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A dynamic programming algorithm for optimal design and operation of tidal power plants

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***Abstract*- In this paper, a dynamic programming algorithm is proposed and demonstrated on a test case to determine the optimum operating schedule of a barrage tidal power plant to maximize the energy generation over a tidal cycle. Since consecutive sets of high and low tides can be predicted accurately for any tidal power plant site, this algorithm can be used to calculate the annual energy generation for different technical configurations of a tidal power plant. Thus an optimal choice of a tidal power plant design can be made from amongst the different design configurations yielding the least cost of energy generation. Since this algorithm determines the optimal time of operation of sluice gate opening and turbine gates opening to maximize energy generation over a tidal cycle, it can also be used to obtain annual schedule of operation of a tidal power plant and minute-to-minute energy generation, for dissemination amongst power distribution utilities.**

Key words: energy resources, hydroelectric power generation, optimal scheduling, dynamic programming, operations research, design optimization

I.INTRODUCTION

Tides are the phenomenon of cyclic rise and fall of ocean and sea levels due to gravitational attraction of the earth's water body by the sun and moon. These cycles are repeated every 12 hours and 25 minutes. The difference between consecutive highs and lows of the ocean and sea levels is known as the tidal range. The mean of these tidal ranges varies quite widely across the earth due to its topography. Large tidal ranges are found in the Bay of Fundy in Canada with mean tidal range of 10 m, the Severn Estuary between England and Wales with mean tidal range of 8 m, the Brittany coast of Northern France with mean tidal range of 7 m, the Gulf of Khambat in western India with mean tidal range of 7 m, the Gulf of Kachchh in western India with mean tidal range of 5 m and Garolim Bay in South Korea with mean tidal range of 5m[2].

There are two primary methods of exploiting the tidal energy. The first method is to construct a barrage at a suitable location across the sea to create an enclosed reservoir (Fig.1). This reservoir is filled and emptied during successive high and low tides using a combination of sluice gates and turbines installed in the barrage. This method taps the potential energy due to the difference in levels of the sea and reservoir. Energy is generated during falling tides, when the reservoir level is higher than the sea level. Power plants using this method have been set up at La Rance (France) with an installed capacity of 240 MW, Annapolis (Canada) with an installed capacity of 20 MW, Wuyantou (China) with an installed capacity of 3 MW, Kislaya Guba(Russia) with an installed capacity of 2 MW and Uldolmok(South Korea) with an installed capacity of 1 MW. The second method is to place floating stream generating devices on the sea, to tap the kinetic energy associated with tidal currents. Prototype devices with capacities ranging from 2 to 300 kW are under various stages of trial in the US, UK, Norway, Italy, Australia and South Korea.

Energy generation utilizing tides is quite attractive on a number of counts. First, the operating cost incurred is only the cost of maintenance of the generating facilities; there is no cost of fuel. Second, this form of energy generation poses little threat to the environment since there are no gaseous emissions or dangers of radioactive fallout. Third, the energy generated can be predicted accurately months in advance, enabling power generation and distribution agencies to plan and schedule activities in advance. This is in sharp contrast to wind generation, where energy generation cannot be predicted accurately in advance even by a few days. Fourth, since the operation of the tidal plant can be planned in advance, the routine maintenance activities can also be planned with good accuracy quite in advance.

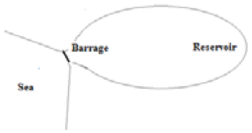

Fig. 1. Barrage Method of Tidal Power Plant

In the barrage method of exploiting tidal energy, the potential head, obtained by the difference of water levels in the sea and the reservoir during rising and falling tides, is used to fill and empty the reservoir (Fig. 2). Sluice gates and turbines are built into the barrage, through which water is allowed to enter or leave the reservoir. Thus when the sea level is falling, water is allowed to flow from the reservoir to the sea through the turbines from time A to time B. The turbines generate electricity, the amount of which is determined by the head obtained by the difference in reservoir and sea levels and the volume of water flowing through the turbines. The flow of water through the turbines is stopped at time B, since the head is not enough for electricity generation. At time C, the sluice gates are opened to allow the water to flow from the sea to the reservoir. The reservoir level thus increases gradually. At time D, the sluice gates are closed since the sea level is going to start falling again.

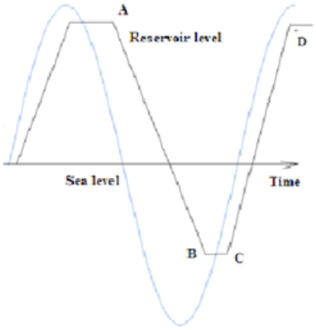

Fig. 2. Typical Tidal Plant Operation

The times A, B, C and D determine the level to which the reservoir is filled and emptied, as well as well the total electricity generation in a given cycle. These timings will depend on the high and low tide levels, reservoir basin characteristics and the characteristics of the turbine and sluice gates. The reservoir basin volume at each basin level decides the amount of rise or fall of reservoir basin level during ingress or exit of water through the turbines and sluice gates in the barrage. The turbine diameter, type and design characteristics decide the maximum water flow possible through the turbine as well as the range of head over which the turbine will generate electricity. The sluice gate area and design characteristics decide the maximum water flow possible.

The time and height of high and low tides in coastal regions can be predicted with a fair degree of accuracy. The Harmonic Method and Response Method are used for the purpose of predicting tides. While both the methods use the past observations of sea level at that location for the calculations, the Harmonic Method requires more than 18.6 years of data, while the Response Method requires only a few months of data. Further, once the time and height of high and low tides are known, the height of tides at any intermediate time between the high and low tides can be calculated by fitting a suitable cosine curve.

There is no research available on the methodology for optimal design of a tidal power plant using dynamic programming. This paper proposes and demonstrates a dynamic programming algorithm to determine the optimum operating schedule of a tidal power plant to maximize the energy generation over a tidal cycle. Since consecutive sets of high and low tides can be predicted accurately for any length of time, this method thereby enables prediction of the annual energy generation. Thus the optimal design of a tidal power plant can be obtained by determining the total energy generation and cost of energy generated through various combinations of design variables such as reservoir basin characteristics and the characteristics of the turbine and sluice gates. This algorithm also determines the optimal time of operation of sluice gate opening and turbine gates opening to maximize energy generation.

The paper is organized as follows: the methodology is described in Section 2, followed by application on a test case obtained in Section 3 and conclusions in Section 4.

II. METHODOLOGY

The problem of determining the optimum operating schedule of a tidal power plant to maximize the energy generation over a tidal cycle, can be viewed as a problem of determining optimal reservoir levels over a tidal cycle. To apply dynamic programming to the problem, a tidal cycle of 12 hours 25 minutes is divided in equal time intervals, each time interval constituting a stage. The problem can thus be reduced to a number of sub-problems, each involving determination of optimal reservoir levels from one stage to the next stage.

The reservoir is always filled during a rising tide utilizing the difference in levels of the reservoir level and sea level. Thus the maximum reservoir level cannot exceed the maximum level of a tide in a tidal cycle. Similarly the reservoir is always depleted during a falling tide utilizing the difference in levels of the reservoir level and sea level. Thus the minimum reservoir level cannot fall below the minimum level of a tide in a tidal cycle. The states at each stage are obtained by dividing the tidal range extending from the minimum to the maximum reservoir levels in equidistant steps. The combination of the number of states and stages needs to be chosen with care, since it decides the accuracy of the solution and the time of computation.

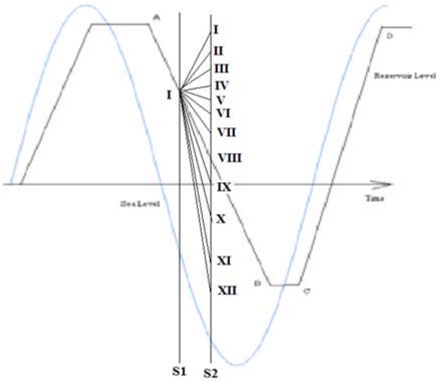

Fig. 3 Dynamic Programming Method

The dynamic programming method involves backward iteration from the last stage till the first stage is reached. For each state in each stage, the path to all states in the next consecutive stage is investigated. For example, the paths from state I of stage S1 to states I,II,III,IV....XII of stage S2 are examined(Fig. 3). Such paths belong to any one the following categories:

(a) The path is possible with generation of electricity with water flowing out of the reservoir through turbines. This situation arises when the path meets the following criteria:

   i. Reservoir level is above the tide level in both the consecutive stages
   ii. Tide level is falling from a particular stage to the consecutive stage
   iii. Reservoir level is falling from a particular stage to the consecutive stage
   iv. Technical conditions related to turbine characteristics are met. Examples of such turbine characteristics are that the head (difference between tide and reservoir levels) should be within maximum and minimum allowed heads for a particular turbine.

   The energy generated in the time period between the consecutive stages is the benefit associated with the path.

(b) The path is possible with the reservoir being filled. This situation arises when the path meets the following criteria:

   i. Reservoir level is below the tide level in both the consecutive stages
   ii. Tide level is rising from a particular stage to the consecutive stage
   iii. Reservoir level is rising from a particular stage to the consecutive stage
   iv. Technical conditions related to sluice gate and turbine characteristics are met. Examples of such turbine characteristics are maximum allowed water flow for a particular turbine and sluice gate combination.

   Since no energy is generated during reservoir filling, the benefit associated with the path is zero. However certain turbines allow generation of electricity for water flow in either direction; the benefit in such cases will be the energy generation in the time period between the consecutive stages during reservoir filling.

(c) The path is possible with reservoir level remaining the same in both the stages. This situation arises when the path meets the following criteria:

   i. Reservoir level remains the same from a particular stage to the consecutive stage
   ii. Reservoir level is either below or above the tide level in both the consecutive stages; or the reservoir level could be below the tide level in a stage and above the tide level in the subsequent stage or vice-versa
   iii. Tide level is rising or falling from a particular stage to the consecutive stage

   Since no energy is generated during reservoir level remaining the same, the benefit associated with the path is zero.

(d) The path is impossible. This situation arises when paths (a) or (b) do not meet the associated criteria. For example, while paths from state I of stage S1 to V,VI,VII,....,XII are possible (implying falling reservoir levels during falling tide), the paths from state I of stage S1 to I,II,III and IV are impossible(implying rising reservoir levels during falling tide). The benefit associated with the path is taken as $-\infty$.

The optimal path for the state under examination is the one that yields the maximum energy benefit. Thus the optimal path from state i of stage s to stage $(s+1)$ is obtained using (1).

$$E_s(i) = \max_{j=1\ to\ n}\left(e_{ij} + E_{s+1}(j)\right) \quad (1)$$

where,

$E_s(i)$: optimal energy generated from state i of stage s to stage S, assuming that the tidal cycle has been partitioned into S equal time periods

$E_{s+1}(j)$: optimal energy generated from state j of stage $s+1$ to stage S, assuming that the tidal cycle has been partitioned into S equal time periods

$E_S(j) = 0, j = 1,2, \ldots, n$

e_{ij} : benefit associated (or energy generated) for a path from state i of stage s to state j of stage $s+1$

n: number of states in each stage

Since the problem satisfies the principle of optimality [1], an optimal path is defined as a succession of states over the stages from a starting reservoir level at the beginning of the tidal cycle, to another finishing reservoir level at the end of the tidal cycle, which results in generation of maximum energy.

III. APPLICATION ON A TEST CASE

An algorithm based on the dynamic programming methodology elaborated on Section 2 was developed as follows. The flow chart of the algorithm is given in Figure 4.

Step 1: Choose the number of stages S and the number of states n. Initialize the energy generated E(S,I) at all the states (I=1,2,....,n) of the last stage S to zero. Let stage S2=S.

Step 2: Check if S2 equals 1. If yes, go to Step 21. If no, go to Step 3.

Step 3: Set Stage S1=S2-1. Calculate the tidal levels T1 and T2 at stages S1 and S2 respectively.

Step 4: Set State I of Stage S1 at 1.

Step 5: Check if I>n. If yes, go to Step 20. If no, go to Step 6.

Step 6:Set State J of Stage S2 at 1.

Step 7: Check if J>n. If yes, go to Step 19. If no, go to Step 8.

Step 8: Calculate the reservoir levels R1 and R2, corresponding to state I at stage S1 and state J at stage S2 respectively. Calculate the average reservoir level R=(R1+R2)/2. Calculate the average tide level T=(T1+T2)/2.

Step 9: Check if T1>T2. If yes, go to Step 15. If no, go to Step 10.

Step 10: Tide level is rising from stage S1 to S2. Check if R2≥R1. If yes, go to Step 11. If no, go to Step 14.

Step 11: Check if R2=R1. If yes, go to Step 13. If no, go to Step 12.

Step 12: Reservoir level is rising from stage S1 to S2. The head is calculated using equation H=T-R. Calculate rate of water flow Q, from the sea to the reservoir through the turbines and sluice gates, corresponding to rise of reservoir level from R1 to R2. Calculate the maximum possible rate of water flow

Qmax corresponding to the turbine and sluice gate design configuration. Check if Q≤ Qmax. If yes, go to Step 13. If no, go to Step 14.

Step 13: Reservoir level remains the same from stage S1 to S2. The path from State I of Stage S1 to Stage S through State J of Stage S2 is possible. Calculate cumulative benefit B(J) for path from State I of Stage S1 to Stage S through State J of Stage S2, as equal to E(S2,J). Increment J by 1. Go to Step 7.

Step 14: The path from State I of Stage S1 to Stage S through State J of Stage S2 is impossible. Calculate cumulative benefit B(J) for path from State I of Stage S1 to Stage S through State J of Stage S2, as equal to (-) Infinity. Increment J by 1. Go to Step 7.

Step 15: Tide level is falling from stage S1 to S2. Check if R1≥R2. If yes, go to Step 16. If no, go to Step 14.

Step 16: Check if R2=R1. If yes, go to Step 13. If no, go to Step 17.

Step 17: Reservoir level is falling from stage S1 to S2. The head H=R-T. Check if turbine generating constraints (such as minimum and maximum head, maximum generating capacity) are satisfied, corresponding to fall of reservoir level from R1 to R2. If yes, go to Step 18. If no go to Step 14.

Step 18: Calculate cumulative benefit B(J) for path from State I of Stage S1 to Stage S through State J of Stage S2, as equal to the sum of E(S2,J) and the energy generated through fall of reservoir level from R1 to R2. If E(S2,J) equals (-) Infinity, then B(J) equals (-) Infinity. Increment J by 1. Go to Step 7.

Step 19: Find the maximum B(J) for all J=1,2,...,n and store it in E(S1,I). The J for which B(J) is maximum is stored in P(S1,I). P(S1,I) indicates the state of Stage S2, which yields the maximum energy generation from State I of Stage S1. If B(J) equals (-) Infinity, for all J=1,2,...,n then (-) Infinity is stored in E(S1,I) and -1 is stored in P(S1,I). Increment I by 1. Go to Step 5.

Step 20: Reduce S2 by 1. Go to Step 2.

Step 21: Find the maximum E(1,J) for all J=1,2,...,n. This is the maximum energy generation possible for a particular tide range, basin configuration, turbine and sluice gate characteristics. The optimal path can be traced using P(1,J) through stages 1 to S.

The scope of error associated with discrete state variables can be reduced to a certain extent, by carrying out a forward iteration after completion of the backward iteration described in Steps 1 to 21 above. During the forward iteration, a search is carried out for a final optimal path only in the vicinity of the approximate optimal path determined by the backward iteration.

The algorithm was applied for determining the optimal design of a tidal power plant at Hansthal creek of the Gulf of Kachchh in the Indian state of Gujarat. The number of stages chosen was 725 (thus ensuring a time period of one minute between consecutive stages) and number of states chosen was 100. The primary data for application of the algorithm are:

- The tide table for Hansthal creek for the year 2011 was generated using WXTide32 software[3]. The tide table was used to obtain a frequency distribution of tide ranges as given in Table 1.

TABLE I
FREQUENCY DISTRIBUTION OF TIDE RANGES

Tide Range (m)	Frequency
1.75-2.25	4

2.25-2.75	38
2.75-3.25	90
3.25-3.75	107
3.75-4.25	161
4.25-4.75	81
4.75-5.25	112
5.25-5.75	53
5.75-6.25	55
6.25-6.75	3

The maximum energy generated is obtained using the algorithm for each turbine and sluice gate configuration and mid-point of each tide range (i.e. for ranges 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5,6 and 6.5 m). The annual energy generation E is thus obtained using the frequency for each tide range.

- The tide level for each stage i is calculated using the equation $MSL + \frac{t}{2}\cos\left(\frac{2\pi i}{S}\right)$ where MSL is the mean sea level (assumed to be 3.5 m), t is the tide range (in metres) and S is the number of stages.
- The duration of all tidal cycles is assumed to be exactly 12 hours 25 minutes (or 745 minutes) .

- The area A of the reservoir is assumed to be 150 sq.km, for all reservoir levels. This assumption simplifies the computation.

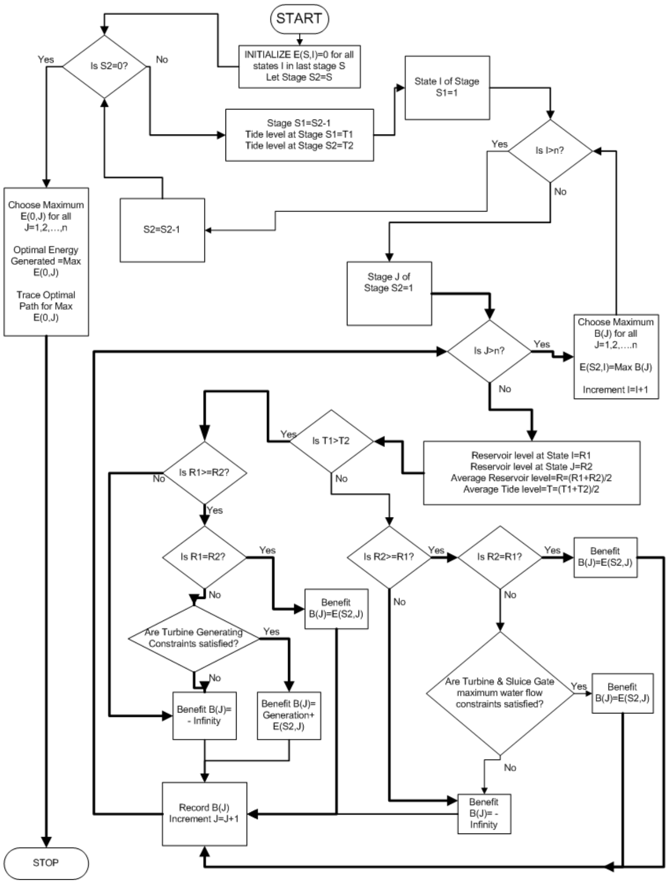

Fig. 4. Flowchart of algorithm

- The maximum rate of water flow possible through the sluice gates and turbines during reservoir filling operation is calculated using the equation $Q_{max=}\ \{(1.6(SG)\sqrt{2gH}) + (4.5D^2\sqrt{H})\}\ m^3/sec$, where SG is the sluice gate area (in sq.m.), g is the acceleration due to gravity (9.81 $m^2\ per\ sec$), H is the head (difference in tide and reservoir levels, in metres) and D is the turbine diameter (assumed 8.5 metres). The actual rate of water flow Q is calculated using the equation $Q = \frac{A(R2-R1)}{(745*60/S)}\ m^3/sec.$ A sluice gate area of 6000 sq.m. was assumed.

- The maximum and minimum reservoir levels are assumed to lie within 95% of maximum and minimum sea levels reached in a tidal cycle.

- The lower and upper permissible limits of head H for energy generation by the turbines are assumed as 1.0 and 5.0 metres respectively.
- The energy generated is calculated using the equation $\frac{745\eta HQg}{60S} KwH$, where η is the generator-turbine efficiency (assumed to be 60%).

- The cost of energy production was calculated using the equation $\frac{400+(1.2*MW)+(0.03*SG)}{E}$, where MW is the installed turbine-generator capacity (in MW) and E is the annual energy production.

The annual energy generated(in GWh) obtained is given in Table 2, along with the relative cost of energy per Kwh for each instance of installed turbine-generator capacity. The optimal installation is 1200 MW.

TABLE II

ANNUAL ENERGY GENERATION AND RELATIVE COST OF ENERGY

Installed Capacity (MW)	Annual Energy Generation(GWh)	Relative cost of energy per Kwh
800	625	1.155
900	759	1.025
1000	829	1.007
1100	891	1.001
1200	948	1.000
1300	1001	1.001
1400	1047	1.013
1500	1093	1.022
1600	1133	1.036

IV.DISCUSSION

The algorithm developed can be further refined by incorporating actual volume-depth characteristics of reservoir, technical constraints and characteristics of turbine-generator and water flow dynamics through the turbines and sluice gates. The algorithm could also be used on a set of tides predicted over a year for obtaining an annual schedule of operation of a tidal power plant and minute-to-minute energy generation programme for dissemination amongst power distribution utilities.

V.REFERENCES

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