New Algorithms for Constrained Rectangular Guillotine Knapsack Problems

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Thesis Summary

In the Unconstrained Rectangular Guillotine Knapsack Problem (RGKP), also referred to in the literature as the two dimensional guillotine cutting problem, we are given one piece of a stock rectangle, and n types of demanded rectangles, with the i^{th} type of demanded rectangle having length l_i , width w_i and value v_i. The problem is to determine the sequence of guillotine cuts on the stock rectangle and on the resulting rectangles so that the total value of the demanded rectangles thus produced is maximized.

If additional constraints are imposed to restrict the maximum number of each of the demanded rectangles which are allowed to be produced, then we get the Constrained Rectangular Guillotine Knapsack Problem (CRGKP), also referred to in the literature as the constrained two dimensional guillotine cutting problem.

One dimensional knapsack problems and rectangular knapsack problems arise as sub-problems when Gilmore and Gomory's (1961, 1963, 1965) column generation method is used to solve one or two dimensional cutting stock problems. One dimensional knapsack problems also arise in the context of project selection and cargo loading. Rectangular knapsack problems have direct applications in material cutting. Their close relationship to resource constrained project scheduling has also been established.

The main focus of this work is the CRGKP, and a new algorithm that yields optimal solutions is presented. This algorithm is seen to be very efficient in comparison with existing algorithms [Christofides and Whitlock (1977), Wang (1983)] for solving the problem. Further it is also shown that the proposed algorithm can handle a much wider range of constraints than the best known existing algorithm. Computational experience on a wide range of test problems has also been presented.

The only known algorithm to solve the general CRGKP as stated above is that of Christofides and Whitlock (1977). Wang (1983) has proposed algorithms which are useful in the specific case where values of the demanded rectangles are directly proportional to their areas. Christofides and

Whitlock's algorithm implicitly enumerates all feasible guillotine cutting patterns by constructing a tree with the stock rectangle at its root, and the sets of rectangles obtainable from the stock rectangle by means of a single guillotine cut as the immediate successors of the root, and so on. The implicit enumeration is guided by an upper bounding function that uses dynamic programming coupled with an allocation routine. The search is organized in a depth-first fashion and the bounds calculated are used to avoid further branching from nodes which are known to yield improved solutions and nodes from which further branching is known to be unprofitable.

Wang's approach enumerates all feasible cutting patters by the reverse process, that is by considering all ways of constructing the stock rectangle starting from the demanded rectangles. Wang's algorithms use certain 'heuristic' measures to restrict the number of patterns constructed. Optimal solutions can be obtained by first generating a good solution manually, and then using either of the algorithms with a suitable heuristic value obtained on the basis if the manually generated solution.

The proposed approach also seeks to construct the stock rectangle starting form the demanded rectangles. However the decision of whether or not to include a given rectangle in a pattern is guided by a measure of desirability of that rectangle. Thus, at any point, the two most promising rectangles are joined together to build a new rectangle. This approach resembles certain heuristic search algorithms studies in the Artificial Intelligence literature. Three measures of attractiveness of rectangles have been suggested and all of them lead to optimal solutions. Two of the measures are dynamic programming and the third measure is derived from the first two. The algorithm can be seen as a "best-first" approach, and is designed to terminate as soon as is possible after an optimal pattern has been constructed.

The approach employed is applicable to one dimensional knapsack problem, and new algorithms which yield optimal solutions to both constrained and unconstrained one dimensional knapsack problems are also presented. However, in their present form, these algorithms are not as efficient as the best known algorithms for these problems.

An extension of the approach employed for CRGKP to the case of multiple stock sheets has also been developed. An application of the proposed algorithm to the general two dimensional cutting stock problems has been suggested.