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# Determining the Optimal Release Times for Movies on Video-on-Demand Websites

**by**

**Megha Sharma**
Associate Professor, IIM Calcutta, D. H. Road, Joka P.O., Kolkata 700 104 India

**Sumanta Basu**
Associate Professor, IIM Calcutta, D. H. Road, Joka P.O., Kolkata 700 104 India

**&**

**Soumyakanti Chakraborty**
Associate Professor, XLRI Xavier School of Management, C.H. Area (East),
Jamshedpur, Jharkhand – 831035, India

# Determining the Optimal Release Time for Movies on Video-on-demand Websites


**Abstract**

The rise of video-on-demand websites (VoDWs) that allow users to watch movies by streaming them online is challenging the conventional thinking around the release of movies via alternative channels. In this paper, we study the decision problem that a movie distributor (MD) faces while selling the digital viewing rights of a movie before its theatrical release. This problem entails determining the optimal release time for the movie on the VoDW, and the optimal fee to be paid. More specifically, we theoretically model the interaction between a MD selling the digital viewing rights of a movie, and several VoDWs willing to buy these rights, each with its own preferred release time. Using generalized forms of utility functions, we identify conditions under which the optimal release time and fee for a MD and a VoDW coincide. For cases where the optimal release time and fee for a MD and all VoDWs differ, we study the negotiation process, and identify the dominant entity (VoDW or MD,) both for complete and partial dominance scenarios. Our model also takes into account the effects of movie quality, piracy and broadband penetration.



**Keywords:** OR in Entertainment, Optimal Release Time, Contract Design, Movies, Video on Demand


## 1 Introduction and Literature Review

Video-on-demand websites (VoDWs), which started as alternative distribution channels for movies, are now rapidly changing the movie distribution business. Generally considered as multi-sided platform businesses, most of these businesses, e.g. Netflix, started as resellers, procuring the digital viewing rights of movies, tv series, etc. These businesses have since vertically integrated, producing movies and tv series themselves. In response, production companies, once content to act as suppliers for VoDWs, have launched their own VoDWs, e.g. HBO.com. As these websites become increasingly vertically integrated, they challenge the movie industry's traditional distribution channels. For example, at the 2017 Cannes Film Festival, Netflix challenged the tradition of a movie needing to be already released in French theaters to be eligible [5]. There is an extensive body of literature on the changing dynamics of platform businesses, and how businesses that start as multi-sided platforms graduate into resellers, or vertically integrated firms [8–10]. This paper, however, focuses on investigating the impact of the rise of VoDWs on the movie distribution business.

VoDWs' biggest impact is generally on the delay between the theatrical and website release dates. This conflict, however, is not a recent trend. Theater owners have always resisted movie distributors (MDs) shortening the release time of movies on alternative channels, fearing

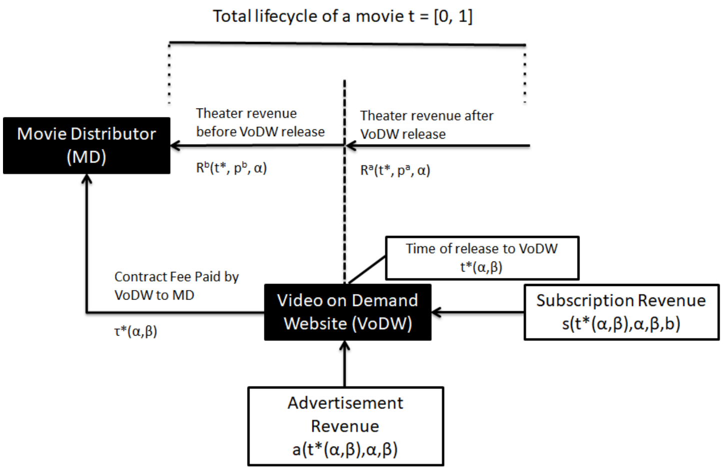



Fig. 1: Business model for one MD and multiple VoDWs

loss of revenue. However, numerous studies have shown the positive impact of shortening the release time. Vogel [21], for example, stated that MDs' revenues from video sales have exceeded those of ticket sales since the 1980s, and [20] in fact revealed an increasing trend to shorten the release time via alternative channels. Most studies, however, have focused on the release time to video. While determining a movie's optimal release time on a VoDW may not appear significantly different from determining it on video, the VoDW business model in fact possesses several differentiating characteristics.

A VoDW is essentially a platform business, which means that the revenue stream is not restricted to subscriber revenues alone; substantial amounts can also be earned from advertising (much more so, comparatively, than movie theaters), leading to lower prices for customers. For VoDWs with lower content aggregation index, whose only source of revenue is subscription fees, exclusive content can be offered, which is difficult for competitors to replicate. VoDWs have, therefore, amassed millions of subscribers in a short space of time, making them extremely powerful, and the norms of movie distribution may soon undergo unprecedented changes [12]. Shorter times between theatrical release and VoDW release may become the norm for all movies, not just those owned by the VoDWs themselves.

In this paper, we study the decision problem that a movie distributor (MD) faces while selling the digital viewing rights of a movie before its theatrical release, determining the optimal release time for the movie on the VoDW, and the optimal fee to be paid. We theoretically model the interaction between a MD selling the digital viewing rights of a movie, and several VoDWs willing to buy these rights, each with its own preferred release time. Using generalized forms of utility functions, we identify conditions under which the optimal release time and fee for a MD and a VoDW coincide. For cases where the optimal release time and fee for a MD and all VoDWs differ, we study the negotiation process, and identify the dominant entity (VoDW or MD), both for partial and complete dominance scenarios. To the best of our knowledge, extant supply-chain literature does not address negotiations regarding the sequential release of a product or service. We also model changes in optimal release time with movie quality, piracy penetration, and broadband penetration.

Figure 1 describes the business model involving one MD and multiple VoDWs. We assume a finite lifecycle for a movie, normalized between $[0, 1]$. Each VoDW is offered a range of release times and corresponding contract fees. Following incentive compatibility (IC), each VoDW chooses the most suitable contract, and the MD must choose the VoDW that maximizes the MD's revenue throughout the lifecycle of the movie, i.e. the sum of ticket sales and the fee paid by the VoDW.

In their 2006 article on research issues in the movie industry, Eliashberg et al. [6] provided an extensive discussion on the literature, and posed research questions for future work. Our paper is relevant to their discussion on theatrical distribution, specifically attempting to answer the question of how optimal launch strategies differ across distribution channels. Further, Vogel [21] stated that distributors' sequential distribution strategies would continue to change. Our work is relevant to this discussion, and may be viewed as an attempt to explore the question of sequential distribution from the perspective of the alternative channel owners, which, to the best of our knowledge, has not been attempted before.

Literature on optimal release timing can be traced back to Frank [7], who theoretically demonstrated that a growing market for an alternative channel shortens the period between theatrical release and the release on the alternative channel. The paper assumed a monotonically decreasing demand function to determine the optimal release time. Lehmann and Weinberg [13] approached the problem from the perspective of the movie producer, and analytically derived the expression for optimal release time. If we consider the release of a movie on an alternative channel as launching a new version of an existing product, [23] demonstrated that it is best done early in the product lifecycle, as long it does not cannibalize the existing product's sales.

The empirical findings from research in this area reflect the theoretical work. Luan and Sudhir [15] showed that, on an average, the optimal release window should be around 2.5 months to maximize revenue. Nelson et al. [16] studied the time between the end of the movie's theatrical run and the release of the movie on DVD. They studied movies from 1998 to 2005, and found that the time reduced quite sharply during this period. The study by Hennig-Thurau [11], on sales data from the US, Germany, and Japan, suggested that simultaneous release in theaters and video may result in a substantial improvement of revenues for the studios in the US, while it would be "devastating" for the theater owners. The improvement of studio revenues was marginal for Germany, and negative for Japan. Their simulations suggested that DVDs should be released three months after the movie release, and that DVD prices should be higher.

Finding the optimal release window is a specific problem, within the category of time consistency problems, first discussed by Coase [4]. Waterman and Weiss [22] used a game-theory approach to study this time consistency problem, and concluded that the movie industry's profit was maximized by a long release window; however, one individual firm could gain by opting for a shorter window, and that this could then become the norm for the industry. Prasad et al. [17] developed a game-theory model considering three classes of customers who had the same expectations regarding the length of the release window. The paper demonstrated that, to maximize profits for a single film, the studio should shorten the window to less than the value that they had previously determined as optimal. However, if this process were repeated, then consumers would adjust their expectations, so the studio would end up shortening the window further, leading to simultaneous release on both channels. However, they showed that, if the studios maximized their profits over multiple movies, then they were able to achieve an equilibrium, without the window shortening over time.

Terrence et al. [20] developed a consumer model by including quality decay of the movie over time, quality/price gap between theater and video channels, and content durability and congestion effects, and demonstrated, among other findings, that when congestion effects are low, films with higher content durability are more likely to be released later via alternative channels, but films with lower content durability should be released using a day-and-date release strategy. A more recent work on release windows by [1] challenged the notion that there is a negative decay in the performance of DVDs as a function of time. The results of this work suggested that it was still optimal to delay the DVD release date to maximize revenue in the DVD channel.

In the context of the coordination problem often experienced in supply-chain contracts [2,3], our paper extends the body of knowledge by investigating a product released sequentially via multiple channels. [14] reviewed supply-chain coordination mechanisms using a framework based on the supply-chain decision structure, and the nature of the demand. In [2], it was shown that, with a standard wholesale price contract, the retailer offered less than the quantity required to maximize the profit of the supply chain, as the retailer would ignore the impact of the decision on the supplier's profits. Numerous studies have explored alternate strategies to incentivize the retailer to increase the order size. Adopting a game-theory approach to the problem, [24] examined option contracts, and demonstrated that they were better than the wholesale price mechanism. The usual assumption of complete information symmetry on the cost structure of the buyer was questioned by [19], who developed a bargaining model that accommodated the information asymmetry of cost structures. [18] focused on the news vendor problem, and offered insights on how supplier pricing policies can serve as coordinating mechanisms.

With reference to the extant literature, our work has an unconventional approach. Our paper deals with an increasingly successful business model, that of VoDWs, which we believe needs to be examined carefully because of its ability to disrupt the distribution business, and also because of its unique characteristics. We believe that our work will add to the literature on optimal timing for movie release via alternative channels by focusing research on alternative channels [6]. This research will also complement the literature on supply-chain coordination by paving the way for new research on supply-chain coordination problems for products that are released sequentially via multiple channels.

## 2 Model

The utility that a VoDW derives from hosting a movie depends on the movie quality (i.e. how good the movie is in attracting and retaining viewers) and the time that elapses between the movie's theatrical release and its VoDW release. We quantify movie quality through a parameter called "success factor" (denoted by $\alpha$, $\alpha \in [0, 1]$). We assume that the rights for VoDW hosting are likely to be negotiated before the movie's theatrical release, as happens for the sale of satellite rights and, therefore, we consider the cast, director etc. as a proxy for the quality of a movie, and as the determinants for $\alpha$.

The revenue generated, and thus the utility derived by a VoDW by hosting a movie, can be categorized into two streams: revenue generated from subscriptions (subscription fee); and revenue generated from advertising shown during the streaming of the movie. Note that the ratio of the revenue generated from subscription and advertising varies significantly among VoDWs, depending on the diversity of the content available. Websites with more diversified content, e.g. Hotstar, generate more revenue from advertising, while VoDWs with more niche content, e.g. Erosnow, generate more revenue from subscriptions. Therefore, we characterize VoDWs using a content aggregation index, $\beta \in [0, 1]$, which is a measure of the diversity of the content available.

The number of subscriptions and, therefore, VoDWs' subscription revenue, also depends on broadband penetration. Therefore, to model the VoDW's utility from subscriptions, we use a parameter called the "broadband penetration index" (denoted by $b$).

We model the utility that a VoDW with content aggregation index $\beta$ derives from hosting a movie with success factor $\alpha$ after time $t$ since its theatrical release, while operating in an area with broadband penetration index $b$ as $U(s(t(\alpha, \beta), \alpha, \beta, b), a(t(\alpha, \beta), \alpha, \beta))$. Here, $s(.)$ denotes the revenue from subscriptions, which is a function of $t, \alpha, \beta$, and $b$. $a(.)$ denotes the revenue from advertising, which is a function of $t, \alpha$ and $\beta$. Note that the time $t$ itself is a function of the success factor of the movie and the content aggregation index (i.e. $\alpha$ and $\beta$).

## 2.1 Notations and Definitions

We now define the parameters and notations used to model the utility functions of the VoDWs and the MDs:

(i) $\alpha$: Success factor of a movie, a function of the cast, director, production company of the movie, etc. $\alpha \in [0, 1]$, where $\alpha = 1$ if the movie has high potential for success and $\alpha = 0$ if it is unlikely to succeed at the box office, only on the cast, production company, etc.

(ii) $\beta$: Content aggregation index of a VoDW. $\beta \in [0, 1]$, where $\beta = 1$ for a highly diversified VoDW, e.g. Hotstar, and $\beta = 0$ for a niche VoDW, e.g. Erosnow.

(iii) $b$: Broadband penetration index. $b \in [0, 1]$, where $b = 1$ for markets with high broadband penetration and $b = 0$ for markets with low broadband penetration.

(iv) $t(\alpha, \beta)$: Time between the theatrical release a movie with success factor $\alpha$ and its release on a VoDW with a content aggregation index of $\beta$. A decision variable for the problem under study.

(v) $s(t(\alpha, \beta), \alpha, \beta, b)$: Subscription revenue earned by a VoDW with content aggregation index $\beta$ by hosting a movie with success factor $\alpha$, after time $t$ since its theatrical release, in a market with broadband penetration index of $b$.

(vi) $a(t(\alpha, \beta), \alpha, \beta)$: Advertising revenue earned by a VoDW with content aggregation index $\beta$ by hosting a movie with success factor $\alpha$, after time $t$ since its theatrical release.

(vii) $U(s(t(\alpha, \beta), \alpha, \beta, b), a(t(\alpha, \beta), \alpha, \beta))$: Utility derived by a VoDW with content aggregation index $\beta$ by hosting a movie with success factor $\alpha$, after time $t$ since its theatrical release, in a market with broadband penetration index of $b$.

(viii) $\tau(\alpha, \beta)$: Amount paid to MD by a VoDW with content aggregation $\beta$ for a movie with success factor $\alpha$.

For brevity, in the remainder of this paper, we refer to the utility derived by a VoDW as $U(s, a)$, unless the complete form is required for understanding. Similarly, we refer to $s(t(\alpha, \beta), \alpha, \beta, b)$ and $a(t(\alpha, \beta), \alpha, \beta)$ as $s(.)$ and $a(.)$ respectively, where showing all the arguments of the functions is not necessary.

We denote the first order partial derivative of a function $f(x_1, x_2, \ldots, x_n)$ with respect to its $i$th argument, i.e. $x_i$, as $f_i(.)$ or $f_i(x_1, x_2, \ldots, x_n)$, depending on the ease of explanation. Similarly, cross derivatives of the function with respect to $i$ and $j$th arguments, i.e. $x_i$ and $x_j$, respectively, is shown as $f_{ij}(.)$ or $f_{ij}(x_1, x_2, \ldots, x_n)$.

## 2.2 Properties of the Functions

In practice, the revenue and utility functions of VoDWs have been observed to follow certain properties. We assume that the utility function that we model in this paper also follows these properties:

(a) $U_1(s, a) \geq 0$; $U_2(s, a) \geq 0$; $U_{11}(s, a) \leq 0$; $U_{22}(s, a) \leq 0$; $U_{12}(s, a) \leq 0$;
The first two conditions imply the obvious properties, that the VoDW's utility is non-decreasing in subscription and advertisement revenues. The next two conditions imply diminishing marginal utility, i.e. the utility increases at a decreasing rate. For example, each additional subscriber must be provided with more incentives, leading to a decrease in the rate of revenue growth. A similar rationale holds for advertising revenue. $U_{12}(s, a) \leq 0$ indicates that rate of change in utility, with respect to change in subscription (or advertising) revenue, reduces as advertising (or subscription, respectively) revenue increases.

(b) Subscription revenue: $s_1(t, \alpha, \beta, b) \leq 0$; $s_2(t, \alpha, \beta, b) \geq 0$; $s_3(t, \alpha, \beta, b) \leq 0$; $s_4(t, \alpha, \beta, b) \geq 0$;
Advertising revenue: $a_1(t, \alpha, \beta) \leq 0$; $a_2(t, \alpha, \beta) \geq 0$; $a_3(t, \alpha, \beta) \geq 0$;
The first set of conditions, $s_1(.), a_1(.) \leq 0$, implies that both subscription and advertising revenues decrease as the release time to VoDW increases. This is observed in practice, as movies released on VoDWs soon after their theatrical release can leverage the promotion created before its theatrical release, and attract more viewers. The second set of conditions, $s_2(t, \alpha, \beta, b) \geq 0, a_2(t, \alpha, \beta) \geq 0$, implies that both subscription and advertising revenues are higher for movies with a higher success factor, $\alpha$. The third set of conditions, $s_3(t, \alpha, \beta, b) \leq 0, a_3(t, \alpha, \beta) \geq 0$, implies that content aggregation $\beta$ positively affects advertising revenue $a(.)$ and negatively affects subscription revenue $s(.)$. This is because, with more content, the search cost for users would increase. Therefore, an increase in content would attract a diverse set of users who would not wish to pay a subscription fee; however, these users would attract advertisers. Finally, $s_4(t, \alpha, \beta, b) \geq 0$ states the obvious, that subscription revenue increases with increase in broadband penetration.

We consider a heterogeneous set of movies and a heterogeneous set of VoDWs, characterized by $\alpha \in [0, 1]$ and $\beta \in [0, 1]$, respectively. For a movie with success factor $\alpha$, the MD offers a set of pairs $(t, \tau)$ for VoDWs to choose from. For every VoDW, we assume that IC and individual

rationality (IR) conditions hold. The IC condition implies that, for a VoDW with content aggregation index $\beta$, given a movie with success factor $\alpha$, the pair $t(\alpha,\beta),\tau(\alpha,\beta)$ maximizes its net utility. Therefore, each VoDW would like to choose a unique pair of time-to-release and contract fee. Moreover, the IR condition implies that this unique pair of time-to-release and contract fee brings a non-negative net utility for the VoDW.
Incentive Compatibility [IC]:

$$U(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta))-\tau(\alpha,\beta)$$
$$\geq U(s(t(\alpha,x),\alpha,\beta,b),a(t(\alpha,x),\alpha,\beta))-\tau(\alpha,x)\forall x\in[\underline{\beta},\overline{\beta}]] \tag{1}$$
$$U(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta))-\tau(\alpha,\beta)\geq 0 \tag{2}$$

## 3 Insights into the VoDW and MD business models

### 3.1 The VoDW business model

Recall that VoDWs are characterized by their content aggregation, measured in terms of content aggregation index $\beta$. We now present insights about the effect that content aggregation has on the release time $t$ and the net utility of the VoDW.

**Lemma 1.** $t_2(\alpha,\beta)\geq 0:$ *Optimal release time is non-decreasing in $\beta$.*

Lemma 1 implies that it is possible for VoDWs with diverse content (i.e. with larger $\beta$) to wait longer for the release of a movie than websites with niche content. It assumes a property of the utility function to be $U_1(s,a)|_\beta > U_1(s,a)|_{\beta+\epsilon}$, emphasizing that the change in VoDW utility caused by a change in subscription revenue is greater when the content aggregation is smaller. The optimal release time of a movie to a VoDW therefore increases with content aggregation index, i.e. VoDWs dedicated to movies prefer a shorter release time, whereas multi-content websites, e.g. Hotstar and Netflix, prefer to a longer release time.

To understand the impact of content aggregation on the decision making of VoDWs, we define the net utility that a VoDW with content aggregation $\beta$ derives from hosting a movie with success factor $\alpha$ as:

$$NU(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta))=U(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta))-\tau(\alpha,\beta)$$

In a decentralized setting, each VoDW will attempt to maximize its net utility by choosing the most suitable release time.

**Lemma 2.** *In an incentive-compatible contract, increasing content aggregation increases net utility of the VoDW only if the change in subscription utility is less than the change in advertising utility.*

Referring to the Appendix for detailed proof, a change in $\beta$ leads to a change in the VoDW's net utility in two ways: change in subscription revenue, i.e. $U_1(s,a)s_3(t,\alpha,\beta,b)$; and change in advertising revenue, i.e. $U_2(s,a)a_3(t,\alpha,\beta)$. In an incentive-compatible contract, the net utility of a VoDW is (non-increasing / increasing) with respect to increasing content aggregation $\beta$, depending on the change in subscription utility (greater than equal to / less than) the change in advertisement utility.

### 3.2 The MD business model

We consider the revenue, and hence the utility, of the MD to be composed of two streams: theater revenue; and the fee paid by the VoDW for the digital rights. We understand that there are other revenue sources, e.g. satellite rights, music rights, etc., but we choose not to consider these as it does not change our problem statement, its treatment, or the results.

Theater revenue depends primarily on the movie quality, i.e. its success factor $\alpha$. However, another important determinant is the prevalence of piracy. For example, in the Indian movie industry, piracy significantly decreases theater revenue. We quantify piracy in the market using a piracy penetration index denoted by $p$. It is important to note that the piracy penetration post-VoDW release is higher than that pre-VoDW release. We indicate the penetration indices as $p^b$ and $p^a$ for pre- and post-release on a VoDW, respectively.

We define a MD's theater revenue per unit of time for a movie with success factor $\alpha$ in a market with piracy penetration $p$ at time $z$ since its theatrical release as $R(z,p,\alpha)$. To account for the probable revenue leakage from theaters due to increased piracy penetration $p$, we assume that $R_2(z,p,\alpha) \leq 0$. We also assume that theater revenue per unit of time is non-increasing over time, i.e. $R_1(z,p,\alpha) \leq 0$, a condition that holds for the majority of movies. Finally, we consider that with increasing success factor $\alpha$, theater revenue per unit of time does not decrease, i.e. $R_3(z,p,\alpha) \geq 0$. We believe that these assumptions are straightforward and do not need further justification.

For a particular movie, with success factor $\alpha$, the MD has to choose a VoDW with an aggregation index of, e.g. $\beta = x$. Therefore, from the multiple VoDWs willing a host a particular movie with success factor $\alpha$, the MD will choose the one that maximizes net profit over time. We develop a closed-form expression for optimal release time (see Proposition 1).

If a movie with success factor $\alpha$ is released on a VoDW with aggregation index $x$ after time $t(\alpha,x)$ since its theatrical release, then the total revenue function of the MD can be written as follows:

$$\Pi_M(x) = \int_0^{t(\alpha,x)} R^b(z,p^b,\alpha)dz + \int_{t(\alpha,x)}^1 R^a(z,p^a,\alpha)dz + \tau(\alpha,x)$$

where $\tau(\alpha,x)$ is the revenue earned by the MD by selling the rights to the VoDW. For a movie with success factor $\alpha$, we now derive the properties for the aggregation index $x^*$ of the optimal VoDW, and the corresponding optimal time $t(\alpha,x^*)$. We present the necessary conditions that $x^*$ must follow in Proposition 1, and in Lemma 3, and prove that the necessary conditions are also sufficient conditions by showing that the MD's total revenue function is concave.

**Proposition 1.** *The optimal time $t(\alpha,x^*)$ to release a movie with success factor $\alpha$ is obtained using the following functional relationship:*

$$R^b(t(\alpha,x^*),p^b,\alpha) - R^a(t(\alpha,x^*),p^a,\alpha) + U_1(s,a)s_1(t,\alpha,x^*,b) + U_2(s,a)a_1(t,\alpha,x^*) = 0$$

*Proof.* The economic rent of content aggregation is defined as:

$$E(\beta) = U(s(t(\alpha, x), \alpha, \beta, b), a(t(\alpha, x), \alpha, \beta)) - \tau(\alpha, x) \tag{3}$$

Following an incentive-compatible contract, the change in economic rent with respect to aggregation index $\beta$ is defined as (see proof of Lemma 2 for detailed steps):

$$E_1(\beta) = U_1(s, a)s_3(t, \alpha, \beta, b) + U_2(s, a)a_3(t, \alpha, \beta) \tag{4}$$

From the properties, $U_1(s, a) \geq 0$; $s_3(t, \alpha, \beta, b) \leq 0$; $U_2(s, a) \geq 0$; and $a_3(t, \alpha, \beta) \geq 0$, we develop two cases, as discussed in Lemma 2, to establish the proof:

1. Case 1: $|U_1(s, a)s_3(t, \alpha, \beta, b)| \leq U_2(s, a)a_3(t, \alpha, \beta)$: Magnitude of change in subscription utility does not exceed the increase in advertising revenue in $\beta$. In this case, the VoDW's economic rent does not decrease in $\beta$. This situation is typically valid for VoDWs with lower subscription revenues.

2. Case 2: $|U_1(s, a)s_3(t, \alpha, \beta, b)| > U_2(s, a)a_3(t, \alpha, \beta)$: Magnitude of change in subscription utility exceeds the increase in advertising revenue in $\beta$. In this case, the VoDW's economic rent decreases in $\beta$. This situation is typically valid for VoDWs with higher subscription revenues.

**Case 1:** The minimum economic rent is defined for a VoDW with $\beta = 0$ as:

$$E(0) = U(s(t(\alpha, 0), \alpha, 0, b), a(t(\alpha, 0), \alpha, 0)) - \tau(\alpha, 0) = E_{min} \tag{5}$$

As $E(\beta)$ increases with aggregation index $\beta$, the following expression holds true:

$$E(\beta) = E_{min} + \int_0^{\beta} [U_1(s, a)s_3(t, \alpha, y, b) + U_2(s, a)a_3(t, \alpha, y)]dy \tag{6}$$

Substituting the expression of $E(\beta)$ from Equation 6 in Equation 3 leads to:

$$\begin{aligned}\tau(\alpha, \beta) =& U(s(t(\alpha, \beta), \alpha, \beta, b), a(t(\alpha, \beta), \alpha, \beta)) - \\ & \{E_{min} + \int_0^{\beta} [U_1(s, a)s_3(t, \alpha, y, b) + U_2(s, a)a_3(t, \alpha, y)]dy\}\end{aligned} \tag{7}$$

The modified total profit expression for a MD after substituting $\tau(\alpha, \beta)$ from Equation 7 is as follows:

$$\begin{aligned}\Pi_M(x) =& \int_0^{t(\alpha,x)} R^b(z, p^b, \alpha)dz + \int_{t(\alpha,x)}^{1} R^a(z, p^a, \alpha)dz + \\ & U(s(t(\alpha, x), \alpha, x, b), a(t(\alpha, x), \alpha, x)) - \\ & \{E_{min} + \int_0^{x} [U_1(s, a)s_3(t, \alpha, y, b) + U_2(s, a)a_3(t, \alpha, y)]dy\}\end{aligned} \tag{8}$$

From the first order condition (FOC) of total profit ($\Pi^M(x)$) maximization for a MD, i.e. differentiating Equation 8 with respect to $x$ and equating to zero,

$$\begin{aligned}\frac{d\Pi_M(x)}{dx} =& t_2(\alpha,x)R^b(t(\alpha,x),p^b,\alpha) - t_2(\alpha,x)R^a(t(\alpha,x),p^a,\alpha)\\ &+ U_1(s,a)s_1(t,\alpha,x,b).t_2(\alpha,x) + U_1(s,a)s_3(t,\alpha,x,b)\\ &+ U_2(s,a)a_1(t,\alpha,x).t_2(\alpha,x) + U_2(s,a)a_3(t,\alpha,x)\\ &- U_1(s,a)s_3(t,\alpha,x,b) - U_2(s,a)a_3(t,\alpha,x) = 0 \end{aligned} \tag{9}$$

From Equation 7, considering $t_2(\alpha,x) \neq 0$, the optimality condition is:

$$R^b(t(\alpha,x^*),p^b,\alpha) - R^a(t(\alpha,x^*),p^a,\alpha) + U_1(s,a)s_1(t,\alpha,x^*,b) + U_2(s,a)a_1(t,\alpha,x^*) = 0 \tag{10}$$

**Case 2:** The maximum economic rent is defined for a VoDW with $\beta = 1$ as:

$$E(1) = U(s(t(\alpha,1),\alpha,1,b),a(t(\alpha,1),\alpha,1)) - \tau(\alpha,1) = E_{max} \tag{11}$$

As $E(\beta)$ decreases with aggregation index $\beta$, the following expression holds true:

$$E(\beta) = E_{max} - \int_\beta^1 [U_1(s,a)s_3(t,\alpha,y,b) + U_2(s,a)a_3(t,\alpha,y)]dy \tag{12}$$

Substituting the expression of $E(\beta)$ from Equation 12 in Equation 3 leads to:

$$\begin{aligned}\tau(\alpha,\beta) =& U(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta)) -\\ &\{E_{max} - \int_\beta^1 [U_1(s,a)s_3(t,\alpha,y,b) + U_2(s,a)a_3(t,\alpha,y)]dy\}\end{aligned} \tag{13}$$

Substituting the expression of $\tau(\alpha,\beta)$ from Equation 13 in the total revenue expression for an MD, we get:

$$\begin{aligned}\Pi_M(x) =& \int_0^{t(\alpha,x)} R^b(z,p^b,\alpha)dz + \int_{t(\alpha,x)}^1 R^a(z,p^a,\alpha)dz +\\ & U(s(t(\alpha,x),\alpha,x,b),a(t(\alpha,x),\alpha,x)) -\\ &\{E_{max} - \int_x^1 [U_1(s,a)s_3(t,\alpha,y,b) + U_2(s,a)a_3(t,\alpha,y)]dy\}\end{aligned} \tag{14}$$

Differentiating Equation 14 with respect to $x$ yields:

$$\begin{aligned}\frac{d\Pi_M(x)}{dx} =& t_2(\alpha,x)R^b(t(\alpha,x),p^b,\alpha) - t_2(\alpha,x)R^a(t(\alpha,x),p^a,\alpha)\\ &+ U_1(s,a)s_1(t,\alpha,x,b).t_2(\alpha,x) + U_1(s,a)s_3(t,\alpha,x,b)\\ &+ U_2(s,a)a_1(t,\alpha,x).t_2(\alpha,x) + U_2(s,a)a_3(t,\alpha,x)\\ &- U_1(s,a)s_3(t,\alpha,x,b) - U_2(s,a)a_3(t,\alpha,x) = 0 \end{aligned} \tag{15}$$

From Equation 15, considering $t_2(\alpha, x) \neq 0$, the optimality condition becomes:

$$R^b(t(\alpha, x^*), p^b, \alpha) - R^a(t(\alpha, x^*), p^a, \alpha) + U_1(s, a)s_1(t, \alpha, x^*, b) + U_5(s, a)a_1(t, \alpha, x^*) = 0 \quad (16)$$

□

The optimality conditions derived above to find the optimal aggregation index $x^*$ and the corresponding optimal release time $t^*$ are valid, as concavity of the total revenue expression is proved in Lemma 3. We present the proof in the Appendix.

**Lemma 3.** *The total revenue function of a MD is concave.*

Using Proposition 1 and Lemma 3, we have addressed the problem, faced by MDs, of choosing the ideal VoDW, when a set of incentive-compatible contracts specifying the optimal release time, and the corresponding contract fee, are given. It is important to note here that the existence of incentive-compatible contracts ensures that, for a movie with a given success factor, there exists a VoDW whose optimal time to host the movie, and the optimal contract fee, are the same as those of the MD, i.e. there is no coordination problem between the MD and the VoDW. In Lemma 4, we explore the behavior of optimal release time $t^*$ with changes in broadband penetration $b$ and piracy penetration $(p^a - p^b)$.

**Lemma 4.** *(a) The optimal release time $t^*$ increases with broadband penetration if the change in marginal subscription revenue increases with time, i.e. $s_{14}(t, \alpha, \beta, b) \geq 0$.*

*(b) The optimal release time $t^*$ increases with increase in piracy penetration.*

Lemma 4(a) highlights a phenomenon that is slightly counter intuitive. It concludes that an increase in broadband penetration does not pose an immediate threat to movie theatre owners, as they may benefit from showcasing the movie exclusively in theatres for longer. This assumes that advertisement revenue surpasses subscription revenue in the long run, and that VoDWs such as Hotstar will dominate the market in future, i.e. $s_{14}(t, \alpha, \beta, b) \geq 0$. Note that the subscription revenue for a movie decreases as release time $t$ increase, $s_1(t, \alpha, \beta, b) \leq 0$. The marginal change in revenue decreases with broadband penetration and, hence, it incentivizes VoDWs to agree to a later release time. Lemma 4(b) emphasizes that a higher piracy penetration forces MDs to delay the release time. This finding is intuitive, as piracy penetration increases once the movie is released and, hence, theatre revenue is affected.

## 4 Coordination between the VoDW and the MD

In this section, we relax the assumption regarding the existence of an incentive-compatible contract, and explore the interaction between the stakeholders if the optimal release times ($t^M$ for the MD and $t^V$ for the VoDW), differ, and study the negotiation process required to reach a release time acceptable to both parties.

We both present conditions under which the eventual release time of a movie will match the optimal release time for either the MD or the VoDW, and conditions under which both the stakeholders will agree to an intermediate release time. We call these scenarios complete dominance, and partial dominance scenarios, respectively. For example, if the MD is the dominant player in a complete dominance scenario, the VoDW will eventually agree to alter the release time to $t^M$, and vice versa. To arrive at the conditions necessary for a complete dominance scenario, we introduce a simple metric called "utility ratio", $\eta$ (defined later), which is derived from the utility function of the VoDW and the revenue function of the MD. We derive numerical thresholds for $\eta$ under which the VoDW or the MD is dominant. We then study the behaviour of $\eta$, and thus, that of the dominance relationship between the two stakeholders, as the success factor ($\alpha$) varies. For the partial dominance scenario, we derive the conditions to help determine the eventual release time for a movie. Finally, we study the optimal release time for the movie, when the objective is to maximize the overall profit throughout the lifecycle of a movie, including net utility and theatre revenue.

For simplicity, we consider the coordination problem between one MD and one VoDW for a movie with success factor $\alpha$. Consider a situation where optimal timings for a movie with success factor $\alpha$, chosen by the VoDW and the MD, are $t^V(\alpha)$ and $t^M(\alpha)$, respectively, with payouts $\tau(t^V(\alpha))$ and $\tau(t^M(\alpha))$. Note that $t^V(\alpha)$ and $t^M(\alpha)$ maximize the net utilities of the VoDW and the MD, respectively, and hence satisfy the following expressions:

$t^V(\alpha)$ is the maximizer for the net utility of the VoDW. That is,

$$t^V(\alpha) = \underset{t(\alpha)}{\arg\max}\ NU(t(\alpha)) = U(s(t(\alpha), \alpha, b), a(t(\alpha), \alpha)) - \tau(t(\alpha)) \tag{17}$$

$t^M(\alpha)$ is the maximizer for the total revenue function of the MD. That is,

$$\begin{aligned} &t^M(\alpha) = \underset{t(\alpha)}{\arg\max}\ \Pi_M(t(\alpha)), \text{ where} \\ &\Pi^M(t(\alpha)) = \int_0^{t(\alpha)} R^b(z, p^b, \alpha)dz + \int_{t(\alpha)}^1 R^a(z, p^a, \alpha)dz + \tau(t(\alpha)) \end{aligned} \tag{18}$$

Therefore, $t^V(\alpha)$ and $t^M(\alpha)$ satisfy the FOC for individual net utility maximization, respectively, as given below:

$$\text{VoDW: } U_1(s, a)s_1(t^V, \alpha, b) + U_2(s, a)a_1(t^V, \alpha) - \tau_1(t^V) = 0 \tag{19}$$

$$\text{MD: } R^b(t^M, p^b, \alpha) - R^a(t^M, p^a, \alpha) + \tau_1(t^M) = 0 \tag{20}$$

Based on values of $t^M(\alpha)$ and $t^V(\alpha)$, there are three possible scenarios that could occur for a given movie. We consider each of these three scenarios and determine the value of $t^*(\alpha)$ at which the movie will eventually be released.

**Scenario 1:** $t^M(\alpha) = t^V(\alpha)$**:** Scenario 1 is ideal for both the stakeholders, and since their individual optimal release times are the same, the movie is released on the VoDW at $t^*(\alpha) = t^M(\alpha) = t^V(\alpha)$. Note that, from Equation 19 and 20, Scenario 1 would occur only if $t^*(\alpha)$ satisfies the optimality conditions for both the VoDW and the MD:

$$R^b(t^*(\alpha), p^b, \alpha) - R^a(t^*(\alpha), p^a, \alpha) + \tau_1(t^*(\alpha)) = U_1(s, a)s_1(t^*(\alpha), \alpha, b) + U_2(s, a)a_1(t^*(\alpha), \alpha) - \tau_1(t^*(\alpha)) = 0 \tag{21}$$

Simplifying equation 21 leads to the following equation:

$$R^b(t^*(\alpha), p^b, \alpha) - R^a(t^*(\alpha), p^a, \alpha) + U_1(s,a)s_1(t^*(\alpha), \alpha, b) + U_2(s,a)a_1(t^*(\alpha), \alpha) = 0 \qquad (22)$$

From Equation 22, the optimal release times for both stakeholders converges when the change in theatre revenue pre- and post-VoDW release exactly matches the change in net utility owing to changes in subscription and advertising revenues.

**Scenarios 2 ($t^M(\alpha) < t^V(\alpha)$) and 3 ($t^M(\alpha) > t^V(\alpha)$):** We refer to these scenarios as coordination scenarios, since the individual optimal release times for the stakeholders do not match, and some coordination effort is required before the movie can be released. In both these scenarios, three possibilities can emerge: 1. the MD agrees to shift its release timing to $t^V$, i.e. $t^*(\alpha) = t^V(\alpha)$; 2. the VoDW agrees to the MD's optimal timing of $t^M$, i.e. $t^*(\alpha) = t^M(\alpha)$; or 3. both stakeholders agree to an intermediate time, i.e. $t^M(\alpha) < t^*(\alpha) < t^V(\alpha)$ (under Scenario 2) or $t^V(\alpha) < t^*(\alpha) < t^M(\alpha)$ (under Scenario 3). The first two cases are complete dominance cases, and the last is one of partial dominance. In Lemma 5, we present conditions which lead to complete and partial dominance cases under Scenario 2.

**Lemma 5.** *(a) One of the stakeholders will agree to release the movie at the optimal time for the other stakeholder, i.e. $t^*(\alpha) = t^M(\alpha)$ or $t^*(\alpha) = t^V(\alpha)$ if*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| \neq |U_1(s,a)s_1(t, \alpha, \beta, b) + U_2(s,a)a_1(t, \alpha, \beta) - \tau_1(t)| \ \forall\, t \in [t^M, t^V]$$

*(b) Both the stakeholders will agree to an intermediate time $t^*(\alpha)$ where $t^M(\alpha) < t^*(\alpha) < t^V(\alpha)$ only if*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| = |U_1(s,a)s_1(t, \alpha, \beta, b) + U_2(s,a)a_1(t, \alpha, \beta) - \tau_1(t)| \neq 0 \ \ at\ t = t^*(\alpha)$$

*and*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| < |U_1(s,a)s_1(t, \alpha, \beta, b) + U_2(s,a)a_1(t, \alpha, \beta) - \tau_1(t)| \ \forall\, t^M \leq t < t^*(\alpha)$$

*and*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| > |U_1(s,a)s_1(t, \alpha, \beta, b) + U_2(s,a)a_1(t, \alpha, \beta) - \tau_1(t)| \ \forall\, t^*(\alpha) < t \leq t^V$$

Note that Lemma 5 presents conditions for complete and partial dominance under Scenario 2 only; a minor variation of this lemma is valid for Scenario 3. For brevity, we do not present it here. In Sections 4.1 and 4.2, we explore each of these scenarios in detail, identifying the required coordination and the optimal release time, which provides a framework for the negotiation process.

## 4.1 Complete Dominance Case:

To understand the negotiation process, consider the total revenue that the MD would realize if the VoDW agrees to release the movie at the MD's optimal timing i.e. $t^M(\alpha)$ instead of its own optimal time $t^V(\alpha)$:

$$D\Pi^M(t^M, t^V) = [\int_0^{t^M(\alpha)} R^b(z, p^b, \alpha)dz + \int_{t^M(\alpha)}^1 R^a(z, p^a, \alpha)dz + \tau(t^M(\alpha))] - [\int_0^{t^V(\alpha)} R^b(z, p^b, \alpha)dz + \int_{t^V(\alpha)}^1 R^a(z, p^a, \alpha)dz + \tau(t^V(\alpha))] \quad (23)$$

Similarly, the difference in net utility that the VoDW would have realized if its optimal release time $t^V(\alpha)$ were chosen instead of $t^M(\alpha)$, is:

$$DU^V(t^V, t^M) = [U(s(t^V(\alpha), \alpha, b), a(t^V(\alpha), \alpha)) - \tau(t^V(\alpha, ))] - [U(s(t^M(\alpha), \alpha, b), a(t^M(\alpha), \alpha)) - \tau(t^M(\alpha))] \quad (24)$$

The stakeholder leading the negotiation would depend upon the values of $D\Pi^M(t^M, t^V)$ and $DU^V(t^V, t^M)$. For example, if $D\Pi^M(t^M, t^V) < DU^V(t^V, t^M)$, VoDW will attempt to convince the MD by matching the additional benefit that the MD would accrue if the deal were struck in favor of the MD. The reverse argument is true when $D\Pi^M(t^M, t^V) > DU^V(t^V, t^M)$.

While these conditions indicate whether the movie will eventually be released at the $t^M(\alpha)$ or at $t^V(\alpha)$, we further explore the changes in each stakeholder's payoff, and derive interesting insights useful for both MDs and VoDWs. For this, we define the change in theatre revenue with change in the MD's release time from $t_1$ to $t_2$ as follows:

$$\Delta R(t_1, t_2) = [\int_0^{t_1} R^b(z, p^b, \alpha)dz + \int_{t_1}^1 R^a(z, p^a, \alpha)dz] - [\int_0^{t_2} R^b(z, p^b, \alpha)dz + \int_{t_2}^1 R^a(z, p^a, \alpha)dz]$$

Similarly, the change in the VoDW's utility with change in release time from $t_1$ to $t_2$ is expressed as:

$$\Delta U(t_1, t_2) = U(s(t_1(\alpha), \alpha, b), a(t_1(\alpha), \alpha)) - U(s(t_2(\alpha), \alpha, b), a(t_2(\alpha), \alpha))$$

It is important to note here that both $\Delta R(t_1, t_2)$ and $\Delta U(t_1, t_2)$ capture the changes in the MD's total theater revenue and the VoDW's utility from advertising and subscriptions, respectively, and these are exclusive of the contract fee $\tau$. In other words, both $\Delta R(t_1, t_2)$ and $\Delta U(t_1, t_2)$ capture only the changes in revenue and utility due to changes in exogenous factors when the release time is changed from $t_1$ to $t_2$. For a given set of values for other parameters, we define a utility ratio parameter $\eta(t_1, t_2)$ that captures the relative change in theatre revenue vis--vis the change in the VoDW's utility when the release time is changed from $t_1$ to $t_2$:

$$\eta(t_1, t_2) = \frac{\Delta R(t_1, t_2)}{\Delta U(t_1, t_2)}$$

$\eta$ emphasizes on the tradeoff between the change in the VoDW's utility and the MD's total revenue when the release time for the movie is changed from $t_1$ to $t_2$. Note that in both Scenarios 2 and 3, either $\Delta U(t^M, t^V)$ or $\Delta R(t^M, t^V)$ will be non-positive and hence $\eta(t^M, t^V) \in (-\infty, 0]$.

The following lemma helps us to understand the relative bargaining power of the stakeholders. Note that we assume stakeholders to be rational and utility-/profit-maximizing.

**Lemma 6.** *(a) When $t^M < t^V$, the condition $\eta(t^M, t^V) > -1$ ensures that the MD will be dominant in the release time negotiations, i.e. $t^*(\alpha) = t^M(\alpha)$, else the VoDW will be dominant, i.e. $t^*(\alpha) = t^V(\alpha)$.*

*(b) When $t^M > t^V$, the condition $\eta(t^M, t^V) < -1$ ensures that the MD will be dominant in the release time negotiations, i.e. $t^*(\alpha) = t^M(\alpha)$, else the VoDW will be dominant , i.e. $t^*(\alpha) = t^V(\alpha)$.*

Lemma 6 establishes a single-parameter criterion, $\eta$, to understand how the two stakeholders' positions change in a complete dominance scenario, resulting in the shift in optimal release time favoring either the MD or the VoDW. We demonstrate the implications of Lemma 6 under the two different scenarios.

Scenario 2 [$t^M(\alpha, \beta) < t^V(\alpha, \beta)$]:
As $R^b(t, p^b, \alpha) > R^a(t, p^a, \alpha)$, it leads to $\Delta R(t^M, t^V) < 0$. From the definition of $\Delta U(t^M, t^V)$, it is obvious that $\Delta U(t^M, t^V) \geq 0$. In this case, the release time optimal for the MD will be retained if $|\Delta R(t^M, t^V)| < \Delta U(t^M, t^V)$, i.e. the VoDW's increase in revenue from shortening the release time to $t^M$ is more than the MD's decrease in revenue from keeping it at $t^M$. Hence, the VoDW will agree to compensate the MD for loss of the theatre revenue, and the minimum amount that MD will expect from the VoDW is $\tau(t^V) + |\Delta R(t^M, t^V)|$. This amount is less than the contract fee that the VoDW would have paid to the MD, i.e. $\tau(t^M)$ and, hence, is also beneficial for the VoDW. This condition is typically applicable for movies with steady performance (good or bad) in theaters, leading to no major impact for a change in the VoDW's release time. Typically, these movies are specific to particular genres, e.g. comedy, where the impact of screen size is minimal. An extreme case would be $\Delta R(t^M, t^V) = 0$, meaning there is no change in theatre revenue. This may be possible if the movie were no longer in theatres. In all the other cases, i.e. $\eta(t^M, t^V) < -1$, the VoDW would be dominant in the negotiations.

Scenario 3 [$t^M(\alpha, \beta) > t^V(\alpha, \beta)$]:
In this scenario, $\Delta R(t^M, t^V) > 0$ and $\Delta U(t^M, t^V) < 0$, yielding $\eta(t^M, t^V) < 0$. In this case, the release time optimal for the MD will also be retained if $t^*(\alpha) = t^M(\alpha)$, if $\Delta R(t^M, t^V) > |\Delta U(t^M, t^V)|$, i.e. the MD's increase in revenue from lengthening the release time to $t^M$ is greater than the VoDW's decrease in revenue. This situation is applicable for movies in the early stages of their lifecycle (e.g. one or two weeks after release), or movies where screen size and ambience play a significant role, e.g. action or sci-fi movies.

Next, we develop closed-form expressions to understand the profit or utility gap, $\Delta U(t^M, t^V)$ and $\Delta R(t^M, t^V)$, between optimality and forced compliance for both stakeholders, and understand how the gaps change with respect to success factor $\alpha$ in both scenarios.

**Lemma 7.** *(a) $\Delta R(t^M, t^V)$ is non-increasing in $\alpha$ for $t^M(\alpha) < t^V(\alpha)$.*

*(b) $\Delta R(t^M, t^V)$ is non-decreasing in $\alpha$ for $t^M(\alpha) > t^V(\alpha)$.*

Lemma 7 indicates how theatre revenue changes with success factor $\alpha$, i.e. the higher the success factor, the higher the theatre revenue losses as the time to release is shortened.

**Lemma 8.** *(a) $\Delta U(t^M, t^V)$ is non-decreasing in $\alpha$ for $t^M(\alpha) < t^V(\alpha)$.*

*(b) $\Delta U(t^M, t^V)$ is non-increasing in $\alpha$ for $t^M(\alpha) > t^V(\alpha)$.*

Lemma 8 indicates how the utility of the VoDW changes with success factor $\alpha$, i.e. the higher the success factor, the more utility increases with an earlier time to release, and decreases with a later time to release.

In Table 1 we summarize the changes in $\Delta R(t^M, t^V)$, $\Delta U(t^M, t^V)$ and $\eta(t^M, t^V)$ with changes in $\alpha$, along with their signs. For example, with increasing $\alpha$, $\Delta R(t^M, t^V)$ is non-increasing, and it takes a negative value whenever $t^M < t^V$. Note that the change in $\eta(t^M, t^V)$ is dependent on absolute values of $\Delta R(t^M, t^V)$ and $\Delta U(t^M, t^V)$.

Tab. 1: Movement of $\eta(t^M, t^V)$ with change in $\alpha$

| | | Scenario 2 ($t^M < t^V$) | Scenario 3 ($t^M > t^V$) |
|---|---|---|---|
| $\Delta R(t^M, t^V)$ | Sign | $\leq 0$ (non-positive) | $\geq 0$ (non-negative) |
| | $\alpha \uparrow$ | non-increasing | non-decreasing |
| $\Delta U(t^M, t^V)$ | Sign | $\geq 0$ (non-negative) | $\leq 0$ (non-positive) |
| | $\alpha \uparrow$ | non-decreasing | non-increasing |
| $\eta(t^M, t^V)$ | Sign | $\leq 0$ (non-positive) | $\leq 0$ (non-positive) |
| | $t^* = t^M$ | $[0, -1]$ | $(-1, -\infty)$ |
| | $t^* = t^V$ | $(-1, -\infty)$ | $[0, -1]$ |

Next, we create an elasticity expression, $\psi$, to understand how the negotiation paradigm changes with success factor $\alpha$. We define the elasticity of the utility ratio as $\psi(\Delta R, \Delta U) = \frac{\Delta R'/\Delta R}{\Delta U'/\Delta U}$ to understand the relative change of theatre revenue with utility of the VoDW. Proposition 2 summarizes the change in the negotiation process with $\alpha$.

**Proposition 2.** *While deciding on release time to the VoDW in a complete dominance scenario for movies with $\alpha \in [0, 1]$, release time will depend on the following conditions. We have segregated the proposition into two parts for easier understanding.*
*Proposition 2.1 for scenario 2 ($t^M < t^V$):*

*(a) If $t^*(\alpha) = t^M(\alpha)$ for a movie with $\alpha = 0$, then optimal release time $t^*(\alpha)$ will continue to be equal to $t^M(\alpha)$ if $\psi(\Delta R, \Delta U) \geq 0$ and $t^M < t^V$.*

*(b) If $t^*(\alpha) = t^V(\alpha)$ for a movie with $\alpha = 1$, then optimal release time $t^*(\alpha)$ will continue to be equal to $t^V(\alpha)$ if $\psi(\Delta R, \Delta U) \leq 0$ if $t^M < t^V$.*

*(c) If $t^*(\alpha) = t^M(\alpha)$ a movie with $\alpha = \hat{\alpha} > 0$ where $\hat{\alpha}$ is the minimum value of $\alpha$ for which $D\Pi^M > DU^V$, then $t^*(\alpha) = t^M(\alpha)$ for movies with $\alpha \in [\hat{\alpha}, 1]$ if $\psi(\Delta R, \Delta U) \geq 0$.*

*Proposition 2.2 for scenario 3 ($t^M > t^V$):*

*(a) If $t^*(\alpha) = t^V(\alpha)$ for a movie with $\alpha = 0$, then optimal release time $t^*(\alpha)$ will continue to be equal to $t^V(\alpha)$ if $\psi(\Delta R, \Delta U) \geq 0$.*

*(b) If $t^*(\alpha) = t^M(\alpha)$ for a movie with $\alpha = 1$, then optimal release time $t^*(\alpha)$ will continue to be equal to $t^M(\alpha)$ if $\psi(\Delta R, \Delta U) \leq 0$.*

(c) *If* $t^*(\alpha) = t^V(\alpha)$ *a movie with* $\alpha = \hat{\alpha} > 0$ *where* $\hat{\alpha}$ *is the minimum value of* $\alpha$ *for which* $D\Pi^M < DU^V$, *then* $t^*(\alpha) = t^V(\alpha)$ *for movies with* $\alpha \in [\hat{\alpha}, 1]$ *if* $\psi(\Delta R, \Delta U) \geq 0$.

*Proof.* For brevity, we prove the proposition only for scenario 2 ($t^M < t^V$). A similar proof is valid when $t^M > t^V$.

Differentiating $\eta(t^M, t^V)$ with respect to $\alpha$ leads to the following expression:

$$\frac{\partial}{\partial \alpha}(\eta(t^M, t^V)) = \frac{\Delta R' \Delta U - \Delta R \Delta U'}{\Delta U^2} \tag{25}$$

where $\Delta R'$ and $\Delta U'$ indicate the derivative of $\Delta R(t^M, t^V)$ and $\Delta U(t^M, t^V)$ with respect to $\alpha$, respectively. Now the condition of $\eta(t^M, t^V)$, increasing in $\alpha$, leads to the following equation:

$$\begin{aligned} &\Delta R' \Delta U - \Delta R \Delta U' \geq 0 \\ &= \frac{\Delta R / \Delta R'}{\Delta U / \Delta U'} \geq 0 \\ &= \psi(\Delta R, \Delta U) \geq 0 \end{aligned} \tag{26}$$

In equation 26, $\psi(\Delta R, \Delta U)$ is defined as the elasticity of the utility ratio that determines the change of $\eta(t^M, t^V)$ with change in $\alpha$. If $\psi(\Delta R, \Delta U) \geq 0$, $\eta(t^M, t^V)$ is non-decreasing with $\alpha$, and vice versa.

In Proposition 2.1(a), $t^*(\alpha) = t^M(\alpha)$ for a movie with $\alpha = 0$, and this convergence occurs only if $\eta \in [0, -1]$. When $\psi(\Delta R, \Delta U) \geq 0$, $\eta(t^M, t^V)$ is non-decreasing with $\psi$, it ensures $\eta$ will remain in $[0, 1]$, and the optimal release time $t^*(\alpha)$ will match with $t^M(\alpha)$.

In Proposition 2.1(b), $t^*(\alpha) = t^V(\alpha)$ for a movie with $\alpha = 1$, and this convergence occurs only if $\eta \in (-1, -\infty)$. When $\psi(\Delta R, \Delta U) \leq 0$, $\eta(t^M, t^V)$ is non-increasing with $\psi$, it ensures $\eta$ will remain in $(-1, -\infty)$, and optimal release time $t^*(\alpha)$ will match with $t^V(\alpha)$.

In Proposition 2.1(c), we define a threshold, $\alpha = \hat{\alpha} > 0$, where $\hat{\alpha}$ is the minimum value of $\alpha$ for which $D\Pi^M > DU^V$, i.e. $t^*(\alpha) = t^M(\alpha)$ and $\eta = -1$. With $\psi(\Delta R, \Delta U) \geq 0$, $\eta$ is non-decreasing in $\alpha$, and hence continues to be in the range $[0, -1]$ to ensure $t^*(\alpha) = t^M(\alpha)$.

A similar proof is valid for Proposition 2.2 in scenario 3 ($t^M > t^V$). □

**Corollary 1.** (a) *If* $t^*(\alpha) = t^M(\alpha)$ *for a movie with* $\alpha = \hat{\alpha} > 0$, *where* $\hat{\alpha}$ *is the minimum value of* $\alpha$ *for which* $D\Pi^M > DU^V$ *when* $t^M > t^V$, *then* $t^*(\alpha) = t^M(\alpha)$ *for all movies with* $\alpha \in [\bar{\alpha}, 1]$, *where* $\bar{\alpha} \leq \hat{\alpha}$ *with increase in piracy penetration* $\theta$.

(b) *If* $t^*(\alpha) = t^V(\alpha)$ *for a movie with* $\alpha = \hat{\alpha} > 0$, *where* $\hat{\alpha}$ *is the minimum value of* $\alpha$ *for which* $D\Pi^M < DU^V$ *when* $t^M < t^V$, *then* $t^*(\alpha) = t^V(\alpha)$ *for all movies with* $\alpha \in [\bar{\alpha}, 1]$, *where* $\bar{\alpha} \leq \hat{\alpha}$ *with increase in piracy penetration* $\theta$.

Corollary 1 indicates the change in market share opportunities for the two stakeholders for movies with varying $\alpha$ through changes in the coordinated release time $t^*$. For example, in scenario 2 ($t^M < t^V$), consider a baseline scenario with $p^b$ and $p^b + \theta$ indicating the piracy penetration pre- and post-VoDWrelease. In the baseline scenario, movies with $\alpha \in [\hat{\alpha}, 1]$ will be released at $t^* = t^V$, and for the rest, $t^* = t^M$. As piracy penetration $\theta$ increases, more movies, i.e. movies with $\alpha \in [\bar{\alpha}, 1]$, will be released in a shorter time period, $t^V$. In scenario 3 ($t^M > t^V$), more movies, i.e. movies with $\alpha \in [\bar{\alpha}, 1]$, will be released at a later time period, $t^* = t^M$.

## 4.2 Partial Dominance and Channel Coordination

Next, we explore the possibility of agreeing on an intermediate time period $t^I \in [t^M, t^V]$ for both scenarios, following the conditions in Lemma 5. Before we discuss the implications of an intermediate time period $t^I$, we explore the problem, from the perspective of a vertically integrated market, of deriving $t^O$, i.e. a release time that maximizes overall market utility throughout the movie's lifecycle, and identify the coordinating conditions between $t^O$ and $t^I$ (for partial dominance scenarios) and $(t^M/t^V)$ (for complete dominance scenarios).

For a vertically-integrated market, total market utility is defined as:

$$\Pi(t) = \int_0^{t(\alpha)} R^b(z, p^b, \alpha)dz + \int_{t(\alpha)}^1 R^a(z, p^a, \alpha)dz + U(s, a)$$

The FOC to maximize market utility yields equation 27 to find the optimal market release time $t^O$:

$$R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + U_1(s, a).s_1(.) + U_2(s, a).a_1(.) = 0 \tag{27}$$

The coordination condition to reach an intermediate time period $t^I \in [t^M, t^V]$ (reproduced from lemma 5 ) is as follows:

$$|R^b(t^I, p^b, \alpha) - R^a(t^I, p^a, \alpha) + \tau_1(t^I)| = |U_1(s, a)s_1(t^I, \alpha, \beta, b) + U_2(s, a)a_1(t^I, \alpha, \beta) - \tau_1(t^I)| \neq 0$$

Proposition 3 elaborates the channel coordinating conditions for the optimal release time both in complete and partial dominance scenarios.

**Proposition 3.** *(a) In complete dominance scenarios, the independently determined release times of the MD or the VoDW, i.e. $t^M$ or $t^V$, will converge to $t^O$ if the change in contract fee with time, i.e. $\tau_1(t)$, matches the change in net utility of the VoDW and theatre revenue with time.*

*(b) In partial dominance scenarios, a mutually agreed intermediate release time $t^I$ will converge with $t^o$ if the change in net utility of the VoDW and theatre revenue with time is half the change in contract fee with time, i.e. $\tau_1(t)/2$.*

*Proof.* Considering the optimality conditions for a decentralized scenario, as mentioned in equations 20 and in 19, and comparing with equation 27 yields:

$$R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) = -[U_1(s, a).s_1(.) + U_2(s, a).a_1(.)] = -\tau(t) \tag{28}$$

In the partial dominance scenario, from the conditions of coordination of $t^I$ (see Lemma 5 ) to $t^o$, we obtain the following condition:

$$R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) = -[U_1(s, a).s_1(.) + U_2(s, a).a_1(.)] = -\tau(t)/2 \tag{29}$$

□

Proposition 3(a) indicates the standard economic maximization criteria of marginal revenue being equal to marginal cost. In this case, the difference between pre- and post-release theatre revenue should be equal to the marginal change in utility with respect to time. The difference in theatre revenue relates to the loss per time period that the MD incurs owing to the online release at time $t$. This loss should match the marginal change in utility of the VoDW, and also the marginal change of the one-time fee $\tau$ offered to the VoDW. Similar conditions prevail for coordination in the partial dominance scenario, where the marginal change in the one-off fee is half that of the complete dominance scenario, owing to the compromise that each stakeholder makes by shifting from their optimal release times.

## 5 Example

In our example, we choose following functional forms, considering the functional properties as discussed in section 2. If not mentioned separately, we follow same set of notations for the expressions as elaborated below:

$$\begin{aligned}
&s(t,\alpha,\beta,b) = 5 - t^2 + \alpha - \beta + t(b-1) - t\beta + t(\alpha - 1)\\
&a(t,\alpha,\beta) = 4 - t^2 + \alpha + \beta - t\beta + t(\alpha - 1)\\
&U(s,a) = \sqrt{s}(25-a) + \sqrt{a}(25-s)\\
&\tau(\alpha,\beta) = \sqrt{3 - \beta(1+t) + t(\alpha-1)} + \sqrt{3 - t^2 + \alpha - t\beta - t}\\
&t(\alpha,\beta) = 1 - \frac{\alpha}{2} + \frac{\beta}{2}\\
&R^b(z,\alpha) = A - Bz - Cp^b + D\alpha\\
&R^a(z,\theta,\alpha) = E - Fz - G(p^b + \theta) + H\alpha
\end{aligned}$$

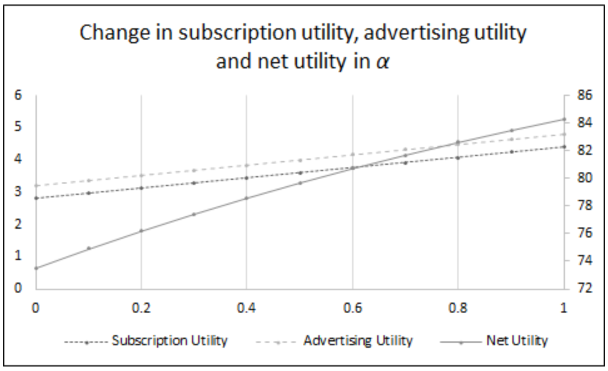
To elaborate on the findings of Lemma 2, we introduce the functional form of net utility as:

$$\sqrt{s}(25-a) + \sqrt{a}(25-s)) - \sqrt{3 - \beta(1+t) + t(\alpha-1)} + \sqrt{3 - t^2 + \alpha - t\beta - t}$$

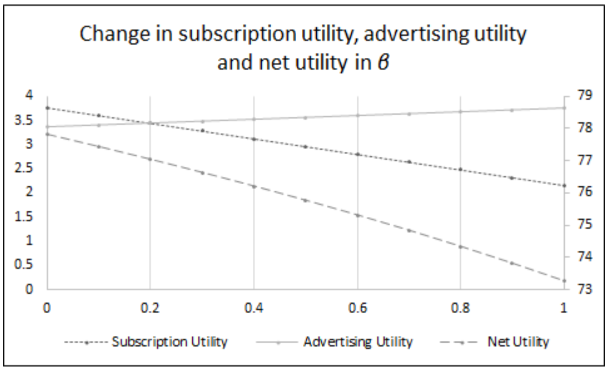
In Figure 2, we illustrate the changes in net utility with changes in $\alpha$, $\beta$ and $b$. The net (subscription and advertising) utility increases with $\alpha$. A similar observation is valid for broadband penetration $b$, as it attracts more users. With change in $\beta$, the change in net utility depends on relative changes in subscription and advertising utilities. In the example chosen, net utility reduces in $\beta$. In a generic setting (see Lemma 2), this will happen if the condition $|U_1(s,a).s_3(t,\alpha,\beta,b)| \leq U_2(s,a).a_3(t,\alpha,\beta)$ holds. In our specific example, this leads to the following condition:

$$t \leq \frac{(\sqrt{s} + \frac{25-s}{2\sqrt{a}}) - (\sqrt{a} + \frac{25-a}{2\sqrt{s}})}{(\sqrt{s} + \frac{25-s}{2\sqrt{a}}) + (\sqrt{a} + \frac{25-a}{2\sqrt{s}})}$$

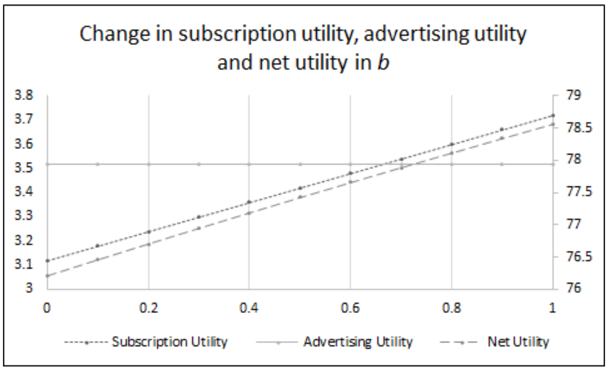
In our example, equation 30 to determine the optimal release time $t^*$, following Proposition 1,




(a) Change in VoDW net utility with $\alpha$




(b) Change in VoDW net utility with $\beta$




(c) Change in in VoDW net utility with broadband penetration $b$

Fig. 2: Change in VoDW utility with $\alpha$, $\beta$ and $b$

is expressed as:

$$(A - Bt - Cp^b + D\alpha) - (E - Ft - G(p^b + \theta) + H\alpha) + (\frac{1}{2\sqrt{s}}(25 - a) - \sqrt{a})(-2t + (b-1) - x) + (\frac{1}{2\sqrt{a}}(25 - s) - \sqrt{s})(-2t - x + (\alpha - 1)) = 0 \tag{30}$$

Replacing $x = 2(t-1) + \alpha$, equation 30 simplifies to:

$$[(A - E) - (Cp^b - Gp^b - G\theta) + \alpha(D - H)] - (B - F)t + [\frac{1}{2\sqrt{(S_1 + S_2 t - t^2)}}(25 - (T_1 + T_2 t - t^2) - \sqrt{(T_1 + T_2 t - t^2)})]((b-1) - \alpha + 2 - 4t) + [\frac{1}{2\sqrt{(T_1 + T_2 t - t^2)}}(25 - (S_1 + S_2 t - t^2) - \sqrt{(S_1 + S_2 t - t^2)})](1 - 4t) = 0 \tag{31}$$

Notations used in equation 31 are as follows: $S_1 = 5 + \alpha - \beta$; $S_2 = \alpha - \beta + b - 1$; $T_1 = 4 + \alpha + \beta$; and $T_2 = \alpha - \beta - 1$.

Because of its algebraic complexity, we were unable to obtain a closed-form root to find $t^*$ from equation 31. Hence, we used numerical analysis techniques in MATLAB to find the value of $t^*$ and its sensitivity with respect to $\alpha$, $b$, and $\theta$. We provide relevant results in Figure 3.

To summarize, $t^*$ increases in $\alpha$. This is understandable, as with increasing $\alpha$, the MD is inclined to postpone the release time to the VoDW to increase theatre revenue. As discussed

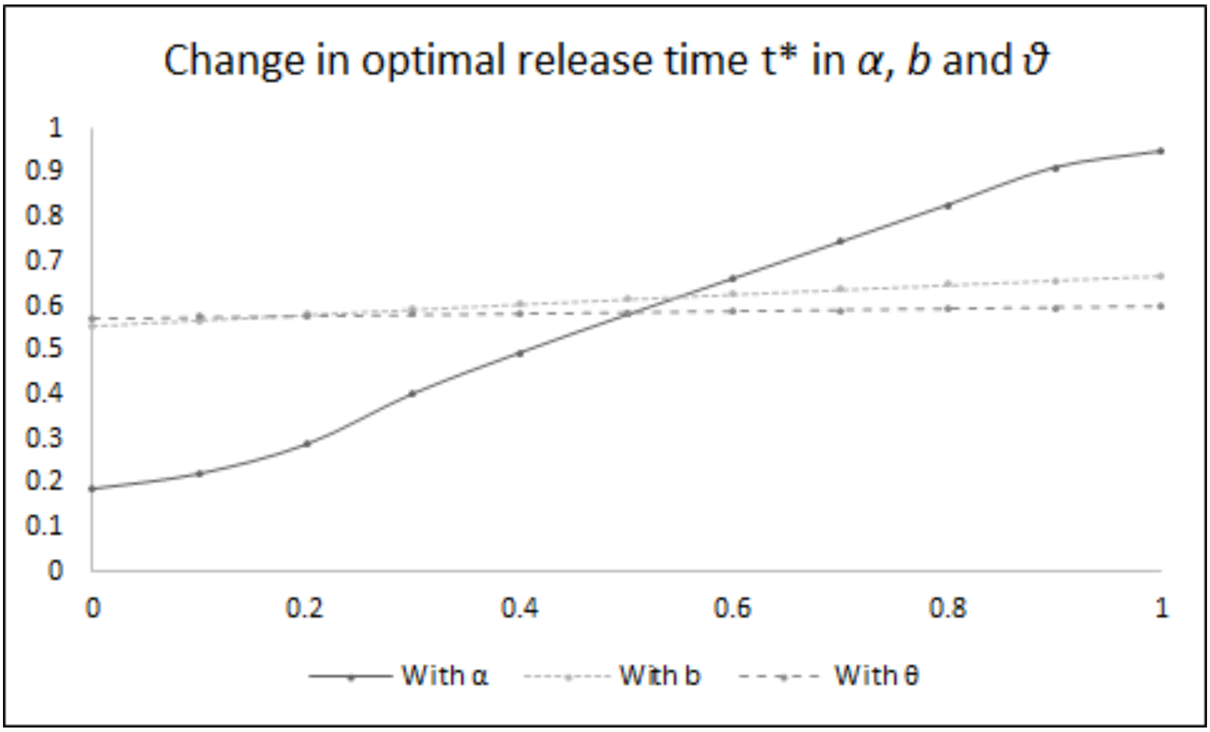



Fig. 3: Change in optimal release time $t$ with $\alpha$, $b$ and $\theta$

in Lemma 4, increasing broadband penetration $b$ increases the release time to the VoDW. In our opinion, this observation is mildly counter intuitive, but in line with practical observations. With increasing broadband penetration $b$, VoDWs with increasing content aggregation will gain a greater share of the market, having less dependency on online movie releases. Hence, VoDWs will not incentivize the MDs sufficiently (by paying higher $\tau$) to keep the online release times closer to theatrical release. Increasing piracy penetration $\theta$ also forces later release times.

## 5.1 Coordination between the MD and the VoDW

In this section, we examine the negotiation process between two stakeholders with different optimal release times, $t^M$ and $t^V$.

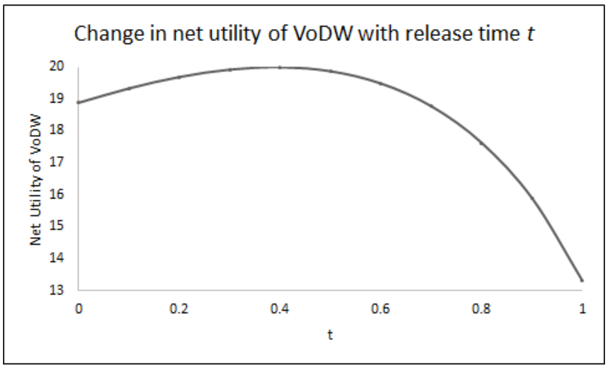
In a decentralized setting, the profit function of the MD is expressed as:

$$
\begin{aligned}
\Pi^M(t) &= \int_0^t (A - Bz - Cp^b + D\alpha)dz + \int_t^1 (E - Fz - Gp^a + H\alpha)dz + \tau(\alpha, \beta) \\
&= (At - \frac{B}{2}t^2 - Cp^b t + D\alpha t) + [E(1-t) - \frac{F}{2}(1-t^2) - G(p^b + \theta)(1-t) + H\alpha(1-t)] \\
&+ \sqrt{3 - \beta(1+t) + t(\alpha - 1)} + \sqrt{3 - t^2 + \alpha - t\beta - t}
\end{aligned}
$$

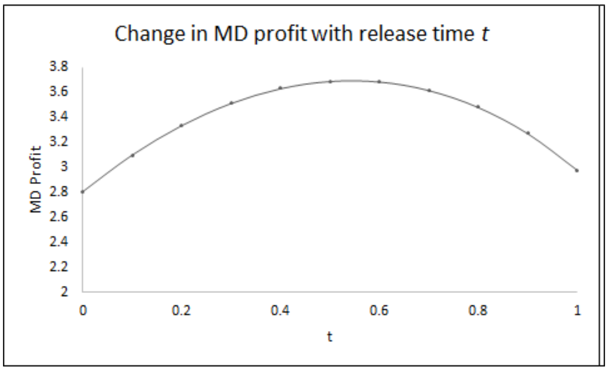
Using the functional forms, we express the net utility of a VoDW as:

$$
U(s,a) - \tau(\alpha,\beta) \quad = (\sqrt{s}(25-a) + \sqrt{a}(25-s)) - \sqrt{3 - \beta(1+t) + t(\alpha-1)} + \sqrt{3 - t^2 + \alpha - t\beta - t}
$$

From the FOC, the profit maximizing condition of a MD in a decentralized setting is as




(a) Change in the VoDW's net utility with release time $t$ and $\alpha = 0.2$, $\beta = 0.4$, $b = 0.3$, $\theta = 0.3$




(b) Change in the MD's profit with release time $t$ and $\alpha = 0.2$, $\beta = 0.4$, $b = 0.3$, $\theta = 0.3$

Fig. 4: Impact of optimal release time $t$, independently decided by the VoDW and the MD, on the MD's profit and the VoDW's net utility

follows:

$$[A - Cp^b + D\alpha - E + F + G(p^b + \theta) - H\alpha] - Bt + 0.5[(3 - \beta) + t(\alpha - \beta - 1)]^{-0.5}(\alpha - \beta - 1)$$
$$+ 0.5[(3 + \alpha) - t(1 + \beta) - t^2]^{-0.5} - (2t + \beta + 1) = 0 \tag{32}$$

From the FOC of the maximization of net utility of a VoDW:

$$[\frac{1}{2\sqrt{(S_1 + S_2 t - t^2)}}(25 - (T_1 + T_2 t - t^2) - \sqrt{(T_1 + T_2 t - t^2)})]((b - 1) - \alpha + 2 - 4t)$$
$$+ [\frac{1}{2\sqrt{(T_1 + T_2 t - t^2)}}(25 - (S_1 + S_2 t - t^2) - \sqrt{(S_1 + S_2 t - t^2)})](1 - 4t)$$
$$+ \frac{\alpha - 1}{2}(3 - t(\alpha - 1))^{-0.5} - \frac{2t + 1}{2}(3 - t^2 + \alpha - t)^{-0.5} = 0 \tag{33}$$

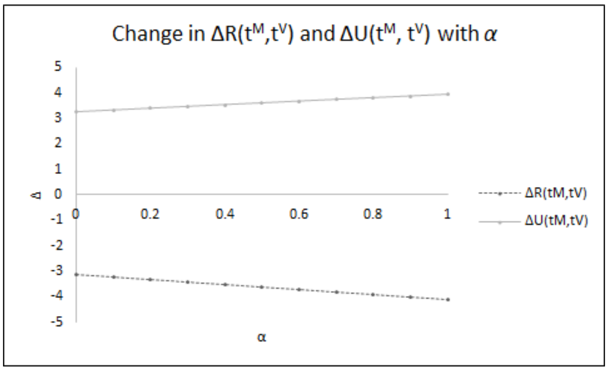
In our example, the changes in the VoDW's net utility, and in the MD's net profit, with respect to release time $t$ are depicted in Figures 4a and 4b, respectively. Both scenarios lead to optimal release times, i.e. $t^M = 0.53$ for the MD and $t^V = 0.4$ for the VoDW, with $\alpha = 0.2$, $\beta = 0.4$, $b = 0.3$ and $\theta = 0.3$.

For the complete dominance scenario, , we illustrate in Figures 5 and 6 changes in $\Delta R(t^M, t^V)$, $\Delta U(t^M, t^V)$ and $\eta(t^M, t^V)$, with $\alpha$ for $t^M < t^V$, and $t^M > t^V$, respectively.

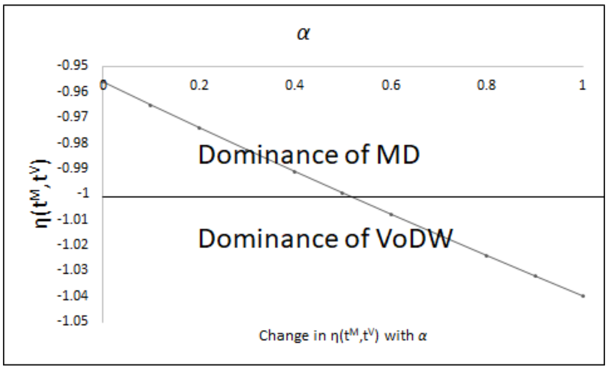
In Figure 5, as explained in Lemma 7 and in Lemma 8, we observe an increase in $\Delta R(t^M, t^V)$ and a decrease in $\Delta U(t^M, t^V)$ with $\alpha$. To understand the dominance of stakeholders with varying $\alpha$, we observe that $\eta(t^M, t^V)$ decreases in $\alpha$ and, after a threshold (in our example, $\alpha$ is 0.5), the dominance shifts from the MD to the VoDW, as depicted in Figure 5b.

Figure 6 illustrates both the non-decreasing nature of $\Delta R(t^M, t^V)$ and $\Delta U(t^M, t^V)$ with changes in $\alpha$ when $t^M > t^V$. The relative dominance in the negotiation process of the VoDW and the MD is shown in Figure 6b, where the MD becomes more dominant in as $\alpha$ increases and $\eta(t^M, t^V)$ decrease. Figures 5 and 6, as summarized in Proposition 2, differentiate clear dominating regions for the VoDW and the MD, based on values of $\alpha$.

In Figures 7a and 7b, we examine the change in threshold metric $\eta(t^M, t^V)$ with changes in broadband $b$ and piracy $\theta$ penetration, when $t^M < t^V$. An increase in broadband penetration $b$

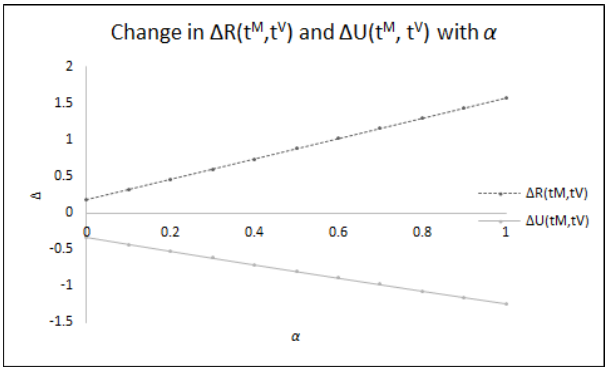



(a) Change in $\Delta R(t^M, t^V)$ and $\Delta U(t^M, t^V)$ with $\alpha$ when $t^M < t^V$

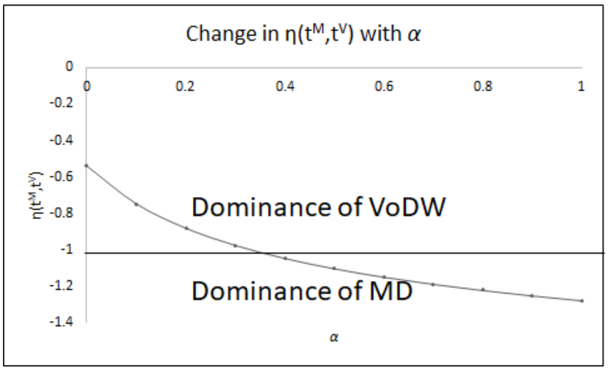



(b) Change in $\eta(t^M, t^V)$ with $\alpha$ when $t^M < t^V$

Fig. 5: Change in $\Delta R(t^M, t^V)$, $\Delta U(t^M, t^V)$ and $\eta(t^M, t^V)$ with change in $\alpha$ when $t^M(0.5) < t^V(0.7)$ with $\beta = 0.5$, $b = 0.5$, $\theta = 0.2$

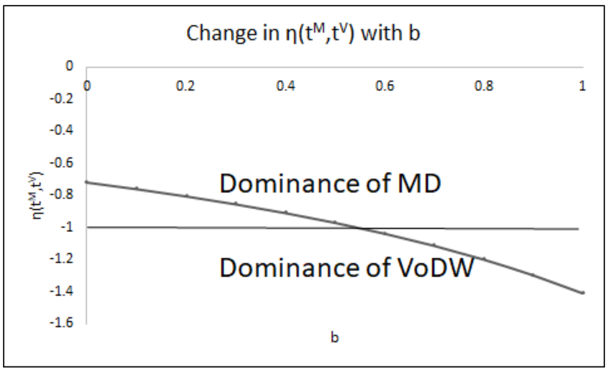



(a) Change in $\Delta R(t^M, t^V)$ and $\Delta U(t^M, t^V)$ with $\alpha$ when $t^M > t^V$

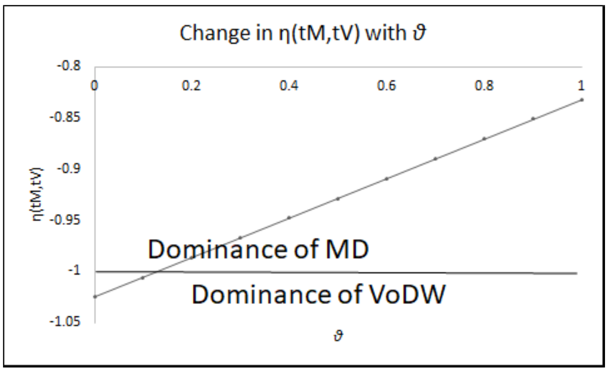



(b) Change in $\eta(t^M, t^V)$ with $\alpha$ when $t^M > t^V$

Fig. 6: Change in $\Delta R(t^M, t^V)$, $\Delta U(t^M, t^V)$ and $\eta(t^M, t^V)$ with change in $\alpha$ when $t^M(0.7) > t^V(0.5)$ with $\beta = 0.5$, $b = 0.5$, $\theta = 0.2$




(a) Change in $\eta(t^M, t^V)$ with $b$ when $t^M < t^V$




(b) Change in $\eta(t^M, t^V)$ with $\theta$ when $t^M < t^V$

Fig. 7: Change in $\eta(t^M, t^V)$with change in $b$ and $\theta$ when $t^M < t^V$ with $\alpha = 0.8$ and $\beta = 0.8$

leads to increased VoDW dominance in negotiations (see figure 7a) and, after a certain threshold ($b = 0.55$ in our example), the VoDW becomes dominant in the negotiation process. The reverse trend is observed in Figure 7b, which shows that, as piracy penetration $\theta$ increases, the MD dominance increases and, after a certain threshold (0.15 in our example), the MD becomes dominant.

## 6 Concluding Discussions

There has been unprecedented growth in broadband penetration in the developing world, particularly in India, following massive investments in infrastructure, and reductions in data charges owing to increased competition. VoDWs have been at the forefront of businesses leveraging this phenomenon. In India, these businesses have seen huge increases in customers numbers, and the largest of them, Hotstar, has witnessed a threefold increase in their customer base to 100 million customers. Relatively new companies have also experienced considerable success; for example, Voot (a Viacom venture), has 25 million customers. Needless to say, this increase in customer numbers has attracted a host of advertisers. Although these companies have a long way to go before they can break even, industry experts have repeatedly expressed their optimism in the potential of these businesses. Businesses such as Amazon Prime and Netflix have also made substantial inroads in the Indian market by competing for the digital rights of blockbuster movies; Amazon is also investing in original content in regional languages. These companies are also increasingly becoming very good hedging options for film producers, as they can recover a substantial portion of their investments even before the movie is released. We believe that the movie industry is headed for significant transformation in the near future, as an increasingly significant segment of the audience will be viewing movies on VoDWs.

This paper examines the developing dynamics of the VoDW business model, and how the rise of this alternative channel affects the sequential distribution of movies, i.e. the release time, and the fee for acquiring the movie's digital rights. It also examines the effects of piracy on VoDWs' business model. We present a model that considers a general form of the utility function for VoDWs both to determine the optimal release time, and the digital rights' acquisition fees, for movies, accounting for piracy, broadband penetration, and movie quality, and also to examine IC, partial, and complete dominance scenarios. Innovative aspects of the model include: a single-parameter criterion, $\eta$, to understand how the two stakeholders' positions change in a complete dominance model; closed-form expressions to understand the profit or utility gap between optimality and forced compliance with changes in movie quality; and an elasticity expression, $\psi$, to understand how the negotiation paradigm changes with movie quality. Using this model, we establish a wide range of valuable insights for VoDW businesses, as well as MDs:

1. 1The optimal release time of a movie to a VoDW increases with content aggregation index, i.e. VoDWs dedicated to movies prefer a shorter release time, whereas multi-content websites, e.g. Hotstar, prefer to a longer release time. The latter scenario also suits theater owners and MDs.

2. Assuming that advertising revenue surpasses subscription revenue in long run, and that VoDWs such as Hotstar will dominate the market, increasing broadband penetration,

perhaps counter-intuitively, leads to a later optimal release time, as the marginal change in revenue decreases with broadband penetration. This is important both for MDs and VoDWs, implying that an increase in broadband penetration does not mean that MDs should release films sooner on VoDWs; it also suggests that VoDWs should not pay excessive fees to show the movie sooner.

3. MDs should delay the release of movie on VoDWs if it will lead to an increase in piracy.

4. In cases where the MD's and the VoDWs' optimal release times differ, our model predicts that, for movies where the box office performance remains steady, the VoDW should be dominant in negotiations, aiming for an earlier release time.

5. For movies where the theater experience is significant (e.g. action or sci-fi), the MD should be dominant in negotiations, aiming for a later release time.

There are a number of questions that future research can address. For example, we have focused on a single MD. It would be interesting to explore how the optimal times to release, and the coordination dynamics, change, if we consider multiple MDs. One other avenue of research would be to look at a VoDW interested in multiple movies by the same MD. It is also quite possible that VoDWs propose to MDs that they procure the digital viewing rights for the next few movies, thus absorbing some of the risks of future movie productions. We hope that this paper will motivate research on determining the optimal times to release and fees for such contracts.

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# Appendix

**Lemma 1.** $t_2(\alpha, \beta) \geq 0$*: Optimal release timings are non-decreasing with* $\beta$*.*

*Proof.* From the IC condition of a VoD with content aggregation $\beta$,

$$\begin{aligned} &U(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta)) - \tau(\alpha,\beta) \geq \\ &U(s(t(\alpha,\beta+\epsilon),\alpha,\beta,b),a(t(\alpha,\beta+\epsilon),\alpha,\beta)) - \tau(\alpha,\beta+\epsilon) \end{aligned} \tag{1}$$

From the IC condition of a VoD with content aggregation $\beta + \epsilon$,

$$\begin{aligned} &U(s(t(\alpha,\beta+\epsilon),\alpha,\beta+\epsilon,b),a(t(\alpha,\beta+\epsilon),\alpha,\beta+\epsilon)) - \tau(\alpha,\beta+\epsilon) \geq \\ &U(s(t(\alpha,\beta),\alpha,\beta+\epsilon,b),a(t(\alpha,\beta),\alpha,\beta+\epsilon)) - \tau(\alpha,\beta) \end{aligned} \tag{2}$$

Adding equations 1 and 2 yields,

$$\begin{aligned} &U(s(t(\alpha,\beta+\epsilon),\alpha,\beta,b),a(t(\alpha,\beta+\epsilon),\alpha,\beta)) - \\ &U(s(t(\alpha,\beta),\alpha,\beta,b),a(t(\alpha,\beta),\alpha,\beta)) \leq \\ &U(s(t(\alpha,\beta+\epsilon),\alpha,\beta+\epsilon,b),a(t(\alpha,\beta+\epsilon),\alpha,\beta+\epsilon)) - \\ &U(s(t(\alpha,\beta),\alpha,\beta+\epsilon,b),a(t(\alpha,\beta),\alpha,\beta+\epsilon)) \end{aligned} \tag{3}$$

To prove by contradiction, we assume that $t_2(\alpha, \beta) < 0$. Hence $t(\alpha, \beta) > t(\alpha, \beta + \epsilon)$. Now considering the contradiction as true with $s_1(t(\alpha, \beta), \alpha, \beta, b) \leq 0$ and $U_1(s, a) > 0$, equation 3 yields $U_1(s, a)|_\beta \leq U_1(s, a)|_{\beta+\epsilon}$, which contradicts the property of the utility function. □

**Lemma 2.** *In an incentive compatible contract, net utility of a VSW is (non-increasing / increasing) with respect to increasing content aggregation parameter* $\beta$ *depending on the change in subscription utility (greater than equal to / less than) the change in advertisement utility.*

*Proof.* Incentive Compatibility [IC] condition of any VoD states that $x = \beta$ is the solution to the following maximization problem of net utility.

$$\max_x U(s(t(\alpha,x),\alpha,\beta,b),a(t(\alpha,x),\alpha,\beta)) - \tau(\alpha,x) \tag{4}$$

From the first order condition:

$$U_1(s,a)s_1(t,\alpha,\beta,b).t_2(\alpha,x) + U_2(s,a)a_1(t,\alpha,\beta).t_2(\alpha,x) = \tau_2(\alpha,x)$$

Substituting $x = \beta$ in the first order equation leads to:

$$U_1(s,a)s_1(t,\alpha,\beta,b).t_2(\alpha,\beta) + U_2(s,a)a_1(t,\alpha,\beta).t_2(\alpha,\beta) = \tau_2(\alpha,\beta) \tag{5}$$

Differentiating net utility with respect to $\beta$ leads to the following expression:

$$\begin{aligned}=&U_1(s,a)s_1(t,\alpha,\beta,b).t_2(\alpha,\beta) + U_1(s,a)s_3(t,\alpha,\beta,b)\\ &+ U_2(s,a)a_1(t,\alpha,\beta).t_2(\alpha,\beta) + U_2(s,a)a_3(t,\alpha,\beta) - \tau_2(\alpha,\beta)\end{aligned} \tag{6}$$

Substituting the expression of $\tau_2(\alpha,\beta)$ from equation 5 into equation 6, change in net utility of VSW with increasing $\beta$ leads to the following expression:

$$= U_1(s,a)s_3(t,\alpha,\beta,b) + U_2(s,a)a_3(t,\alpha,\beta)$$

If $|U_1(s,a)s_3(t,\alpha,\beta,b)| \geq U_2(s,a)a_3(t,\alpha,\beta)$, then the net utility is non-increasing with increasing $\beta$.
If $|U_1(s,a)s_3(t,\alpha,\beta,b)| < U_2(s,a)a_3(t,\alpha,\beta)$, then the net utility is increasing with increasing $\beta$. □

**Lemma 3.** *The profit function of MD is concave.*

*Proof.* While defining economic rent $E(\beta)$ to incorporate in the profit expression of MD, we have considered two cases depending on whether $E(\beta)$ is non-decreasing or decreasing with increasing aggregation index $\beta$. In this case, we prove the lemma considering Case 1 and similar derivation will be valid for Case 2.

The optimality condition of is reproduced as:

$$R^b(t(\alpha,x^*),p^b,\alpha) - R^a(t(\alpha,x^*),p^a,\alpha) + U_1(s,a)s_1(t,\alpha,x^*,b) + U_2(s,a)a_1(t,\alpha,x^*) = 0 \tag{7}$$

Following second order condition, differentiating equation 7 with respect to x yields,

$$\begin{aligned}\frac{d^2\Pi_M(x)}{dx^2} =& t_2(\alpha,x)R_1^b(t(\alpha,x),p^b,\alpha) - t_2(\alpha,x)R_1^a(t(\alpha,x),p^a,\alpha)\\ &+ [U_{11}(s,a)s_1(t,\alpha,x,b).t_2(\alpha,x) + U_{11}(s,a)s_3(t,\alpha,x,b)]s_1(t,\alpha,x,b)\\ &+ [U_{12}(s,a)a_1(t,\alpha,x,b).t_2(\alpha,x) + U_{12}(s,a)a_3(t,\alpha,x,b)]s_1(t,\alpha,x,b)\\ &+ [s_{11}(s,a)t_2(\alpha,x) + a_{13}(t,\alpha,x,b)]U_1(s,a)\\ &+ [U_{21}(s,a)s_1(t,\alpha,x,b).t_2(\alpha,x) + U_{21}(s,a)s_3(t,\alpha,x,b)]a_1(t,\alpha,x)\\ &+ [U_{22}(s,a)a_1(t,\alpha,x,b).t_2(\alpha,x) + U_{22}(s,a)a_3(t,\alpha,x,b)]a_1(t,\alpha,x)\\ &+ [a_{11}(s,a)t_2(\alpha,x) + a_{13}(t,\alpha,x)]U_2(s,a)\end{aligned} \tag{8}$$

After simplification, we represent equation 8 as:

$$\begin{aligned}\frac{d^2\Pi_M(x)}{dx^2} =& t_2(.)[R_1^b(.) - R_1^a(.)] + [s_1(.).t_2(\alpha,x) + s_3(t,\alpha,x,b)]U_{11}(s,a)s_1(.)\\ &+ [a_1(.).t_2(.) + a_3(.)]U_{12}(s,a)s_1(.) + [s_{11}(.)t_2(\alpha,x) + s_{13}(.)]U_1(s,a)\\ &+ [s_1(.).t_2(.) + s_3(.)]U_{21}(s,a)a_1(.) + [a_1(.).t_2(.) + a_3(.)]U_{22}(s,a)a_1(.)\\ &+ [a_{11}(s,a)t_2(\alpha,x) + a_{13}(.)]U_2(s,a) \qquad (9)\end{aligned}$$

Considering the functional properties and assuming $|a_1(t,\alpha,x)t_2(\alpha,x)| > a_3(t,\alpha,x)$, $\frac{d^2\Pi_M(x)}{dx^2} \leq 0$ and hence concavity of the profit function is proved. To understand the assumption, we fragment the influence of $\beta$ on advertisement revenue in two parts: i) advertisement revenue decreases with increase in $\beta$ and ii) The associated condition implies that the magnitude of change in advertisement utility because of change in release time is higher than the change in advertisement utility over content aggregation. Similar proof is valid for case 2. □

**Lemma 4.** *(a) The optimal release time $t^*$ increases with broadband penetration if change in marginal subscription revenue increases with time, i.e. $s_{14}(t,\alpha,\beta,b) \geq 0$.*

*(b) The optimal release time $t^*$ increases with increase in piracy penetration.*

*Proof.* Proof of Lemma 4 (a):
The optimality condition of is reproduced as:

$$R^b(t(\alpha,x^*),p^b,\alpha) - R^a(t(\alpha,x^*),p^a,\alpha) + U_1(s,a)s_1(t,\alpha,x^*,b) + U_2(s,a)a_1(t,\alpha,x^*) = 0 \quad (10)$$

Using Implicit Function Theorem (IFT) with the optimality expression represented as $F(x^*,b) = 0$ leads to $\frac{\partial x^*}{\partial b} = \frac{\partial F/\partial b}{\partial F/\partial x^*}$. From lemma 3, it is proved that $\frac{\partial F}{\partial x^*} \leq 0$. Differentiating $F(x^*,b)$ with respect to broadband penetration $b$ yields:

$$\frac{\partial F}{\partial b} = s_1(t,\alpha,\beta,b)U_{11}(s,a)s_4(t,\alpha,\beta,b) + U_1(s,a)s_{14}(t,\alpha,\beta,b) + a_1(,\alpha,\beta)U_{12}(s,a)s_4(t,\alpha,\beta,b) \quad (11)$$

Considering the functional properties as: $s_(.) \leq 0, U_{11}(.) \leq 0, s_4(.) \geq 0, U_1(.) \geq 0, a_1(.) \leq 0, U_{12}(.) \leq 0$ with the additional assumption as $s_{14}(t,\alpha,x^*,b) \geq 0$ yields $\frac{\partial F}{\partial b} \geq 0$.

Considering $\frac{\partial F}{\partial x^*} \leq 0$ and $\frac{\partial F}{\partial b} \geq 0$, IFT yields that $\frac{\partial x^*}{\partial b} \geq 0$. From Lemma 1, $t(\alpha,x^*)$ increases with increase in $x^*$.

Proof of Lemma 4 (b):
To understand the effect of piracy, we represent the piracy penetration after movie release as $p^a = p^b + \theta$ with $\theta \geq 0$ as a continuous variable indicating the increase in piracy penetration after movie release. Incorporating $\theta$ into the optimality expression results into the following modification:

$$R^b(t(\alpha,x^*),p^b,\alpha) - R^a(t(\alpha,x^*),p^b+\theta,\alpha) + U_1(s,a)s_1(t,\alpha,x^*,b) + U_2(s,a)a_1(t,\alpha,x^*) = 0 \quad (12)$$

Using IFT with the optimality expression represented as $F(x^*, \theta) = 0$ leads to $\frac{\partial x^*}{\partial \theta} = \frac{\partial F/\partial \theta}{\partial F/\partial x^*}$. From lemma 3, it is proved that $\frac{\partial F}{\partial x^*} \leq 0$. Differentiating $F(x^*, b)$ with respect to increase in piracy penetration $\theta$ yields:

$$\frac{\partial F}{\partial \theta} = -R_2^a(t, \theta, \alpha) \tag{13}$$

As $R_2^a(t, \theta, \alpha) \leq 0$, this results into $\frac{\partial x^*}{\partial \theta} \geq 0$. From Lemma 1, $t(\alpha, x^*)$ increases with increase in $x^*$. □

**Lemma 5.** *(a) One of the stakeholders will agree to release the movie at the optimal time for the other stakeholder, i.e.* $t^*(\alpha) = t^M(\alpha)$ *or* $t^*(\alpha) = t^V(\alpha)$ *if*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| \neq |U_1(s, a)s_1(t, \alpha, \beta, b) + U_2(s, a)a_1(t, \alpha, \beta) - \tau_1(t)| \; \forall \, t \in [t^M, t^V]$$

*(b) Both the stakeholders will agree to an intermediate time* $t^*(\alpha)$ *where* $t^M(\alpha) < t^*(\alpha) < t^V(\alpha)$ *only if*

$$R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t) = U_1(s, a)s_1(t, \alpha, \beta, b) + U_2(s, a)a_1(t, \alpha, \beta) - \tau_1(t) \neq 0 \;\; at \; t = t^*(\alpha)$$

*and*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| < |U_1(s, a)s_1(t, \alpha, \beta, b) + U_2(s, a)a_1(t, \alpha, \beta) - \tau_1(t)| \; \forall \, t^M \leq t < t^*(\alpha)$$

*and*

$$|R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)| > |U_1(s, a)s_1(t, \alpha, \beta, b) + U_2(s, a)a_1(t, \alpha, \beta) - \tau_1(t)| \; \forall \, t^*(\alpha) < t \leq t^V$$

*Proof.* Proof of Lemma 5(a):
Differentiating the profit expression of MD with respect to $t$ yields:

$$\frac{d\Pi^M(t)}{dt} = R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha) + \tau_1(t)$$

Differentiating the net utility of VSW with respect to $t$ yields:

$$\frac{dNU(t)}{dt} = U_1(s, a)s_1(t, \alpha, \beta, b) + U_2(s, a)a_1(t, \alpha, \beta) - \tau_1(t)$$

Considering complete dominance, two possibilities of convergence will emerge in scenario 2 ($t^M < t^V$) or in scenario 3 ($t^M > t^V$). Either MD agrees to shift its release timing to $t^V$, i.e. $t^*(\alpha) = t^V(\alpha)$, or VoD agrees to MD's optimal timing of $t^M$, i.e. $t^*(\alpha) = t^M(\alpha)$. In both scenarios, MD will change its release time to $t^V$ if $|\frac{d\Pi_M(t)}{dt}| < |\frac{NU(t)}{dt}| \; \forall t \in [t^M, t^V]$, i.e. the change in MD's profit with time is less than the change in net utility. Hence VSW can always offer $|\frac{d\Pi_M(t(\alpha))}{dt}|$ for postponing the release time by $dt$ to MD and avoid the net loss $|\frac{NU(t(\alpha))}{dt}| - |\frac{d\Pi_M(t(\alpha))}{dt}|$.

The reverse situation is valid, i.e. $t^*(\alpha) = t^M(\alpha)$, if $|\frac{d\Pi_M(t(\alpha))}{dt}| > |\frac{NU(t(\alpha))}{dt}| \; \forall t \in [t^M, t^V]$. Combining both cases of complete dominance, one of the stakeholders will agree to change the release time optimal for the other stakeholder if

$$|R^b(t,p^b,\alpha)-R^a(t,p^a,\alpha)+\tau_1(t)| \neq |U_1(s,a)s_1(t,\alpha,\beta,b)+U_2(s,a)a_1(t,\alpha,\beta)-\tau_1(t)| \; \forall \; t \in [t^M,t^V]$$

Proof of Lemma 5(b):
This proof is an extension of the proof shown for part (a). Considering the case where $t^M(\alpha) < t^*(\alpha) < t^V(\alpha)$ under scenario 2, MD will postpone its optimal release time to $t^I$ because $|\frac{d\Pi_M(t)}{dt}| < |\frac{NU(t)}{dt}| \; \forall t \in [t^M,t^I)$. Similarly VoD website will prepone its optimal release time to $t^I$ because $|\frac{d\Pi_M(t)}{dt}| > |\frac{NU(t)}{dt}| \; \forall t \in (t^I,t^V)$. At $t^* = t^I$, both stakeholders will not have any incentive to move out as their gain and loss will match with each other. These three conditions are narrated in Lemma. □

**Lemma 6.** *(a) When $t^M < t^V$, the condition $\eta(t^M,t^V) > -1$ ensures that MD will take a lead in the release time negotiation i.e. $t^*(\alpha) = t^M(\alpha)$, else VoD website will lead the negotiation process, i.e. $t^*(\alpha) = t^V(\alpha)$.*

*(b) When $t^M > t^V$, the condition $\eta(t^M,t^V) < -1$ ensures that MD will take a lead in the release time negotiation i.e. $t^*(\alpha) = t^M(\alpha)$, else VoD website will lead the negotiation process, i.e. $t^*(\alpha) = t^V(\alpha)$.*

*Proof.* Difference between the unrealized profits by the two stakeholders is defined as:

$$D\Pi^M - DU^V = \Delta R(t^M,t^V) + \Delta U(t^M,t^V)$$

MD will take primary lead if $D\Pi^M - DU^V > 0$. This condition translates to the following expression:

$$\Delta R(t^M,t^V) + \Delta U(t^M,t^V) > 0$$

For scenario (a), i.e. $t^M < t^V$, $\Delta R(t^M,t^V) < 0$ and $\Delta U(t^M,t^V) > 0$ leads to the following condition: $\Delta U(t^M,t^V) > |\Delta R(t^M,t^V)|$ or $\eta(t^M,t^V) > -1$.

For scenario (b), i.e. $t^M > t^V$, $\Delta R(t^M,t^V) > 0$ and $\Delta U(t^M,t^V) < 0$ leads to the following condition: $\Delta R(t^M,t^V) > |\Delta U(t^M,t^V)|$ or $\eta(t^M,t^V) < -1$. □

**Lemma 7.** *(a) $\Delta R(t^M,t^V)$ is non-increasing in $\alpha$ when $(t^M(\alpha,\beta) < t^V(\alpha,\beta))$.*

*(b) $\Delta R(t^M,t^V)$ is non-decreasing in $\alpha$ when $(t^M(\alpha,\beta) > t^V(\alpha,\beta))$.*

*Proof.* Difference in theatre revenue due to change in VSW release time is specified as:

$$\Delta R(t^M,t^V) = \int_{t^V}^{t^M} R^b(z,p^b,\alpha)dz - \int_{t^V}^{t^M} R^a(z,p^a,\alpha)dz \tag{14}$$

Differentiating Equation 14 with respect to $\alpha$ yields:

$$\begin{aligned}
=&[R^b(t^M,p^b,\alpha) - R^a(t^M,p^a,\alpha)]t_1^M(\alpha) - [R^b(t^V,p^b,\alpha) - R^a(t^V,p^a,\alpha)]t_1^V(\alpha) \\
&+ \int_{t^V}^{t^M} (R_3^b(z,p^b,\alpha) - R_3^a(z,p^a,\alpha))dz
\end{aligned} \tag{15}$$

To understand the sign of equation 15, we discuss some properties of theatre revenue expression. We will use these properties in subsequent lemmas whenever required.

(i) At any time period $t$, theatre revenue generated before releasing the movie to VSW will always be higher than the scenario where the movie is released in VSW, i.e. $R^b(t, p^b, \alpha) > R^a(t, p^a, \alpha)$.

(ii) Rate difference between the revenues generated at time period $t$ and period $t + \delta$ with $\delta > 0$ follows the condition: $(R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha)) > (R^b(t+\delta, p^b, \alpha) - R^a(t+\delta, p^a, \alpha))$. This condition emphasizes the fact that there will be a greater theatre revenue loss with increasingly smaller time window given for VSW release.

We also recognize that the change of theatre revenue is influenced by $\alpha$ in two ways:

(a) $(R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha))t_1(\alpha)$: Indirect effect of $\alpha$ with the change in theatre revenue observed because of change in release time

(b) $(R_3^b(t, p^b, \alpha) - R_3^a(t, p^a, \alpha))$: Direct effect of $\alpha$ to the change in theatre revenue because of VSW release at any time period $t$. Please note that this expression will be non-negative with the change more evident, i.e $>> 0$ for successful movies.

We also assume that the direct change in theatre revenue due to change in $\alpha$ is greater than the indirect change, i.e. $(R_3^b(t, p^b, \alpha) - R_3^a(t, p^a, \alpha)) \geq |(R^b(t, p^b, \alpha) - R^a(t, p^a, \alpha))t_1(\alpha)|$. With this statement, it is straightforward to conclude that $\Delta R(t^M, t^V)$ is non-increasing in $\alpha$ when $(t^M(\alpha, \beta) < t^V(\alpha))$.

In the scenario when $(t^M(\alpha) > t^V(\alpha))$, $\Delta R(t^M, t^V)$ is non-decreasing in $\alpha$ following the same rationale. $\square$

**Lemma 8.** *(a) $\Delta U(t^M, t^V)$ is non-decreasing with increasing $\alpha$ when $t^M(\alpha, \beta) < t^V(\alpha, \beta)$.*

*(b) $\Delta U(t^M, t^V)$ is non-increasing with increasing $\alpha$ when $t^M(\alpha, \beta) > t^V(\alpha, \beta)$.*

*Proof.* We define $\Delta U(t^M, t^V) = U(s, a)|_{t=t^M} - U(s, a)|_{t=t^V}$. Differentiating this expression with respect to $\alpha$ yields,

$$\begin{aligned}=&[U_1(s, a)s_1(t^M, \alpha, b)t_1^M(\alpha) + U_1(s, a)s_2(t^M, \alpha, b) + U_2(s, a)a_1(t^M, \alpha, b)t_1^M(\alpha) + U_2(s, a)a_2(t^M, \alpha)] \\ &- [U_1(s, a)s_1(t^V, \alpha, b)t_1^V(\alpha) + U_1(s, a)s_2(t^V, \alpha, b) - U_2(s, a)a_1(t^V, \alpha, b)t_1^V(\alpha) + U_2(s, a)a_2(t^V, \alpha)]\end{aligned}$$

The expression above is rearranged to:

$$\begin{aligned}=&[U_1(s, a)s_1(t^M, \alpha, b)t_1^M(\alpha) - U_1(s, a)s_1(t^V, \alpha, b)t_1^V(\alpha)] && (16)\\ &+ [U_1(s, a)s_2(t^M, \alpha, b) - U_1(s, a)s_2(t^V, \alpha, b)] && (17)\\ &+ [U_2(s, a)a_1(t^M, \alpha, b)t_1^M(\alpha) - U_2(s, a)a_1(t^V, \alpha, b)t_1^V(\alpha)] && (18)\\ &+ [U_2(s, a)a_2(t^M, \alpha) - U_2(s, a)a_2(t^V, \alpha)] && (19)\end{aligned}$$

Scenario 2 $(t^M(\alpha, \beta) < t^V(\alpha, \beta))$:
In this proof, we assume that the direct effect of number of subscribers ($s$) or number of

advertisements ($a$) overcomes the change in VSW utility because of indirect effect due to change in release time $t$, i.e. $|s_2(t,\alpha,b)| > |s_1(t,\alpha,b)t_1(\alpha)|$ or $|a_2(t^M,\alpha)| > |a_1(t^M,\alpha,b)t_1^M(\alpha)|$. For example, whenever there is an increase in number of subscribers $s$, utility of VSW increases irrespective of the marginal change in utility due to change in time, i.e. $s_1(t,\alpha,b)t_1(\alpha)$.

The assumption above facilitates us to show the magnitude of expression in line number 17 is greater than the expression in line number 16 and similarly expression in line number 19 is greater than the expression in line number 18. Following the functional properties, i.e. $U_1(.), U_2(.), s_2(.), a_2(.) \geq 0$ with $U_{11}(.), s_{12}(.)$ and $a_{12}(.) \leq 0$, we conclude that the change of $\Delta U(t^M, t^V)$ is non-decreasing in $\alpha$.

Scenario 3 ($t^M(\alpha,\beta) > t^V(\alpha,\beta)$):
Similar line of proof can be pursued to prove the non-increasing nature of $\Delta U(t^M, t^V)$ in $\alpha$ for this scenario. □

**Corollary 1.** *(a) If $t*(\alpha) = t^M(\alpha)$ for a movie with $\alpha = \hat{\alpha} > 0$ where $\hat{\alpha}$ is the minimum value of $\alpha$ for which $D\Pi^M > DU^V$ when $t^M > t^V$, then $t^*(\alpha) = t^M(\alpha)$ for all movies with $\alpha \in [\bar{\alpha}, 1]$ where $\bar{\alpha} \leq \hat{\alpha}$ with increase in piracy penetration $\theta$.*

*(b) If $t*(\alpha) = t^V(\alpha)$ for a movie with $\alpha = \hat{\alpha} > 0$ where $\hat{\alpha}$ is the minimum value of $\alpha$ for which $D\Pi^M < DU^V$ when $t^M < t^V$, then $t^*(\alpha) = t^V(\alpha)$ for all movies with $\alpha \in [\bar{\alpha}, 1]$ where $\bar{\alpha} \leq \hat{\alpha}$ with increase in piracy penetration $\theta$.*

*Proof.* We prove this corollary considering scenario 2 ($t^M < t^V$) and a similar line of proof will be valid for scenario 3 ($t^M > t^V$). Please note that in scenario 2, $t^* = t^V$ whenever $\eta = \frac{\Delta R}{\Delta U} \in (-1, -\infty)$ with $\Delta R \leq 0$ and $\Delta U \geq 0$. We have already identified that $\eta$ is non-increasing in both $\alpha$ and $\theta$.

$\Delta R(t^M, t^V)$ is represented as follows:

$$\begin{aligned}\Delta R(t^M, t^V) = &\int_{t^V}^{t^M} R^b(z, p^b, \alpha)dz - \int_{t^V}^{t^M} R^a(z, p^a, \alpha)dz \\ &\int_{t^V}^{t^M} R^b(z, p^b, \alpha)dz - \int_{t^V}^{t^M} R^a(z, p^b + \theta, \alpha)dz\end{aligned} \tag{20}$$

Differentiating $\Delta R(t^M, t^V)$ with respect to $\theta$ yields:

$$= -\int_{t^V}^{t^M} R_2^a(z, \theta, \alpha)dz \tag{21}$$

Previous proposition in the main paper describes that $[\hat{\alpha}, 1]$ movies will have $t^* = t^V$ as $\eta$ is non-increasing in $\alpha$. Hence movies with $\alpha \in [0, \hat{\alpha})$ will have $t^* = t^M$ with $\eta(t^M(\hat{\alpha}), t^V(\hat{\alpha})) = -1$ with piracy penetration $p^b$ and $p^b + \theta$ respectively before that after the movie release to VoD. Assume there exists an $\bar{\alpha} < \hat{\alpha}$ for which $\eta(t^M(\bar{\alpha}), t^V(\bar{\alpha})) > -1$ as $\eta$ is non-increasing in $\alpha$. Keeping other parameters unchanged and without loosing on generality, we can state that

$$\eta(t^M(\bar{\alpha}), t^V(\bar{\alpha})) = -1$$

with an increase in post release piracy penetration $\theta$, i.e. $\theta + \epsilon$ with $\epsilon > 0$. It shows that with an increase in $\theta$, $t^* = t^V$ is valid for all movies with $\alpha \in [\bar{\alpha}, 1]$ because the corresponding $\eta$ values lie in the range $(-1, -\infty)$. For scenario 3 ($t^M > t^V$), similar approach can be followed to prove. $\square$