

Chapter 1

INTRODUCTION

1.1 Background

This thesis attempts to enhance the body of knowledge regarding quantitative equity (stocks) portfolio selection. A major step in quantitative management of investment portfolios was made by Harry Markowitz in 1952. The ideas introduced in his work have come to build the foundations of what is popularly referred to as *Modern Portfolio Theory*. Today, decades later, those very same principles form the basis for financial models that are constantly being reinvented to incorporate new findings. This work tries to advance the state of knowledge in this regard by (i) enhanced understanding of the performance and properties of mean-risk models when realistic investment constraints are factored in, and (ii) developing agent-based approach for portfolio selection.

Investment portfolio selection and management is a well researched area and several models have been proposed to select an investment portfolio. Most popular of them are the mean-risk models with variance as a measure of risk, as initially proposed by Markowitz. Since Markowitz's work, several studies have enriched the mean-variance models incorporating many practical constraints. Many researchers have also proposed alternative ways of quantifying risk which led to the development of several other risk measures. In particular with the increasing regulatory restrictions, financial institutions were forced to quantify and report the possible loss the investors could face in extremely unfavorable cases. This prompted the development of risk measures concerned only with extremely unfavorable results, or in other words with the left tail of portfolio return distribution. These alternative risk measures are gaining increasing acceptability in the investment industry. The use of a single risk measure for portfolio selection has however received criticisms for its simplicity and inability to capture other aspects of portfolio return distribution. The use of two risk measures is hence an obvious step forward to overcome some of the shortcomings and a few studies have looked into such models. Simultaneously, there have been efforts to build portfolio selection models using tools that simulate the actions and interactions of autonomous investors. Agent

based model is one such tool that has been used primarily for stock trading to generate buy and sell signals with very restrictive applications to portfolio selection.

In this thesis the specific problems of interest to us are (i) to examine the mean-risk model with two risk measures - variance and Conditional Value-at-Risk (CVaR) with a few realistic features such as long-short equity investing, threshold and cardinality restrictions, (ii) to study a few interesting properties of long-short mean-risk models in presence of CVaR risk measure, and (iii) to develop agent based models for portfolio selection in presence of cardinality and threshold constraints and study their performance.

1.2 Problem description

An investor wants to invest some money in the stock market. Since there are too many stocks, he has to decide on the stocks he should choose to purchase (or, in finance jargon, take a long position). He also needs to decide how to apportion the money among the chosen stocks. The investment in any stock does not guarantee him a known fixed return since the price of the stock at the end of investment period is unknown. The decision needs to be made today, in the absence of full information, and one can only wait until the end of investment period, when the missing information – the future stock prices - is revealed in order to evaluate it.

One useful way that aids in decision making process is to take a discrete view of the uncertainty in the future prices of the stocks by specifying a set of states or scenarios that might occur. Such discrete states may be simulated by means of some mathematical models or by considering the daily price movements of the stocks in the market for the last few years. Obviously, investor's goal is to choose a portfolio that would maximize his expected return – subject to a 'reasonable' risk – if he holds on to the portfolio for some period of time in future. The problem faced by the investor is known as the *investment portfolio selection problem*.

The risk involved in any investment choice is the risk of losing money or, at least the

risk of not gaining as much as with other choices. This risk has been quantified and viewed in a number of different ways. Traditional investors like to view the variability in stock returns as the risk while regulators prefer the loss in extremely bad cases as the measure of risk. In any case the investor needs to take the best decision with an acceptable level of risk.

In a practical portfolio planning process the investment decision to be taken by an investor is not simple, and is influenced by several other constraints. The investor generally wants to be fully invested. With the recent advancements in the stock markets around the world, many stock exchanges are now allowing investors to engage in 'short selling'. Short selling means that an investor has a negative view about a stock that its price would fall in future and he tries to benefit from this situation without really owning the stock. The investor borrows the stock from someone (may be from another investor who has a different view) with a promise to return back the stock in future. He then sells the stock. In future when the stock price falls, he buys the stock and returns it to the lender. In this way he makes a profit. 'Short selling' also involves risk that the investor views may prove wrong and the stock price increases instead of decreasing. This calls for some protection in the form of regulatory restrictions to ensure that the lender of the stock is protected.

Another practical motivation may be to invest only in a fixed number of stocks that is, the number of stocks to hold in the portfolio is fixed. This is the cardinality constrained version of the investment portfolio selection problem. Yet another practical planning may call for avoiding buying/selling or holding stocks in a very small proportion. If a stock is bought/sold or held in the portfolio it is always done in a proportion of money that is above a minimum value. This is the threshold constrained investment portfolio selection problem.

1.3 Thesis contribution

In this dissertation we have been able to put together the following things

- a. We reviewed different models and practical constraints used in investment portfolio selection and management.
- b. We studied long-short portfolio optimization in the presence of two risk measures; namely variance and Conditional Value at Risk (CVaR), and asset choice constraints of (i) buy, sell and holding thresholds and (ii) cardinality restrictions on the number of stocks to be held in the portfolio. The mean-variance-CVaR model improves upon the classical mean-variance model by controlling both the variance and CVaR of the resulting return distribution. Our long-short extension to the mean-variance-CVaR model incorporates many financial institutions' practices in respect of the "short" decisions. We highlight that introducing short selling leads to superior choice of portfolios, with higher expected return and much lower risk exposures, as characterized by CVaR and variance. We further analyze the effects of applying buy and sell thresholds and cardinality restrictions on the number of stocks. Such constraints are of practical importance but make the efficient frontier discontinuous. When stocks' returns are represented as discrete random variables, the formulation leads to a Quadratic Mixed Integer Program (QMIP). We conclude that the long-short model with cardinality constraint is superior to the long only model even without cardinality constraint. The models are tested on real data drawn from the FTSE 100 index.
- c. We carried out an analytical study of the mean-CVaR and mean-variance-CVaR long-short portfolio selection models. The model formulations incorporate margin trading and regulations for short selling. The conditions for simultaneous holding of long and short positions on the same stock in an efficient portfolio are worked out. In addition, the study illustrates that in an ideal case having predetermined margin requirements on long and short positions, is not necessarily a good proposition. Empirical illustration based on FTSE 100 index data is provided in support of the properties found.
- d. We developed a multi-agent model for the portfolio selection problem in presence of cardinality restriction on the number of stocks to be held in the portfolio. A system of agents divides the initial wealth and follows individual investment strategies starting with pseudo-random portfolios. Periodically, the agents share information

about their performances, and can switch portfolios. A final cardinality constrained portfolio is constructed by consolidating individual portfolios formed by the agents based on the past performance of the stocks. The study proposes the multi-agent model as an alternative to other popular decision models in finance in solving the cardinality constrained portfolio selection problem. The portfolio suggested by the agent based model has been found to frequently perform better than the portfolios suggested by mean-variance models when tried out in real market (using FTSE 100 and Nikkei 225 index data).

- e. We developed a multi-agent model for portfolio selection to consider threshold restriction on the stocks to be held in the portfolio. The threshold restriction imposes an implicit weak cardinality restriction on the chosen portfolio. In the suggested framework, the strategies to be followed by the agents have to be different from those used while constructing the cardinality constrained portfolio. The portfolio suggested by the agent based model frequently outperforms the portfolios suggested by mean-variance models when tried out in real market (using FTSE 100 and Nikkei 225 index data).

1.4 Thesis Structure

We study the relevant literature in Chapter 2. The research problems of interest, covered in future chapters are pointed out.

Chapter 3 focuses on long-short portfolio selection model in presence of variance and CVaR as the risk measures of choice. In addition, realistic constraints like regulatory, investing, threshold and cardinality restrictions are considered for practical portfolio planning.

In Chapter 4, we present model formulations for the mean-CVaR and mean-variance-CVaR models under margin trading and short selling and study a few interesting properties of the efficient portfolios.

In Chapter 5, we set out the motivation for our work on multi-agent models for portfolio

selection. We carry out a detailed empirical investigation into the multi-agent models suggested in a related work by Parkes and Huberman (Parkes and Huberman 2001).

In Chapter 6, we build a multi-agent model for cardinality constrained portfolio selection problem and test it on real data from FTSE 100 and Nikkei 225.

In Chapter 7, we develop a multi-agent model to consider threshold constrained version of the portfolio selection problem. This model is also tested on FTSE 100 and Nikkei 225.

Chapter 8 concludes the thesis and gives directions for future research.