sigma)$ as U_H and $U_H(q'(\rho), \rho, \sigma)$ as U'_H for notational convenience]

From the first order condition of **[ICC-H]**,

$$\frac{\partial U'_H}{\partial q'}.\frac{\partial q'}{\partial \rho} + \frac{\partial U'_H}{\partial \rho} - \frac{\partial \tau'}{\partial q'}.\frac{\partial q'}{\partial \rho} = 0 \quad (39)$$

Using Equations 19 and 39, Equation 38 is reduced to:

$$\frac{\partial Y_H}{\partial \rho} = \frac{\partial U'_H}{\partial \rho} - \frac{\partial U_H}{\partial \rho} \quad (40)$$

As $\frac{\partial^2 U_H}{\partial q \partial \rho} > 0$, so $\frac{\partial U'_H}{\partial \rho} > \frac{\partial U_H}{\partial \rho}$

So, $\frac{\partial Y_H}{\partial \rho} > 0$ (Hence proved) □

Proposition 5. *For limiting conditions of customer variability, i.e.* $\lim_{\sigma\to 0}$ *and* $\lim_{\sigma\to\infty}$*, two revenue responses* ρ^c_L *and* ρ^c_H *can be found with the option to choose between two contracts: combined fee contract and usage based contract such that:*

(a) $U_L(q'(\rho), \rho) - T - \tau'(q'(\rho)) \geqslant U_L(q(\rho), \rho) - \tau(q(\rho))\ \forall\ \rho \geqslant \rho^c_L$ *and* $U_L(q'(\rho), \rho) - T - \tau'(q'(\rho)) < U_L(q(\rho), \rho) - \tau(q(\rho))\ \forall\ \rho < \rho^c_L$ *where* ρ^c_L *is defined as* $min\{\rho : U_L(\rho) - U_L(q(\rho), \rho) + \tau(q(\rho)) - \tau'(q'(\rho)) = T\}$

(b) $U_H(q'(\rho), \rho) - T - \tau'(q'(\rho)) \geqslant U_H(q(\rho), \rho) - \tau(q(\rho))\ \forall\ \rho \geqslant \rho^c_H$ *and* $U_H(q'(\rho), \rho) - T - \tau'(q'(\rho)) < U_H(q(\rho), \rho) - \tau(q(\rho))\ \forall\ \rho < \rho^c_H$ *where* ρ^c_H *is defined as* $min\{\rho : U_H(\rho) - U_H(q(\rho), \rho) + \tau(q(\rho)) - \tau'(q'(\rho)) = T\}$

(c) $\rho^c_H \geqslant \rho^c_L$

Proof of parts (a) and (b). For $\lim_{\sigma\to 0}$, online platform providers with revenue response $\rho \geqslant \rho^c_L$ will go for combined fee contract because of the property described in Lemma 6 [part(a)]. Similarly for $\lim_{\sigma\to\infty}$, online platform providers with revenue response $\rho \geqslant \rho^c_H$ will go for combined fee contract because of the property described in Lemma 6 [part(b)]. □

Proof of part (c). Lets assume that $\rho^c_H < \rho^c_L$. From the property of ρ^c_L,

$$\lim_{\sigma\to 0} U(q'(\rho^c_L), \rho^c_L) - T - \tau'(q'(\rho^c_L)) \geqslant \lim_{\sigma\to 0} U(q(\rho^c_L), \rho^c_L) - \tau(q(\rho^c_L)) \quad (41)$$

$$\lim_{\sigma\to 0} U(q'(\rho^c_L), \rho^c_L) - \lim_{\sigma\to 0} U(q(\rho^c_L), \rho^c_L) \geqslant T + \tau'(q'(\rho^c_L)) - \tau(q(\rho^c_L)) \quad (42)$$

As $q'(\rho_L^c) > q(\rho_L^c)$ and $\frac{\partial^2 U}{\partial q \partial \sigma}$, Equation 42 can be expressed as:

$$\lim_{\sigma\to\infty} U(q'(\rho_L^c), \rho_L^c) - \lim_{\sigma\to\infty} U(q(\rho_L^c), \rho_L^c) < T + \tau'(q'(\rho_L^c)) - \tau(q(\rho_L^c))$$

As $\rho_H^c < \rho_L^c$,

$$\lim_{\sigma\to\infty} U(q'(\rho_H^c), \rho_H^c) - \lim_{\sigma\to\infty} U(q(\rho_H^c), \rho_H^c) < T + \tau'(q'(\rho_H^c)) - \tau(q(\rho_H^c)) \quad (43)$$

Equation 43 contradicts the basic property of ρ_H^c, hence $\rho_H^c \geqslant \rho_L^c$. □

We now compare the threshold revenue responses of platform providers at limiting conditions of user variability for two different set of contracts offered to online platform providers: first with usage based and fixed fee and second with usage based and combined. Lemma 7 shows our findings.

Lemma 7. *(a)* $\rho_H^c \geqslant \rho_H$

(b) $\rho_L^c \geqslant \rho_L$

Proof of part (a). Lets assume $\rho_H > \rho_H^c$. From the property of ρ_H,

$$V_H(\rho) - T < U_H(q(\rho), \rho) - \tau(q(\rho)) \;\forall\; \rho < \rho_H \quad (44)$$

So, $\rho = \rho_H^c$ will give the following equation,

$$V_H(\rho_H^c) - U_H(q(\rho_H^c), \rho_H^c) + \tau(q(\rho_H^c)) < T \quad (45)$$

Again from the property of ρ_H^c,

$$U_H(q'(\rho_H^c), \rho_H^c) - U_H(q(\rho_H^c), \rho_H^c) + \tau(q(\rho_H^c)) - \tau'(q'(\rho_H^c)) \geqslant T \quad (46)$$

Now, $V_H(\rho_H^c) = \lim q(\rho) \to \infty U_H(q(\rho), \rho_H^c)$ and $\frac{\partial U}{\partial q} \geqslant 0$

Therefore $V_H(\rho_H^c) \geqslant U_H(q'(\rho_H^c), \rho_H^c)$ and $\tau'(q'(\rho_H^c)) \geqslant 0$. As inequality in Equation 46 is valid, Equation 45 cannot be true and it is proved by contradiction. □

Proof of part (b). Lets assume $\rho_L > \rho_L^c$. From the property of ρ_L,

$$V_L(r) - T < U_L(q(\rho), \rho) - \tau(q(\rho)) \;\forall\; \rho < \rho_L \quad (47)$$

So, $\rho = \rho_L^c$ will give the following equation,

$$V_L(\rho_L^c) - U_L(q(\rho_L^c), \rho_L^c) + \tau(q(\rho_L^c)) < T \quad (48)$$

Again from the property of ρ_L^c,

$$U_L(q'(\rho_L^c), \rho_L^c) - U_L(q(\rho_L^c), \rho_L^c) + \tau(q(\rho_L^c)) - \tau'(q'(\rho_L^c)) \geqslant T \quad (49)$$

Now, $V_L(\rho_H^c) = \lim q(\rho) \to \infty U_L(q(\rho), \rho_L^c)$ and $\frac{\partial U}{\partial q} \geqslant 0$

Therefore $V_L(\rho_L^c) \geqslant U_L(q'(\rho_L^c), \rho_L^c)$ and $\tau'(q'(\rho_L^c)) \geqslant 0$

As inequality in Equation 49 is valid, Equation 48 cannot be true and it is proved by contradiction. □

The interesting result is that platform providers will shift to the combined fee contract at higher values of revenue responses for both the limiting cases. As we have already seen that the impact of introduction of a fixed fee is always profit improving for the IaaS provider, the profits will only increase if there is an additional usage based fee. Therefore, offering a combined fee contract has two effects on the profits of an IaaS provider:

(i) The revenues would go up because of the extra earnings from the usage based component.

(ii) The revenues would reduce because less number of platform providers would adopt the contract.

It would thus make sense for IaaS providers to offer a combined fee contract in lieu of a fixed fee contract if the first effect compensates for the loss due to the second effect.

7 Conclusion and future directions

This work is an attempt to address the issue of pricing IaaS. We focus on online two-sided platform providers - one of the most important customer segments for IaaS providers. The pricing decision is modeled by introducing two important factors - revenue response and the user variability - which affect the utility that a platform provider derives from IaaS offerings. We first determined the conditions which determine the attractiveness of different pricing contracts for the platform providers. We then showed that the platform providers with higher user variability opt for a fixed fee contract at higher values of revenue response. Next, the option of adding a small usage based fee to the fixed fee is analyzed to find out the effect of this change on the decision of platform providers. Our results indicate that platform providers shift to the combined fee contract at higher values of revenue responses compared to the fixed fee contract. In future we are planning to determine the exact pricing structure of the IaaS providers. In practice, Amazon and Rackspace, two of the most important IaaS providers, offer a pure usage based contract and a combined fee contract. Interestingly, Amazon guarantees resource availability for subscribers of their combined contract [`http://aws.amazon.com/ec2/purchasing-options/`]. Rackspace offers a free advisory service with its combined contract [`http://www.rackspace.com/cloud/cloud_hosting_products/servers/pricing`]. These special offers provide an incentive to purchase the combined contract at relatively lower values of revenue response. In the case of Amazon's EC2, the combined fee contract is an attractive option for platform providers with high values of revenue response as a failure to access computing resources might result in a considerable loss of revenues. The same is true for special advisory services that Rackspace offers. We plan to incorporate these special incentives in our model to gain a deeper understanding of this issue.

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